On the physical origin of dark matter density profiles

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ABSTRACT
The radial mass distribution of dark matter haloes is investigated within the framework of the spherical infall model. We present a new formulation of spherical collapse including non-radial motions, and compare the analytical profiles with a set of high-resolution N-body simulations ranging from galactic to cluster scales. We argue that the dark matter density profile is entirely determined by the initial conditions, which are described by only two parameters: the height of the primordial peak and the smoothing scale. These are physically meaningful quantities in our model, related to the mass and formation time of the halo. Angular momentum is dominated by velocity dispersion, and it is responsible for the shape of the density profile near the centre. The phase-space density of our simulated haloes is well described by a power-law profile, \( \rho/\sigma^3 = 10^{1.46 \pm 0.04} (\rho_c/v_{\text{esc}}^3)(r/r_{\text{vir}})^{-1.90 \pm 0.05} \). Setting the eccentricity of particle orbits according to the numerical results, our model is able to reproduce the mass distribution of individual haloes within 20 per cent accuracy.

Key words: galaxies: haloes – cosmology: theory – dark matter.

1 INTRODUCTION
The hierarchical clustering paradigm states that the growth of cold dark matter (CDM) haloes proceeds by accretion of smaller units from the surrounding environment, either by continuous infall or by discrete merging events.

Cosmological N-body simulations are a valuable tool in the study of the mass distribution of dark matter haloes and its evolution in the non-linear regime. In the early numerical work of Quinn, Salmon & Zurek (1986) and Frenk et al. (1988), haloes showed an isothermal density profile \( \rho \propto r^{-2} \) on the scales that could be resolved. Dubinski & Carlberg (1991) and Crone, Evrard & Richstone (1994), amongst others, had enough resolution in their simulations to detect evidence of departure from a pure power law. Later on, Navarro, Frenk & White (1996, 1997, hereafter NFW) found that the density profile could be fitted by a simple analytical function

\[
\rho(r) = \frac{\rho_c}{(r/r_c)(1 + r/r_c)^2} 
\]

in terms of a characteristic density \( \rho_c \) and a characteristic radius \( r_c \). This profile is steeper than isothermal at large radii and shallower near the centre. The logarithmic slope of the density profile, \( \alpha(r) \equiv d \log(\rho)/d \log(r) \), tends to \( \alpha = -3 \) for \( r \to \infty \) and \( \alpha = -1 \) for \( r \to 0 \). It corresponds to the isothermal case, \( \alpha = -2 \), at the characteristic radius only. Navarro et al. (1997) further showed that the two free parameters in equation (1) are not independent. Should this be true, the final mass distribution of objects of different scales could be described in terms of a one-parameter family of analytical profiles.

Similar results have been found in independent simulations with much higher mass and force resolution than those analysed in the original NFW paper. However, there is still some controversy about the innermost value of \( \alpha \) and its dependence on resolution. Moore et al. (1998, 1999), Ghigna et al. (1998, 2000) and Fukushige & Makino (1997, 2001) find steeper density profiles near the centre \( (\alpha \sim -1.5) \), whereas other authors (Jing & Suto 2000; Klypin et al. 2001; Ricotti 2003) claim that the actual value of \( \alpha \) may depend on halo mass, merger history, and substructure. Power et al. (2003) pointed out that the logarithmic slope becomes increasingly shallower inwards, with little sign of approaching an asymptotic value at the resolved radii. In that case, the precise value of \( \alpha \) at a given cut-off scale would not be particularly meaningful. This result has been later confirmed by Fukushige, Kawai & Makino (2003) and Hayashi et al. (2003), and it is predicted by several analytical models (e.g. Taylor & Navarro 2001; Hoefl, Mückel & Gottlöber 2004).

Observed rotation curves of dwarf spiral and low surface brightness (LSB) galaxies (e.g. Flores & Primack 1994; Moore 1994; Burkert 1995; Kravtsov et al. 1998; Borriello & Salucci 2001; de Blok et al. 2001; de Blok & Bosma 2002; Marchesini et al. 2002) seem to indicate that the shape of the density profile at small scales is significantly shallower than what is found in numerical simulations. This discrepancy has been often signalled as a genuine crisis of the CDM scenario, and several alternatives have been suggested, such as warm (Colín, Avila-Reese & Valenzuela 2000; Sommer-Larsen & Dolgov 2001), repulsive (Goodman 2000), fluid (Peebles 2000),

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dark matter.

Unfortunately, it has proved remarkably hard to establish the inner
slope of the dark matter distribution observationally (see, e.g. Swaters et al.
2003). Some authors (van den Bosch & Swaters 2001; Jimenez, Verde
& Oh 2003; Swaters et al. 2003) claim that a cuspy density profile
with $\alpha \sim -1$ is consistent with current observations, although a shallower
slope is able to explain them as well. Yet, a value as steep as $\alpha = -1.5$ can be
confidently ruled out in most cases. According to Hayashi et al. (2003), only
about 30 per cent of the rotational curves are actually inconsistent with the simulation
data.

On cluster scales, X-ray analyses have led to wide-ranging re-
sults, from $\alpha = -0.6$ (Ettori et al. 2002) to $\alpha = -1.2$ (Lewis, Buote
& Stocke 2003) or even $\alpha = -1.9$ (Arabadjis, Bautz & Garmire 2002).
Measurements based on gravitational lensing yield conflicting
estimates as well, either in rough agreement with the results of numerical
simulations (e.g. Dahle, Hannestad & Sommer-Larsen 2003; Gavazzi et al.
2003), or finding much shallower slopes, $\alpha = -0.5$ (e.g. Sand, Treu & Ellis
2002; Sand et al. 2003), although the latter results have been disputed by independent analyses of the same
data (Bartelmann & Meneghetti 2003; Dalal & Keeton 2003).

A conclusive theoretical prediction of the central mass distribu-
tion of CDM haloes is therefore an important check for any model of
structure formation. The controversy regarding the ‘universal’
density profile and its logarithmic slope at the centre has stimu-
lated a great deal of analytical work. On one hand, we would like to
find out not only the actual shape of the profile, but also the physical
mechanisms behind the ‘universality’ observed in numerical N-body simulations.
On the other hand, it would be interesting to find and explain correlations with the virial mass, environment, pri-
mordial power spectrum or even the nature of dark matter particles in
the halo.

A number of plausible arguments about the radial structure of
haloes have been proposed (e.g. Padmanabhan 1996; Avila-Reese, Firmani
& Hernández 1998; Henriksen & Widrow 1999; Kull 1999; Engi-
neer, Kanekar & Padmanabhan 2000; Lokas 2000; Subramanian, Cen
& Ostriker 2000). Several authors (e.g. Salvador-Sole, Solanes
& Manrique 1998; Syer & White 1998; Nusser & Sheth 1999;
Manrique et al. 2003) argue that the central density profile is
linked to the merging history of dark matter substructure, and
baryons have been invoked both to flatten (e.g. El-Zant, Shlosman
& Hoffman 2001; El-Zant et al. 2003) and to steepen (Blumenthal et al.
1986) the dark matter profile.

In this paper, we present an analytical model for the assembly of
CDM haloes based on the spherical collapse paradigm. Following
the spirit of Hoffman & Shaham (1985), we will assume that objects
do not form around local maxima of the primordial density field,
but of the smoothed density field. We argue that all information
about the initial conditions below the smoothing scale is lost during
the merging process. We will show that setting the smoothing scale
for a halo of a given mass is equivalent to specifying its formation
time. Initial conditions are computed following the Gaussian ran-
dom peaks statistics described by Bardeen et al. (1986, hereafter
BBKS), and angular momentum is included in a phenomenological
way. Our theoretical predictions are then compared with the results
of high-resolution numerical simulations, showing that this model
is able to reproduce accurately the mass distribution of individual
objects.

The paper is structured as follows. Details of our implementation
of spherical collapse are given in Section 2. Numerical experiments
are described in Section 3, where both the mass and velocity dis-
tributions are investigated. We discuss our results in Section 4, and
Section 5 summarizes our main conclusions.

2 SPHERICAL COLLAPSE

The assembly of dark matter haloes is a highly non-linear process,
and strong simplifying assumptions must be made in order to tackle
the problem analytically. Traditionally, there are two complemen-
tary paradigms: the spherical infall model (Gunn & Gott 1972)
and the Press–Schechter formalism (Press & Schechter 1974), in
which mergers play a dominant role (see, e.g. Nusser & Sheth 1999;
Manrique et al. 2003).

In this paper, we will follow the first approach. Our aim is to
calculate the present-day mass distribution arising from a spher-
cally symmetric perturbation of the primordial density field. The
dynamics of an overdense mass shell in a $\Lambda$CDM universe are briefly
reviewed in Section 2.1. The equations of motion are integrated nu-
merically until turn-around, and then a simple prescription based
on adiabatic invariance is proposed to account for secondary infall
and shell crossing. Section 2.2 describes our choice of initial condi-
tions, according to the peak formalism developed by BBKS. The
initial density profile is set by two free parameters, the height of the peak, $\nu$, and the smoothing scale, $R_s$. Finally, we study in Sec-
tion 2.3 the effect of angular momentum (i.e. non-radial motions)
by considering the orbital eccentricity, $e$, as a third free parameter
of the model.

2.1 The model

The simplest way of addressing the problem of structure formation
is to assume spherical symmetry. As shown by Tolman (1934) and
Bondi (1947), a spherically symmetric solution of the Einstein equa-
tions can be easily interpreted in terms of Newtonian dynamics. The
equation of motion for a Lagrangian shell enclosing a mass $M$ can be
derived from the conservation of energy

$$\epsilon(r) \equiv \frac{E(r)}{m} = \frac{r^2}{2} - \frac{GM(r) + (4\pi/3)\rho_0 r^3}{r} = \epsilon_0(r)$$

where $r(t)$ is the Lagrangian coordinate, $M(r) = M(r_0)$ is the mass
contained within the shell, and $\rho_0 = \Lambda /8\pi G$ is the vacuum energy
density associated with a cosmological constant. Throughout this
paper, the subscript ‘i’ will be used to denote initial conditions.

If we assume the universe to be homogeneous, the mass is given by

$$M_i = \frac{4\pi}{3} \Omega_m \rho_c r_i^3.$$
Energy conservation leads to the equation of motion yielding enough values of shell would be similar to that of a Friedmann universe. For high around radius \( \Omega_1 \)

In a homogeneous universe, \( \alpha(t) \) is independent of \( r_i \) and plays the role of a uniform expansion factor. Our model assumes that structures grow from spherically symmetric perturbations, defined as

\[
\Delta_i(r_i) = \frac{M(r_i)}{4\pi r_i^2 \rho_i^*} - 1
\]

with \( 0 < \Delta_i \ll 1 \). These perturbations are introduced at an early epoch \( a_i \), by slightly displacing the spherical shells of matter. Keeping only terms to first order in \( \Delta_i(r_i) \), the new positions and velocities are given by

\[
r'_i \simeq r_i \left[ 1 - \frac{1}{3} \Delta_i(r_i) \right] ; \quad v'_i \simeq H r_i \left[ 1 - \frac{2}{3} \Delta_i(r_i) \right].
\]

The mass enclosed by the perturbed shell is still \( M_i \), and the initial specific energy (using \( \Omega_i^2 \simeq 1 \) and \( \Omega_i \simeq 0 \)) is \( \epsilon_i \simeq -\frac{\dot{r_i}}{2} \Delta_i(H r_i)^2 \). Energy conservation (2) leads to the equation of motion

\[
-\frac{5}{12} \Delta_i \simeq \frac{\dot{r_i}}{H r_i} = \Omega_i^2 r_i - \Omega_i \ln^2 r_i.
\]

According to this equation, the evolution of a single spherical shell would be similar to that of a Friedmann universe. For high enough values of \( \Delta_i \), the shell reaches a maximum radius \( r_m \) at a turn-around time \( t_m \) and then recollapses. In an Einstein–de Sitter universe (\( \Omega_i = 1, \Omega_i = 0 \)), equation (7) can be solved analytically, yielding

\[
r_m = \frac{3r_i}{5 \Delta_i}, \quad t_m = \frac{\pi}{2H(5\Delta_i/3)^{1/2}}.
\]

This is also a valid approximation for shells reaching their turn-around radius \( r_m \) before the cosmological constant term starts to dominate the expansion. Since the shells need at least another \( t_m \) to vitalize, expression (8) can be applied in a \( \Lambda \)CDM universe to estimate the maximum expansion radius and time for the innermost part of a virialized halo. For the outer shells, the equation of motion (7) must be integrated numerically in order to find the trajectories up to the maximum radius.

In the absence of shell-crossing, shells would reach the origin at \( T = 2t_m \). Since they are assumed to be composed of collisionless CDM particles, they would simply pass through the centre, describing an oscillatory motion with amplitude \( r_m \) and period \( T \). However, equation (7) holds as long as the enclosed mass \( M_i \) remains constant. As a shell recollapses, its particles will cross the orbits of inner shells, and energy will no longer be a constant of motion.

After turn-around, our model assumes that the particles belonging to a shell oscillate (or, more generally, orbit) within the gravitational potential of the dark matter halo. Since CDM particles are expected to spend most of the time in the outermost regions of their orbits (particularly when angular momentum is taken into account), we approximate the mass distribution of the halo by a simple power law

\[
M(r) = M_i \left( \frac{r}{r_m} \right)^{\alpha_i}(r_m)
\]

where \( \alpha_i(r) \equiv \frac{d \log M_i(r)}{d \log r} \) is the local value of the logarithmic slope of the mass profile, evaluated at the maximum radius of the orbit.

At first sight, this might seem similar to the classical approach based on self-similarity (Bertschinger 1985), but in that case the final mass distribution is indeed assumed to be a power law, whereas in our model this ansatz is only an approximation to compute the local potential. The final mass profile is obtained self-consistently, adding the contributions from all shells, each of them with an individual value of \( \alpha_i(r_m) \).

The probability of finding a particle inside radius \( r \) is proportional to the fraction of time it spends within \( r \):

\[
P(r, r_m) = \frac{1}{t_m} \int_0^r \frac{dx}{v_i(x)} = \frac{1}{t_m} \int_0^r \frac{dx}{\sqrt{\Phi(r_m) - \Phi(x)}}.
\]

We evaluate numerically the value of \( \alpha_i(r_m) \) in order to compute the potential. Taking different prescriptions, such as an isothermal profile (\( \alpha_i = 1 \) for every shell) or a Keplerian potential (\( \alpha_i = 0 \)) does not lead to qualitative variations in the probability \( P(r, r_m) \) and the resulting mass distribution.

If phase mixing is considered to be efficient, particles initially on the same shell will be spread out from \( r = 0 \) to \( r = r_m \), a short time after \( t_m \). For the sake of simplicity, we will consider that phase mixing is instantaneous, so immediately after turn-around the shell is transformed into a density distribution whose cumulative mass is proportional to \( P(r, r_m) \).

This means that recently accreted particles contribute to the mass within \( x_m < r_m \) (i.e. shell-crossing). For shells whose maximum radius was \( x_m \), the enclosed mass changes from \( M_i(x_m) \) to

\[
M_i(x_m) = M_i(x_m) + M_{\text{add}}(x_m)
\]

where \( M_{\text{add}}(x_m) \) accounts for particles belonging to outer shells. To compute \( M_{\text{add}}(x_m) \) (see Zaroubi & Hoffman 1993), we must integrate the contribution of every shell whose maximum radius is larger than \( x_m \), up to the current turn-around radius \( R_m \):

\[
M_{\text{add}}(x_m) = \int_{x_m}^{R_m} \frac{dM_i(r)}{dr} P(x_m, r) \, dr.
\]

To compute the evolution of the shell after shell-crossing, we apply adiabatic invariance (Gunn 1977). If the potential evolves on a time-scale much longer than the orbital period of the inner particles, their dynamics admits an adiabatic invariant

\[
J_r = 1 \int_0^{r_m} v_i(r) \, dr
\]

also known as the radial action. For a power-law potential, the radial action \( J_r \) is proportional to \( \sqrt{x_m} M_i(x_m) \). When we increase the mass by an amount \( M_{\text{adj}} \), the maximum radius of the inner shell must decrease in order to keep \( J_r \) constant. The final radius, \( x_0 \), is given by the implicit equation \( x_0 = F(x_0) x_m \), where

\[
F(x_0) = \frac{M_i}{M_i + M_{\text{adj}}(x_0)} = \frac{M_i}{M_i + M_{\text{adj}}(F(x_0) x_m)}
\]

and whose solution must be obtained numerically for each shell.

To summarize, the numerical procedure to compute the final radius \( r(t) \) of a Lagrangian shell of matter involves the following steps:

(i) Set \( M_i \) (or, equivalently, \( r_i \)). Start by the outer shell.
(ii) Integrate the equation of motion (7) up to \( t_m \).
(iii) If \( t_m > t_0 \), the shell is still expanding: \( r = r(t_0) \).
(iv) If \( t_m < t_0 \), solve (14) to compute \( r_0 = F(r_0) r_m \), and add the contribution of this shell to \( M_{\text{adj}}(r) \).
(v) Repeat for the next shell towards the centre.
2.2 Initial conditions

The model described above allows us to compute the mass distribution arising from a primordial fluctuation \( \Delta_i(r_i) \), but does not say anything about the shape of this function or its physical origin. Nevertheless, it is important to note that the final density profile is entirely determined by this initial condition. Thus, in the spherical collapse paradigm, the case for a universal density profile can be reformulated in terms of universality in the primordial fluctuations that set \( \Delta_i(r_i) \). More precisely, the final density profile may be characterized by (at most) the same number of free parameters as the initial condition \( \Delta_i(r_i) \).

Hoffman & Shaham (1985) suggested that, according to the hierarchical scenario of structure formation, halos should collapse around maxima of the smoothed density field. The statistics of peaks in a Gaussian random field has been extensively studied in the classic paper by BBKS. A well-known result is the expression for the radial density profile of a fluctuation centred on a primordial peak of arbitrary height:

\[
\frac{\langle \delta(r) \rangle}{\sigma_0} = \nu \psi(r) - \frac{\theta(y, \gamma \nu)}{\sqrt{1 - \gamma^2}} \left[ y^2 \psi(r) + \frac{R_i^2}{3} \nabla^2 \psi(r) \right] \tag{15}
\]

where \( \psi(r) \equiv \xi(\sigma) / \sigma_0 \) is the normalized two-point correlation function, \( \sigma_0 = \xi(0)^{1/3} \) is the rms density fluctuation, and \( \nu \psi_0 \) is the height of the peak. The quantities \( y = \sigma_0^2 / \sigma_2 \) and \( \gamma = \sqrt{2} \gamma \) are related to the moments of the power spectrum,

\[
\sigma_j^2 = \frac{1}{2\pi^2} \int_0^\infty \frac{P(k)k^{2j+1}}{\sigma_j} \, dk, \tag{16}
\]

and the function \( \theta(y, \gamma \nu) \) parametrizes the second derivative of the density fluctuation near the peak. BBKS suggest the approximate fitting formula

\[
\theta(y, \gamma \nu) \approx \frac{3(1 - y^2) + (1.216 - 0.9y^2) \exp \left( -\frac{\gamma \nu}{\sigma_0} \right)}{3(1 - y^2) + 0.45 + (1.216 - 0.9y^2)^{1/2} + \frac{\gamma \nu}{\sigma_0}} \tag{17}
\]

valid to 1 per cent accuracy in the range \( 0.4 < y < 0.7 \) and \( 1 < \gamma \nu < 3 \), which is the scale relevant for both galaxies and galaxy clusters.

Expression (15) is often quoted in the literature (e.g. Hoffman 1988; Del Popolo et al. 2000; Lokas & Hoffman 2000; Hotelski 2002) as the initial condition \( \Delta_i(r_i) \). However, we argue that \( \langle \delta(r) \rangle \) denotes (see BBKS) the mean profile of the smoothed density field

\[
\delta_i(r) = \int W_i(r - x) \delta(x) \, dx \tag{18}
\]

where the function \( W_i(r - x) \) is a smoothing kernel that depends on a certain filtering scale \( R_i \).

The smoothed profile \( \langle \delta_i(r) \rangle \) given by (15) is in general not equal to the mean value of the actual overdensity, \( \delta(r) \), that must be integrated to compute \( \Delta_i(r_i) \):

\[
\Delta_i(r_i) = 4\pi \int_0^{r_i} \delta(x)x^2 \, dx \tag{19}
\]

Comparing this expression with (18), we see that \( \Delta_i(r_i) \) is equivalent to \( \delta_i(0) \) as long as \( W_i \) is taken to be a spherical top hat of radius \( r_i \).

To sum up, we are interested in the physical density profile \( \Delta_i(r_i) \) around a local maximum of the smoothed density field. To locate the maximum, we use a Gaussian smoothing kernel

\[
W_i(r - x) = (2\pi R_i^2)^{-3/2} \exp \left( -\frac{|r - x|^2}{2R_i^2} \right) \tag{20}
\]

in order to avoid the oscillations in Fourier space that arise from a top-hat filter. We set the scale of the fluctuation, \( R_i \), and impose the condition that \( r = 0 \) is a maximum of \( \delta_i(r) \).

Then, we compute \( \Delta_i(r_i) = \delta_i(0) \) by applying a top hat smoothing of radius \( r_i \). BBKS show that the probability distribution of \( \delta_i(0) \) is a Gaussian with mean

\[
\langle \delta_i(0) \rangle = \nu \psi_0 - \frac{\gamma \theta(y, \gamma \nu)}{1 - \gamma^2} \sigma_0^2 \left( 1 - \frac{\sigma_0^2}{\sigma_2} \right). \tag{21}
\]

The moments

\[
\sigma_j^2 = \frac{1}{2\pi^2} \int_0^\infty P_i(k)k^{2j+1} \, dk \tag{22}
\]

are computed from

\[
P_i(k) = P(k) \exp \left[ -\frac{(kR_i)^2}{2} \right] \tag{23}
\]

and

\[
P_i(k) = P(k) \exp \left[ -\frac{(kR_i)^2}{2} \right] \frac{3|\sin(k \gamma_i) - k \gamma_i \cos(k \gamma_i)|}{(k \gamma_i)^3} \tag{24}
\]

where \( P(k) \) is the \( \Lambda \) CDM power spectrum that we used to generate the initial conditions for our simulations, evaluated at time \( a_i \).

2.3 Angular momentum

Two decades after the seminal paper by Gunn & Gott (1972), it was pointed out that angular momentum would prevent the orbits of dark matter particles from reaching the origin (e.g. Gurevich & Zybin 1988; White & Zariatsky 1992; Sikivie, Tkachev & Wang 1995).

For purely radial orbits, the mass in the centre is dominated by \( M_{\text{cl}} \) (i.e. particles from the outer shells) when the profile at turnaround is shallower than isothermal (Fillmore & Goldreich 1984). The final density distribution is valid to 1 per cent accuracy in the range \( 0.4 < y < 0.7 \) and \( 1 < \gamma \nu < 3 \), which is the scale relevant for both galaxies and galaxy clusters.

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To sum up, we are interested in the physical density profile \( \Delta_i(r_i) \) around a local maximum of the smoothed density field. To locate the maximum, we use a Gaussian smoothing kernel

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\]

where \( P(k) \) is the \( \Lambda \) CDM power spectrum that we used to generate the initial conditions for our simulations, evaluated at time \( a_i \).

\[
\min r = \frac{1 - \epsilon}{1 + \epsilon} r_{\text{max}} \tag{26}
\]

where \( r_{\text{max}} \) is computed from adiabatic invariance (14), following the procedure explained in Section 2.1. Angular momentum is taken into account by adding the usual term \( j^2/(2r^2) \) to the gravitational potential \( \Phi(r) \) in equation (10). Although this changes the actual value of the radial action, \( J_r \) is still proportional to \( \sqrt{m \mathcal{M}(r_{\text{max}})} \) and (14) can be used to compute the collapse factor \( F(r) \). The assumption of spherical symmetry leads to angular momentum conservation, which implies constant \( \epsilon \) during the contraction.
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3 NUMERICAL EXPERIMENTS

In order to test the analytical model, we carried out a series of high-resolution N-body simulations of cluster formation with the adaptive mesh code ART (Kravtsov, Klypin & Khokhlov 1997). Slightly lower spatial resolution experiments have been run from the same initial conditions with the Tree-SPH gasodynamical code GADGET (Springel, Yoshida & White 2001; Springel & Hernquist 2002). The radial structure of both dark and baryonic components has been addressed in Ascasibar et al. (2003). For a detailed description of the numerical experiments, the reader is referred to Ascasibar (2003).

A sample of cluster-size haloes has been selected from an initial low-resolution (128³ particles) pure N-body simulation of a 80 h⁻¹ Mpc box in a ΛCDM universe (Ω_m = 0.3; Ω_Λ = 0.7; h = 0.7; σ_v = 0.9). Higher resolution has been achieved by means of the multiple-mass technique (see Klypin et al. 2001, for details), using three levels of mass refinement. This is equivalent to an effective resolution of 512³ CDM particles (3.1 × 10⁶ h⁻¹ M⊙) in the highest refinement level. The minimum cell size allowed in the ART runs was 1.2 h⁻¹ kpc. All simulations were started at z = 50.

This procedure has been applied to 15 objects, ranging from 3 × 10¹³ to 2 × 10¹⁴ h⁻¹ M⊙. In order to explore a broader mass range, a smaller box (25 h⁻¹ Mpc) has been simulated, and six galaxy-mass haloes have been added to the cluster sample. Mass resolution is 1.2 × 10⁶ h⁻¹ M⊙ for these objects, with a minimum cell size of 0.2 h⁻¹ kpc.

All CDM haloes have been classified according to their dynamical state according to a substructure-based criterion. We use the BOUND DENSITY MAXIMA galaxy finding algorithm (see e.g. Colín et al. 1999; Klypin et al. 1999) with a maximum aperture of 200 h⁻¹ kpc, and look for the two most massive subhaloes within the virial radius. We label as a major merger any object in which the second subhalo is heavier than 50 per cent of the mass of the main one; if the mass is between 10 and 50 per cent, the object is classified as a minor merger; otherwise, it is assumed to be a relaxed system in virial equilibrium.

The centre of mass of each halo is found by an iterative procedure. Starting with an initial guess, we compute the centre of mass within a sphere of 500 h⁻¹ kpc. We move the sphere to the new centre of mass and repeat until convergence is reached. Then, we decrease the radius of the sphere by 10 per cent. The process continues until the sphere contains 200 dark matter particles, which we consider our resolution limit (Klypin et al. 2001).

The virial mass and radius are defined by an overdensity of ~300 with respect to the mean density of the universe (i.e. ~100 times the critical density). NFW profiles are fitted to the cumulative mass between 0.1rₗ₁ and rₘ using both ρₗ and rₗ in logarithmic steps. The virial radii inferred from the best-fitting values of these parameters are within 10 per cent of the ones measured directly from the mass profile. For each object, we also compute the spin parameter within rₘ,

\[ \lambda = \frac{J}{GM^{5/2}} \]

assuming that E ≈ −K by virtue of the virial theorem. A summary of the physical properties of our 19 dark matter haloes can be found in Table 1.

<table>
<thead>
<tr>
<th>rₘ</th>
<th>Mₘ</th>
<th>Nₘ</th>
<th>c</th>
<th>λₘ</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.241</td>
<td>222.16</td>
<td>716.649</td>
<td>9.55</td>
<td>0.056</td>
<td>Minor</td>
</tr>
<tr>
<td>1.126</td>
<td>165.95</td>
<td>535.311</td>
<td>8.42</td>
<td>0.039</td>
<td>Relaxed</td>
</tr>
<tr>
<td>1.201</td>
<td>201.36</td>
<td>649.561</td>
<td>8.55</td>
<td>0.048</td>
<td>Minor</td>
</tr>
<tr>
<td>1.042</td>
<td>131.51</td>
<td>424.223</td>
<td>6.36</td>
<td>0.047</td>
<td>Minor</td>
</tr>
<tr>
<td>0.957</td>
<td>101.88</td>
<td>328.645</td>
<td>9.68</td>
<td>0.049</td>
<td>Relaxed</td>
</tr>
<tr>
<td>0.787</td>
<td>56.66</td>
<td>182.774</td>
<td>4.16</td>
<td>0.044</td>
<td>Major</td>
</tr>
<tr>
<td>0.877</td>
<td>77.41</td>
<td>252.924</td>
<td>3.43</td>
<td>0.058</td>
<td>Major</td>
</tr>
<tr>
<td>0.872</td>
<td>77.07</td>
<td>248.622</td>
<td>11.35</td>
<td>0.017</td>
<td>Relaxed</td>
</tr>
<tr>
<td>0.932</td>
<td>94.10</td>
<td>303.556</td>
<td>3.16</td>
<td>0.106</td>
<td>Major</td>
</tr>
<tr>
<td>0.852</td>
<td>71.89</td>
<td>231.905</td>
<td>8.89</td>
<td>0.041</td>
<td>Relaxed</td>
</tr>
<tr>
<td>0.757</td>
<td>50.42</td>
<td>162.659</td>
<td>5.36</td>
<td>0.036</td>
<td>Major</td>
</tr>
<tr>
<td>0.648</td>
<td>31.63</td>
<td>102.027</td>
<td>8.22</td>
<td>0.043</td>
<td>Minor</td>
</tr>
<tr>
<td>0.588</td>
<td>23.63</td>
<td>76.229</td>
<td>5.62</td>
<td>0.083</td>
<td>Minor</td>
</tr>
<tr>
<td>0.244</td>
<td>1.69</td>
<td>1407.151</td>
<td>10.88</td>
<td>0.037</td>
<td>Relaxed</td>
</tr>
<tr>
<td>0.234</td>
<td>1.49</td>
<td>1241.135</td>
<td>9.55</td>
<td>0.038</td>
<td>Minor</td>
</tr>
<tr>
<td>0.219</td>
<td>1.22</td>
<td>1017.428</td>
<td>13.88</td>
<td>0.029</td>
<td>Relaxed</td>
</tr>
<tr>
<td>0.159</td>
<td>0.47</td>
<td>389.370</td>
<td>13.48</td>
<td>0.053</td>
<td>Minor</td>
</tr>
<tr>
<td>0.159</td>
<td>0.47</td>
<td>389.370</td>
<td>12.28</td>
<td>0.080</td>
<td>Minor</td>
</tr>
<tr>
<td>0.120</td>
<td>0.20</td>
<td>167.384</td>
<td>13.81</td>
<td>0.053</td>
<td>Relaxed</td>
</tr>
</tbody>
</table>
part, as well as beyond the virial radius. The raggedness at large mass distribution, although some deviations occur at the innermost figure, the NFW formula provides a reasonable approximation to the best-fitting characteristic density and radius. As can be seen in the figure, the average over relaxed haloes is shown in Fig. 2. All pro-

3.1 Mass distribution

The spherically averaged mass distribution of our numerical CDM haloes is shown in Fig. 2. All profiles have been rescaled by their best-fitting characteristic density and radius. As can be seen in the figure, the NFW formula provides a reasonable approximation to the mass distribution, although some deviations occur at the innermost part, as well as beyond the virial radius. The raggedness at large \( r \) is mainly due to substructure, while the differences at small \( r \) might reflect either genuine deviations from the NFW formula or a systematic dependence on halo mass or environment.

In Fig. 3, the logarithmic slopes of both mass and density profiles are plotted according to our dynamical classiﬁcation. We do not ﬁnd any evidence for asymptotic behaviour up to the resolution limit, in agreement with recent numerical studies (Fukushige et al. 2003; Hayashi et al. 2003; Navarro et al. 2003; Power et al. 2003).

As pointed out by several authors (e.g. Jing & Suto 2000; Klypin et al. 2001; Ascasibar et al. 2003), the mass distribution near the centre might depend on the dynamical state of the halo. At the resolution of the present simulations, relaxed haloes are well described by the NFW model, but merging systems display steeper pro-

\[
c(M, a) = \frac{K}{a(M)} \frac{a}{a_c(M)}
\]

(28)

where the collapse time \( a_c \) is deﬁned as the epoch at which the typical collapsing mass, \( M_c(a_c) \), equals a ﬁxed fraction \( F \) of the halo mass at epoch \( a \). According to Bullock et al. (2001), we set \( K = 3 \) and \( F = 0.001 \) in order to account for the massive haloes. The scatter around the relation is ﬁtted by \( \Delta \log(c) \simeq 0.11 \) (Colin et al. 2003).

We also plot the model by Eke, Navarro & Steinmetz (2001), which computes the collapse time from

\[
D(a_c)\sigma_{\text{eff}}(M_c) = \frac{1}{C_\sigma}
\]

(29)

where \( D(a) \) is the linear growth factor, \( M_c \) is the mass within \( r = 2.17r_s \) (maximum circular velocity for a NFW proﬁle), \( C_\sigma \simeq 28 \) and

\[
\sigma_{\text{eff}}(M) = -\frac{d\ln[\sigma(M)]}{d\ln(M)}\sigma(M).
\]

(30)

In this model, the concentration is given by

\[
c(M, a) = \left[ \frac{\Omega(a) \Delta[a_c(M)]}{\Delta(a) \Omega[a_c(M)]} \right]^{1/3} \frac{a}{a_c(M)}.
\]

(31)

Our sample of dark matter haloes is consistent with the expected trend, although our cluster-size haloes seem to be slightly more
concentrated than the theoretical model. On the other hand, major mergers display systematically lower concentrations, since their collapse time is very close to the present. However, the value of $a_e$ is not well defined in a merging system. In particular, during the first stages of the merger the profile corresponds to an old object, perturbed by an approaching satellite. It is only after virialization that the profile relaxes to its final form, and $a_e$ increases accordingly.

3.2 Angular momentum

In addition to the radial mass distribution, the kinematic structure of simulated haloes can offer interesting insights into the formation of galaxies and galaxy clusters. In particular, we are interested in the specific angular momentum of dark matter particles in order to set the eccentricity in the analytical model.

We separate the velocity field into a random component (i.e. velocity dispersion) and ordered rotation (i.e. average $j$). The velocity dispersion of a collisionless CDM halo is related to the total mass distribution by virtue of the Jeans equation (see, e.g. Binney & Tremaine 1987). For a spherically symmetric system with isotropic velocity dispersion, neglecting infall,

$$\frac{1}{\rho} \frac{d}{dr} \left( \rho \sigma_r^2 \right) = -\frac{GM}{r^2}$$

where $\rho$ is the local density, $\sigma_r$ is the radial velocity dispersion, and $M$ is the mass enclosed within radius $r$. An approximate velocity dispersion profile could be derived by substituting a given mass distribution (e.g. NFW) and setting an arbitrary normalization $\rho(0)$. The contribution of random motions to the angular momentum of the CDM particles would be given by the tangential velocity dispersion. In the isotropic case, this amounts to $\langle \dot{j}^2 \rangle = 2r^2 \sigma_r^2(r)$. Note that random motions do not contribute to the total angular momentum of the halo (i.e. $\langle j \rangle = 0$).

A more empirical approach to the velocity dispersion profile has been followed by Taylor & Navarro (2001). They realized that the coarse-grained phase-space density of a sample of numerical galaxy-size haloes followed a power law

$$\frac{\rho}{\sigma^3} \propto r^\beta$$

with $\beta = -1.875$ over more than two and a half decades in radius. Rasia, Tormen & Moscardini (2003) obtained a similar result for cluster-size haloes, although their best-fitting slope is $\beta = -1.95$. The normalization, though, has not been given in any of the two studies.

We have investigated the phase-space structure of the CDM haloes in our sample, including both galaxies and galaxy clusters. The average profile $\rho/\sigma^3$ is plotted in Fig. 5. We find that all our haloes are well described by

$$\frac{\rho}{\sigma^3} = 10^{1.46 \pm 0.04} \frac{v_{\text{vir}}}{v_{\text{vir}}} \left( \frac{r}{r_{\text{vir}}} \right)^{-1.90 \pm 0.05}$$

where $v_{\text{vir}}^2 \equiv GM_{\text{vir}}/r_{\text{vir}}$. This result is in fair agreement with the slopes reported by Taylor & Navarro (2001) and Rasia et al. (2003). The scatter around the average profile is remarkably low, taking into account that our haloes span four orders of magnitude in mass, and that we considered all systems (even major mergers) in the analysis. Indeed, we do not find any evidence that the slope of the relation depends on mass or dynamical state. Therefore, we claim that expression (34) can be regarded as a ‘universal’ phase-space density profile with only one free parameter, which is the virial radius of the halo.

We now show in Fig. 6 that angular momentum is indeed dominated by random motions. We compare the contribution to the tangential velocity of CDM particles from $\langle j \rangle$ and $\langle j^2 \rangle$ over spherical shells. The specific angular momentum grows roughly linearly with radius (i.e. approximately constant $v_{\text{vir}}$), in agreement with previous numerical work (Barnes & Efstathiou 1987; Bullock et al. 2001; Chen & Jing 2002; van den Bosch et al. 2002). The average bulk rotation velocity of our halos (solid symbols in Fig. 6) is about $0.1v_{\text{vir}}$, although we find a significant increase near the centre. Nevertheless, it is worth mentioning that the separation in bulk and random velocity is prone to numerical errors at the innermost regions.

The spin parameter at the virial radius (see Table 1) is typically $\approx 0.047$, consistent with previous studies (e.g. Colín et al. 2003). We do not find any obvious dependence on dynamical state.
eccentricity up to the virial radius,\(^1\) while relaxed systems are more consistent with a power-law profile.

The eccentricity profile of our haloes is plotted in Fig. 7. For each object, we compute the spherically symmetric potential derived from its mass distribution. The pericentric and apocentric radii of every CDM particle are estimated from its position and velocity, and the eccentricity is computed as

\[ e = \frac{r_{\text{max}} - r_{\text{min}}}{r_{\text{max}} + r_{\text{min}}} . \]  

(35)

The general trend is that particle orbits are slightly more radial as we move out to the current turn-around radius of the halo, \(R_{\text{ta}}\). Moreover, we find a systematic dependence on the dynamical state. As can be seen in Fig. 8, major mergers are well described by constant eccentricity up to the virial radius,\(^1\) while relaxed systems are more consistent with a power-law profile. Minor mergers are somewhat in the middle. The average profile can be fitted by a power law, but the slope is shallower than for relaxed systems.

A least-squares fit to the relaxed population yields

\[ e(r_{\text{max}}) \simeq 0.8 \left( \frac{r_{\text{max}}}{R_{\text{vir}}} \right)^{0.1} \]  

(36)

for \(r_{\text{max}} < 0.1 R_{\text{vir}}\). We note that our approximation to compute the eccentricity breaks down at larger radii, since the mass distribution (and thus, the gravitational potential) are by no means static beyond the virial radius.

\(^1\) For our dark matter haloes, \(r_{\text{vir}}/R_{\text{ta}}\) is typically of the order of 0.2–0.3.

4 MODEL RESULTS

We now attempt to reproduce the density profiles of our simulated haloes with the model described in Section 2. In order to keep the number of free parameters to a minimum, we assumed a constant eccentricity, \(e \simeq 0.5\), which corresponds to the average over the whole radial range. This value implies that particles are able to sink into the dark matter potential up to one third of their maximum radius, or, equivalently, that all mass within radius \(r\) comes from shells whose turn-around radius was \(<3r\). The effect of a power-law profile will be considered in Section 4.2.

4.1 Constant eccentricity

Once the eccentricity is set to \(e = 0.5\), the model has two free parameters, which describe the primordial density peak: its height, \(\nu\), and the smoothing scale, \(R_t\). These parameters set the mass and formation time of the halo. For a given mass, a higher peak on a smaller scale corresponds to an earlier formation time and a steeper density profile near the centre.

We generated a logarithmically spaced grid of mass profiles for \(0.8 < \nu < 8.0\) and \(0.08 < R_t < 7.0\), and then performed a \(\chi^2\) minimization for each halo, considering data points within \(0.1 < r/R_{\text{vir}} < 1\). Best-fitting values of \(R_t\) and \(\nu\) are plotted in Fig. 9, as well as the areas where \(\sqrt{\chi^2/(\text{dof})} \leq 0.12\). Most cluster-size haloes can be described as very high-\(\nu\) peaks, while galaxies seem to have

Figure 7. Orbit eccentricity of CDM particles as a function of apocentric radius. Dotted line marks \(e = 0.5\); dashed line, expression (36).

Figure 8. Same as Fig. 7, separating relaxed haloes (left), minor (middle) and major (right) mergers.

Figure 9. Best-fitting values of the peak height, \(\nu\), and the smoothing scale, \(R_t\). Black squares represent relaxed haloes, open squares are used for minor mergers, and stars for major merging systems. Shaded areas indicate \(\sqrt{\chi^2/(\text{dof})} \leq 0.12\). Dashed lines are drawn at constant \(r_{\text{vir}}\).
collapsed around less extreme fluctuations. This means that clusters are expected to form earlier than galaxies, in the sense that the seeds of their dark matter haloes are already in place at a higher redshift.

An exception to this rule are major mergers. While some of these systems still have an early collapse time, some others have collapsed approximately at the present epoch. The first class corresponds to objects that have not relaxed yet, and their core still corresponds to the old halo. As relaxation completes and substructure is erased, the best-fitting parameters move along the lines of constant mass in the $v$–$R_t$ diagram. The smoothing scale rises sharply, and $v$ decreases accordingly.

As can be seen in Fig. 9, there is nevertheless a certain degeneracy between $R_t$ and $v$, which prevents a reliable determination of the formation time. The exact value of the best-fitting parameters may vary within the shaded area, depending on the details of the fit. For instance, we tend to get higher peaks on smaller scales as we give more weight to the inner parts of the profile.

This can be understood when we compare the numerical data with the results of our spherical collapse model. We chose to fit the radial range $0.1 < r/r_{vir} < 1$ in order to test the quality of the extrapolations, both towards the centre and to large radii. Individual circular velocity profiles are shown in Fig. 10, together with the best fits provided by our model and the NFW formula.

Although the mass distribution is fairly well described in general terms, the central density is usually underestimated by both the spherical collapse model and the NFW formula (in qualitative agreement with recent numerical studies, see, e.g. Navarro et al. 2003). When we fit the innermost parts, we are biased towards more concentrated distributions in either case. The best-fitting values of the free parameters ($c$ and $r_{vir}$, or $R_t$ and $v$) are relatively robust as long as most of the halo mass is enclosed. However, they become rather unrealistic if, for example, only data within $0.1 r_{vir}$ are considered, indicating that neither prescription provides a complete description of the halo profiles.

At large radii, the spherical collapse model reproduces the upturn in the circular velocity, while this quantity drops to zero for a NFW profile. This is due to the fact that the NFW density vanishes at infinity, while it tends asymptotically to the mean value in the spherical collapse model. Adding a constant density background to the NFW formula is enough to bring the circular velocity profile in agreement with the numerical data.

We assess the accuracy of both the NFW profile and the spherical collapse model in Fig. 11, where the quantity

$$\frac{\Delta M}{M} = \frac{M_{\text{model}} - M_{\text{data}}}{M_{\text{data}}}$$

(37)

is plotted as a function of radius.

The most important difference between both models is that the accuracy of the NFW fit improves significantly for relaxed systems, particularly concerning the extrapolation towards $r \to 0$. The fits based on our spherical collapse model are on average less accurate, and the scatter between individual dark matter haloes (i.e. the $1 \sigma$ error bars) is a little bit larger. Yet, the average uncertainty is always lower than 20 per cent for relaxed objects, and only slightly larger for merging systems, where the prediction of the spherical collapse model is indeed very similar to the NFW fit.

4.2 Variable eccentricity

Although constant eccentricity might provide a useful approximation for merging systems, it is evident from Fig. 8 that it is not a good description of relaxed haloes, where the eccentricity profile $e(r)$ increases significantly from the centre to the turn-around radius. Moreover, the assumption of constant eccentricity leads to systematic differences between the mass distribution predicted by our model and the numerical data, as shown in Fig. 11.

Therefore, we would like to investigate the consequences of using a more realistic prescription for $e(r)$. For constant eccentricity, high values of $e$ lead to steeper density profiles in the central regions, whereas the outer parts of the halo remain largely unaffected (see Fig. 1). We plot in Fig. 12 the density profile resulting from our power-law fit (36) for $v = 3$ and $R_t = 1 h^{-1}$ Mpc. The mass distribution is very similar to that obtained with $e = 0.5$, but the shape is slightly different. There is a small increase in the density between 10 and $100 h^{-1}$ kpc, but the profile flattens near the centre due to the more circular orbits.

This flattening can be clearly seen on the bottom panel of Fig. 12, where we plot the logarithmic slope of the density profile. It is interesting to note that the spherical collapse model predicts a finite density at $r = 0$ when non-radial motions are included, albeit the size of the ‘core’ is extremely small. As was discussed in Section 3.1, a similar trend is shown by the relaxed haloes in our sample.

At our resolution limit, the net effect of using expression (36) is just to increase the density at small radii. Then our model gives a somewhat poorer description of merging systems, but the quality of the fit improves considerably for relaxed haloes. As can be seen in Fig. 13, the accuracy of our model is comparable to the NFW formula when a realistic prescription is used for the eccentricity of particle orbits.

5 DISCUSSION AND CONCLUSIONS

The radial density profile of dark matter haloes has been investigated within the framework of the spherical collapse theory. We have shown that the model described in Section 2 is able to reproduce the mass distribution of realistic CDM haloes. Although the final profile cannot be cast in a simple analytical form, it provides not only a more phenomenological fit, but a physically motivated description of the density distribution in terms of the primordial initial conditions.

However, it is not easy to understand how the assumption of spherical symmetry could be able to describe the hierarchical assembly of cosmological structures. Instead of continuous infall of spherical shells, the formation of CDM haloes observed in numerical experiments takes place in a discrete and anisotropic way. Most of the matter is accreted in clumps, along the preferred directions set by the filamentary large-scale structure.

None the less, the very complicated coalescence process looks very regular in energy space (Zaroubi, Naim & Hoffman 1996). Moreover, Moore et al. (1999) have shown that the final density profile is not very sensitive to the details of the merging history, by comparing the mass distribution of a galaxy cluster halo with a resimulation in which the power spectrum was truncated at ~4-Mpc scales.

We suggest, as a plausible explanation for the success of the spherical infall model, that merging is implicitly taken into account through the smoothing scale $R_t$. Cosmological structures do not form around maxima of the primordial density field, but of the smoothed density field (Hoffman & Shaham 1985). Some memory of the initial conditions will be lost during major mergers, particularly within the innermost regions of the resulting halo. Contrary to the common view (see, e.g. BBKS), we argue that $R_t$ has a precise physical interpretation; below the mass-scale defined by $R_t$, all information about...
the primordial substructure would have been erased by relaxation processes.

In the outer regions, matter is accreted in a more gentle way. Minor mergers do not significantly alter the dynamical structure of the halo. The mass of infalling clumps is much less than the average over a spherical shell at large radii. Thus, the density profile and the accretion rate are determined by the amount of matter available to the halo, which is ultimately set by the primordial initial conditions.

Nevertheless, there is still an additional degree of freedom, related to the magnitude and distribution of angular momentum within the dark matter halo. Angular momentum sets the shape of the density profile at the inner regions. For purely radial orbits, the core is dominated by particles from the outer shells. As the angular momentum increases, these particles remain closer to their maximum radius, resulting in a shallower density profile.

We found that angular momentum is dominated by the tangential component of the velocity dispersion. Random motions are well described by a ‘universal’ phase-space density profile over several orders of magnitude in radius. This profile is a power law with slope $\beta \simeq -1.9$, in agreement with the results of Taylor & Navarro (2001).
value of $e = 0.5$ as a first-order approximation. However, relaxed haloes are better described by a power-law profile in terms of their turn-around radius. Although the details of the mass distribution are sensitive to the prescription assumed for the angular momentum, the spherical collapse model provides a valuable tool to predict the structure of virialized dark matter haloes, given the power spectrum of primordial fluctuations. Indeed, we claim that the physical origin of the mass distribution observed at the present day is related to the shape of the primordial density peaks. ‘Universal’ profiles with two free parameters arise naturally from Gaussian random peak statistics, since the primordial fluctuations are fully specified by their height, $v_f$, and smoothing scale, $R_s$.

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