

## A TRANSFER MATRIX TECHNIQUE FOR EVALUATING THE NATURAL FREQUENCIES AND CRITICAL SPEEDS OF A ROTOR WITH MULTIPLE FLEXIBLE DISKS

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### EXTENDED ABSTRACT

Modern turbomachinery is used to provide power for a wide range of applications, from steam turbines for electrical power plants to the turbopumps used in the Space Shuttle Main Engine. Such devices are subject to a variety of dynamical problems, including vibration, rotordynamical instability, and shaft whirl. In order to properly design and evaluate the performance and stability of turbomachinery, it is important that appropriate analytical tools be available that allow for the study of potentially important dynamical effects. This research effort is concerned with developing a procedure to account for disk flexibility which can readily be used for investigating how such effects might influence the natural frequencies and critical speeds of practical rotor systems.

In the present work, a transfer matrix procedure is developed in which the disk flexibility effects are accounted for by means of additional terms included in the transfer matrix formulation. In this development, the shaft is treated as a discrete system while the disk is modelled as a continuous system using the governing partial differential equation. Based on this governing equation, an equivalent inertial moment  $M_k^*$ , which is the generalized dynamic force coupling between shaft and disk, is then derived. Analysis shows that only the disk modes of one nodal diameter contribute to the inertial moment,  $M_k^*$ , and thus influence the natural frequencies of the rotor system.

To determine the  $M_k^*$ , the modal expansion method is employed and the governing partial differential equation of the disk is transformed to a set of decoupled forced vibration equations in the generalized coordinates. The  $M_k^*$  are then calculated in terms of modal shapes, natural frequencies, and material and geometric parameters which can be found in the literature or can be obtained from experiments. Finally the  $M_k^*$  are incorporated into the point transfer matrix. By so doing, the properties of quick computational speed and ease of use are retained and the complexity of solving partial differential equations is avoided. This allows the present procedure to be easily applied to practical engineering problems. This is especially true for multiple flexible disk rotor systems.

As an example, three different cases for a simplified model of the Space Shuttle Main Engine (SSME) High Pressure Oxygen Turbo-Pump (HPOTP) rotor have been studied using this procedure. Some of the more interesting results obtained in this example study are enumerated below.

- 1.) Disk flexibility can introduce additional natural frequency(s) to a rotor system.
- 2.) Disk flexibility can cause shifting of some of the natural frequencies.
- 3.) As disk flexibility is increased, lower natural frequencies of the rotor system will be influenced.
- 4.) At certain rotor speeds, disk flexibility may cause the disappearance of a natural frequency.
- 5.) The axial position of the disk on the rotor shaft has a significant effect on the degree of this influence.