Is there supernova evidence for dark energy metamorphosis?

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ABSTRACT

We reconstruct the equation of state \( w(z) \) of dark energy (DE) using a recently released data set containing 172 Type Ia supernovae (SNe) without assuming the prior \( w(z) \gtrless -1 \) (in contrast to previous studies). We find that DE evolves rapidly and metamorphoses from dust-like behaviour at high \( z \) (\( w \simeq 0 \) at \( z \sim 1 \)) to a strongly negative equation of state at present (\( w \lesssim -1 \) at \( z \simeq 0 \)). DE metamorphosis appears to be a robust phenomenon which manifests for a large variety of SNe data samples provided one does not invoke the weak energy prior \( \rho + p > 0 \).

Invoking this prior considerably weakens the rate of growth of \( w(z) \). These results demonstrate that DE with an evolving equation of state provides a compelling alternative to a cosmological constant if data are analysed in a prior-free manner and the weak energy condition is not imposed by hand.

Key words: methods: statistical – cosmological parameters – cosmology: theory.

1 INTRODUCTION

One of the most tantalizing observational discoveries of the past decade has been that the expansion of the Universe is speeding up rather than slowing down. An accelerating universe is strongly suggested by observations of Type Ia high redshift supernovae (SNe) provided these behave as standard candles. The case for an accelerating universe is further strengthened by the discovery of cosmic microwave background (CMB) anisotropies on degree scales (which indicate \( \Omega_{\text{rad}} \simeq 1 \)) combined with a low value for the density in clustered matter \( \Omega_m \simeq 1/3 \) deduced from galaxy redshift surveys. All three sets of observations strongly suggest that the Universe is permeated by a relatively smooth distribution of dark energy (DE) which dominates the density of the Universe \( \Omega_{\text{DE}} \simeq 2\Omega_m \simeq 2/3 \) and whose energy momentum tensor violates the strong energy condition \( \rho + 3p > 0 \) so that \( w_{\text{DE}} = p/\rho < -1/3 \).

Although a cosmological constant \( (w = -1) \) provides a plausible answer to the conundrum posed by DE, it is well known that the unevolving cosmological constant faces serious ‘fine tuning’ problems because the ratio between \( \rho_A / \Lambda / 8\pi G \) and the radiation density, \( \rho_r \), is already a miniscule \( \rho_A / \rho_r \sim 10^{-54} \) at the electroweak scale \( (T \sim 100 \text{ GeV}) \) and even smaller \( \rho_A / \rho_r \sim 10^{-323} \) at the Planck scale \( (T \sim 10^{18} \text{ GeV}) \). This issue is further exacerbated by the ‘cosmological constant problem’ which arises because the \( \Lambda \)-term generated by quantum effects is enormously large \( \rho_A \gtrsim m^4_P \), where \( m_P \simeq 1.2 \times 10^{19} \text{ GeV} \) is the Planck mass (Zeldovich 1968; Weinberg 1989).

Although the cosmological constant problem remains unresolved, the issue of fine tuning which plagues \( \Lambda \) has led theorists to explore alternative avenues for DE model building in which either DE or its equation of state are functions of time. [Following Sahni et al. (2003) we shall refer to the former as ‘quiescence’ and to the latter as ‘kinescence’.] Inspired by inflation, the first DE models were constructed around a minimally coupled scalar field (quintessence) whose equation of state was a function of time and whose density dropped from a large initial value to the small values which are observed today (Peebles & Ratra 1988; Wetterich 1988). (‘Tracker’ quintessence models had the advantage of allowing the current accelerating epoch to be reached from a large family of initial conditions (Caldwell, Dave & Steinhardt 1998).)

Half a decade after SNe-based observations pointed to the possibility that we may be living in an accelerating Universe, the theoretical landscape concerning DE has evolved considerably (see the reviews Sahni & Starobinsky 2000; Carroll 2001; Peebles & Ratra 2002; Sahni 2002; Padmanabhan 2003). In addition to the cosmological constant and quintessence, the current paradigm for DE includes the following interesting possibilities.

(i) \( \text{DE with} \ w \lesssim -1 \) (Chiba, Okabe & Yamaguchi 2000; Caldwell 2002; McInnes 2002; Sahni & Shtanov 2003; Alam & Sahni 2003; Caldwell, Kamionkowski & Weinberg 2003; Carroll, Hoffman & Trodden 2003; Frampton 2003; Frampton & Takahashi 2003; Singh, Sami & Dadhich 2003; Johri 2003).

(ii) \textbf{The Chaplygin gas}, the equation of state of which drops from \( w = 0 \) at high redshifts to \( w \simeq -1 \) today (Kamenshchik, Moschella & Pasquier 2001).

(iii) \textbf{Braneworld models} in which the source for cosmic acceleration rests in the gravity sector rather than in the matter sector of
the theory (Deffayet, Dvali & Gabadadze 2002; Sahni & Shtanov 2003; Maeda, Mizuno & Torii 2003).

(iv) DE models with negative potentials (Felder et al. 2002; Kallosh et al. 2002; Alam, Sahni & Starobinsky 2003).


(vii) DE driven by quantum effects (Sahni & Habib 1998; Parker & Raval 1999a,b).

(viii) DE with a late-time transition in the evolution of state (Bassett et al. 2002; Corasaniti et al. 2003).

(ix) Unified models of DE and inflation (Peebles & Vilenkin 1999; Copeland, Liddle & Lidsey 2001; Sahni, Sami & Souradeep 2002) etc.

Faced with the current plethora of DE scenarios the concerned cosmologist is faced with two options:

(i) she can test every single model against observations, or
(ii) she can take a more flexible approach and determine the properties of DE in a model-independent manner.

In this paper we proceed along route (ii) and demonstrate that model independent reconstruction brings us face to face with exciting new properties of DE.

Applying the techniques developed in Saini et al. (2000), Sahni et al. (2003) to a new data set consisting of 172 SNe from Tonry et al. (2003) and an additional 22 SNe from Barris et al. (2004) we show that the DE equation of state which best fits the data evolves from $w \approx 0$ at $z \approx 1$ to $-1.2 < w \leq -1$ today. An evolving equation of state of DE is favoured by the data over a cosmological constant for a large region in parameter space.

### 2 Model Independent Reconstruction of DE

SN observations during the previous decade have been pioneered by two teams: the High-z Supernova Search Team (HST) (Riess et al. 1998) and the Supernova Cosmology Project (SCP) (Perlmutter et al. 1999). The enormous efforts made by these two teams have changed the theory of DE, but physical interpretations of DE (quintessence, Chaplygin gas, Brane worlds, etc.) can show significant evolution in $w(\tau)$ over sufficiently large look-back times.

In this paper we shall reconstruct the properties of DE without assuming any priors on the cosmic equation of state. The dangers of imposing priors on $w(\tau)$ have been highlighted in Maor et al. (2002) and several of our subsequent results will lend support to the conclusions reached in this paper.

#### 2.1 Cosmological reconstruction of $w(\tau)$

Cosmological reconstruction is based on the observation that, in a spatially flat universe, the luminosity distance and the Hubble parameter are related through the equation (Starobinsky 1998; Huterer & Turner 1999; Nakamura & Chiba 1999):

$$H(z) = \left[ \frac{d}{dz} \left( \frac{d_L(z)}{1+z} \right) \right]^{-1}.$$  \hspace{1cm} (3)

Thus, knowing $d_L$, we can unambiguously determine the Hubble parameter as a function of the cosmological redshift. Next, the Einstein equations

$$H^2 = \frac{8\pi G}{3} \left[ \rho_m + \rho_\text{DE} \right],$$

$$q = -\frac{\dot{a}}{aH} - \frac{4\pi G}{3H^2} \sum_i (\rho_i + 3p_i),$$  \hspace{1cm} (4)

are used to determine the energy density and pressure of DE:

$$\rho_\text{DE} = \rho_{\text{critical}} - \rho_m = \frac{3H^2}{8\pi G} [1 - \Omega_m(x)],$$

$$p_\text{DE} = \frac{H^2}{4\pi G} \left( q - \frac{1}{2} \right).$$  \hspace{1cm} (5)

where $\rho_{\text{critical}} = 3H^2/8\pi G$ is the critical density of a Friedmann–Robertson–Walker (FRW) universe. The equation of state of DE

$$w_{\text{eff}} = p_{\text{DE}}/\rho_{\text{DE}} = \frac{H^2}{4\pi G} \left( q - \frac{1}{2} \right).$$

are obtained for a new sample of high-z SNe by SCP (Knop et al. 2003).\footnote{One way around this difficulty is to define observables solely in terms of $H$ and its derivatives (called ‘Statefinders’ in Sahni et al. 2003). A detailed discussion of these issues can be found in Alam et al. (2003).}

1 It is interesting that, when no priors are set on $\Omega_m$, the DE equation of state becomes virtually unbounded from below and has a 99 per cent confidence level of being $\leq -1$ (Knop et al. 2003).\footnote{One way around this difficulty is to define observables solely in terms of $H$ and its derivatives (called ‘Statefinders’ in Sahni et al. 2003). A detailed discussion of these issues can be found in Alam et al. (2003).}

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This prior was assumed both in the analysis of the SN data set as well as in the 2dFGRS study (Tonry et al. 2003; Knop et al. 2003). Aside from the cosmological constant ($w = -1$), the topological defect models alluded to earlier and the sine-hyperbolic scalar field potential (Sahni & Starobinsky 2000; Urena-Lopez & Matos 2000; Sahni et al. 2003) no viable DE models exist with the property $w = constant$. Indeed, most models of DE (quintessence, Chaplygin gas, Brane worlds, etc.) can show significant evolution in $w(\tau)$ over sufficiently large look-back times.

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One route towards the meaningful reconstruction of $w(z)$ lies in inventing a sufficiently versatile fitting function for either $d_L(z)$ or $H(z)$. The parameters of this fitting function are determined by matching to SN observations and $w(z)$ is determined from (3) and (6). Our reconstruction exercise will be based upon the following flexible and model independent Ansatz for the Hubble parameter (Sahni et al. 2003)

$$H(x) = H_0 \left[ \Omega_m x^3 + A_0 + A_1 x + A_2 x^2 \right]^{1/2}, \quad (7)$$

where $x = 1 + z$. This Ansatz for $H(z)$ is exact for the cosmological constant $w = -1$ ($A_1 = A_2 = 0$) and for DE models with $w = -2/3$ ($A_0 = A_2 = 0$) and $w = -1/3$ ($A_0 = A_1 = 0$). It has also been found to give excellent results for DE models in which the equation of state varies with time including quintessence, Chaplygin gas, etc. (Sahni et al. 2003; Alam et al. 2003). The Ansatz (7) is equivalent to the following expansion for DE:

$$\rho_{DE} = \rho_{\Lambda 1} \left( A_0 + A_1 x + A_2 x^2 + A_3 x^3 \right), \quad (8)$$

where $\rho_{\Lambda 1} = 3 H_0^2 / (8 \pi G)$ is the present-day critical density. The condition $A_3 \geq 0$ allows $\rho_{DE}$ to mimic the properties of dark matter at large redshifts ($A_1 \ll 1$ follows from large scale structure constraints). From (7) and (8) we find $\Omega_m = \Omega_{DE} + A_3$, i.e. the value of $\Omega_m$ in (7) can be slightly larger than $\Omega_{DE}$ in this case.

Substituting (7) into the expression for the luminosity distance we get

$$d_L(z) = \frac{c}{H_0} \int_0^{1+z} \frac{dx}{\sqrt{\Omega_m x^3 + A_0 + A_1 x + A_2 x^2}}. \quad (9)$$

The parameters $A_0$, $A_1$ and $A_2$ are determined by fitting (9) to SN observations using a maximum likelihood technique. This Ansatz has only three free parameters ($\Omega_m$, $A_1$, $A_2$) as $A_0 + A_1 x + A_2 x^2 = 1 - \Omega_m$ for a flat universe. A note of caution: because the Ansatz (8) is a truncated Taylor expansion in $x = 1 + z$, its range of validity is $z \lesssim v$, and consequently the Ansatz-derived $H(z)$ and $d_L(z)$ should not be used at higher redshifts.

Note that the weak energy condition for DE $\rho_{DE} \geq 0$, $\rho_{DE} + p_{DE} \geq 0$ has the following form for the Ansatz (7):

$$A_0 + A_1 x + A_2 x^2 \geq 0, \quad A_1 + 2 A_2 x \geq 0, \quad (10)$$

provided we assume that the $\Omega_m x^3$ term in (7) is totally due to non-relativistic dark matter and does not include any contribution from DE. The demand that the weak energy condition (WEC) (10) be satisfied for all $x \geq 0$ (i.e. in the past as well as in the future) requires $A_0$, $A_1$ and $A_2$ to be non-negative. However, the demand that the WEC (10) be satisfied in the past ($x \geq 1$) but not necessarily in the future leads to the somewhat weaker constraint

$$A_1 + 2 A_2 \geq 0, \quad A_2 \geq 0. \quad (11)$$

[Models in which $\rho_{DE}(z) < 0$ for $z < 0$ and which violate the WEC in the future have been discussed in Felder et al. (2002); Kallosh et al. (2002); Alam et al. (2003).]

The presence of the term $\Omega_m x^3$ in (7) has two important consequences: (i) it ensures that the Universe transits to a matter-dominated regime at early times ($z \gg 1$), and (ii) it allows us to incorporate information (available from other data sets) regarding the current value of the matter density in the Universe. This information can be used to perform a maximum likelihood analysis with the introduction of suitable priors on $\Omega_m$. In further analysis we will assume that $\Omega_m x^3$ term in (7) does not include any contribution from DE.

We have also studied simple extensions of the Ansatz (7) by adding new terms $A_1 x^{-1}$ and $A_1 x^4$. The $A_1 x^{-1}$ term allows $w(z)$ to become substantially less than $-1$, thereby providing greater leeway to phantom models. The $A_1 x^4$ term allows DE to evolve towards equations of state which are more stiff than dust ($w = 0$); its role is therefore complementary to that of $A_1 x^{-1}$. Despite the inclusion of these new terms, our best fit to the SN data presented below does not change significantly (choosing $A_1 = 0.0003$ and $A_4 = 0.008$), which points to the robustness of the Ansatz (7) for the given data set.

We should add that for our reason for choosing an Ansatz to fit $H(z)$ rather than some other cosmological quantity was motivated by the fact that the Hubble parameter is directly related to a fundamental physical quantity – the Ricci tensor – and is therefore likely to remain meaningful even when other quantities (such as the equation of state) become ‘effective’. [This happens for instance, in the case of the Braneworld models of DE discussed in Deffayet et al. (2002) and Sahni & Shtanov (2003).]

The rationale for choosing a three-parameter Ansatz for $H(z)$ is the following. The observed luminosity distance determined using Type Ia SNe is rather noisy, therefore in order to determine the Hubble parameter from $d_L(z)$ and following that the equation of state, one must take two derivatives of a noisy quantity. This difficulty can be tackled in two possible ways: (i) either one smoothes the data over some interval $\Delta t$ (binning is one possibility), or (ii) we may choose to smooth ‘implicitly’ by parameterizing $H(z)$ through an appropriate fitting function. The number of free parameters $N$ in the fit to $H(z)$ will be related to the smoothing interval $\Delta z$ through $\Delta z = z_{max} / N$. Increasing $N$ implies decreasing $\Delta z$ which results in a rapid growth of errors through $\Delta H(z) \propto (\Delta z)^{-3/2}$, and $\Delta w(z) \propto (\Delta z)^{-5/2}$ (Tegmark 2002), therefore in order not to lose too much accuracy in our reconstruction we considered three-parameter fits for $H(z)$ in our paper [these correspond to two-parameter fits for $w(z)$].

We now test the usefulness of the Ansatz (7) in reconstructing different DE models. The Ansatz returns exact values for $\Lambda$CDM, and $w = -1/3$, $w = -2/3$ quintessence models. In Fig. 1 we show the accuracy of the Ansatz (7) when applied to several other DE models such as tracker quintessence, the Chaplygin gas and super-gravity (SUGRA) models. We plot the deviation of $\log(d_L H_0)$ (which is the measured quantity for SNe) obtained with the Ansatz (7) from the actual model values. Clearly the Ansatz performs very well over a significant redshift range for $\Omega_m = 0.3$ (Also see Appendix B). In fact, in the redshift range where SNe data is available, the Ansatz recovers these models of DE with less than 0.5 per cent errors. However it would be appropriate to add a note of caution at this point. Although fig. 1 clearly demonstrates the usefulness of the Ansatz for some DE models, its performance vis-a-vis other models of DE is by no means guaranteed. By its very construction the Ansatz (7) is expected to have limitations when describing models with a fast phase transition (Bassett et al. 2002) as well as rapidly oscillating quintessence models (Sahni & Wang 2000). (The Ansatz (7) can give reasonable results even for these models provided the resulting DE behaviour is suitably smoothed.) For this reason, although the bulk of our analysis will be carried out using (7), we shall supplement it when necessary with other fitting functions, which will provide us with an
2.1.1 Methodology

For our primary reconstruction, we use a subset of 172 Type Ia SNe, obtained by imposing constraints \(A_{\chi} < 0.5\) and \(z > 0.01\) on the 230 SNe sample, as in the primary fit of Tonry et al. (2003). For the Ansatz (9), we require to fit four parameters: \((H_0, \Omega_{\text{om}}, A_1, A_2)\). We may use prior information on \(H_0\) (\(H_0 = 72 \pm 8\) km s\(^{-1}\) Mpc\(^{-1}\)) Freedman et al. 2001) and \(\Omega_{\text{om}}(\Omega_{\text{m0}}h = 0.2 \pm 0.03;\) Percival et al. 2001).\(^4\)

The measured quantity for this data is \(y = (\log(d_L H_0))\), therefore the likelihood function is given by

\[
L = N \exp \left( -\frac{x^2}{2} \right),
\]

\[
x^2 = \sum_{i=1}^{172} \frac{y_i - y_{\text{m}}(H_0, \Omega_{\text{om}}, A_1, A_2)}{\sigma_i}^2,
\]

where \(N\) is a normalization constant. Therefore, the probability distribution function in the four-space \((H_0, \Omega_{\text{om}}, A_1, A_2)\) is

\[
P(H_0, \Omega_{\text{om}}, A_1, A_2) \propto \exp \left( -\frac{x^2}{2} \right) \Pr(\Omega_{\text{m0}}h) \Pr(H_0).
\]

where \(\Pr\) refers to the priors applied on the parameters of the system.

\(^4\) One should note, however, that the prior on \(\Omega_{\text{om}}\) is not model independent since it relies on the \(\Lambda\)CDM model to project from redshift space to real space. Results coming from the use of this prior should therefore not be taken too literally in the present context. See Kunz et al. (2004) for an interesting discussion of related issues.

Our goal is to reconstruct cosmological parameters such as the equation of state \(w(z) = w(z; \Omega_{\text{om}}, A_1, A_2)\), therefore we marginalise over \(H_0\) and obtain the probability distribution function in the \((\Omega_{\text{om}}, A_1, A_2)\) space:

\[
P(\Omega_{\text{om}}, A_1, A_2) = \int P(H_0, \Omega_{\text{om}}, A_1, A_2) \, dH_0.
\]

In order to do this, we have to define the bounds of a four-dimensional volume in \((H_0, \Omega_{\text{om}}, A_1, A_2)\). The bounds of \(H_0\) are taken at 5 \(\sigma\) of the Hubble Space Telescope (HST) prior. For \(\Omega_{\text{om}}\), the natural choice is 0 \(\leq\) \(\Omega_{\text{om}}\) \(\leq\) 1. It is not immediately obvious what the bounds should be for \(A_1, A_2\). We choose a sufficiently large rectangular grid for \(A_1, A_2\) (roughly corresponding to \(-6 \leq w_0 \leq 5\)) which includes most known models of DE. This bound is merely a matter of convenience and does not affect our results in any way. After marginalization, we have a grid in \((\Omega_{\text{om}}, A_1, A_2)\) space on which \(P(\Omega_{\text{om}}, A_1, A_2)\) is specified at each point. We may now proceed in two ways. First, we may choose to fix \(\Omega_{\text{om}}\) at a suitable constant value (e.g. \(\Omega_{\text{om}} = 0.3\)) thereby obtaining a grid in the \((A_1, A_2)\) plane with \(P\) (the probability if \(\Omega_{\text{om}}\) is known to be an exact value) defined at each point. For a particular redshift, we may then calculate \(w(z; \Omega_{\text{om}}, A_1, A_2)\) at each point of the grid. This would yield results that would hold true if \(\Omega_{\text{om}}\) were known exactly. Instead of using the exact value of \(\Omega_{\text{om}}\), we may use the prior information about it available to us (\(\Omega_{\text{m0}}h = 0.2 \pm 0.03\), and calculate \(w(z; \Omega_{\text{om}}, A_1, A_2)\) at each point of a three-dimensional grid, the probability \(P\) at each point being known. Therefore, at any given redshift \(z\), \(w(\Omega_{\text{om}}, A_1, A_2)\) can be tagged with a numerical value \(P(\Omega_{\text{om}}, A_1, A_2)\). Starting from the best-fitting \(w(z)\) (the value at the peak of the probability distribution), we may move down on either side till 34 per cent of the total area is enclosed under the curve, thus obtaining asymmetric 1\(\sigma\) bounds on \(w(z)\). The 2\(\sigma\), 3\(\sigma\) bounds can be similarly obtained.

2.1.2 Results

We first show preliminary results for which the matter density is fixed at a constant value of \(\Omega_{\text{om}} = 0.3\). A detailed look at the \(x^2\) surface in the \((A_1, A_2)\) plane (Fig. 2) reveals the existence of two minima in \(x^2\), a shallower one close to \(\Lambda\)CDM \((A_1 = 0.177, A_2 = -0.119, w_0 = -1.03, x^2 = 1.0402)\), and a deeper minimum at \(A_1 = -4.360, A_2 = 1.829, w_0 = -1.33, x^2 = 1.0056\). We would like to draw the reader’s attention to the fact that imposing the prior \(w(z) \geq -1\) \((z \geq 0)\) amounts to disallowing a significant region of parameter space (the unshaded region in Fig. 2). Consequently, an analysis which assumes \(w(z) \geq -1\) loses all information about the region 2\(\sigma\) around the deeper minimum! Because we have no reason (observational or theoretical) to favour either minimum over the other, we shall always choose the deeper minimum as our best fit in all the subsequent calculations.

In Fig. 3, we plot the deviation of the squared Hubble parameter \(H^2/H_0^2\) from \(\Lambda\)CDM over redshift for the best fit. We note that the quantity \(H^2/H_0^2\) has a simple linear relationship with the parameters of the fit (equation 7), therefore the errors in this quantity increase with redshift. Another quantity of interest is the energy density of DE. For this Ansatz, \(\rho_{\text{DE}} = \rho_{\text{DE}}/\rho_{\text{tot}} = A_0 + A_1 x + A_2 x^2\) (where \(\rho_{\text{tot}} = 3H_0^2/8\pi G\) is the present day critical density). Fig. 4 shows the logarithmic variation of \(\rho_{\text{DE}}\) with redshift. In this figure too the errors increase with redshift. An interesting point to note is that initially, DE density decreases with redshift, showing the phantom-like nature \((w < -1)\) of DE at lower redshifts of \(z \lesssim 0.25\), while at higher redshifts the DE density begins to track the matter density.
Before moving on to the second derivative of the luminosity distance (e.g. the equation of state) we may obtain more information from the DE density by considering a weighted average of the equation:

$$1 + \bar{w} = \frac{1}{\Delta \ln(1 + z)} \int [1 + w(z)] \frac{dz}{1 + z}, \quad (16)$$

where $\Delta$ denotes the total change of the variable between integration limits. This quantity can be elegantly expressed in terms of the difference in energy densities over a range of redshift as

$$1 + \bar{w} = \frac{1}{3} \Delta \ln(\rho_{DE}/\rho_{0c}).$$

Thus the variation in the DE density depicted in Fig. 4 is very simply related to the weighted average equation of state!

In Table 1 we show the values of $\bar{w}$ obtained using different ranges in redshift for our best fit with corresponding $1\sigma$ and $2\sigma$ errors. We have taken the ranges of integration to be approximately equally spaced in $\ln(1 + z)$, with the upper limit set by the furthest SN known at present. The values of $\bar{w}$ may be calculated using equation (16) (which uses the second derivative of the luminosity distance), or they can simply be read off from Fig. 4 using equation (17). From this table, a ‘metamorphosis’ in the properties of DE occurring somewhere between $z \sim 0$ and $z \sim 1$ can be clearly seen [note that, effectively, one needs to differentiate $d_L(z)$ only once to come to this conclusion].

We now reconstruct the equation of state of DE which, for the Ansatz (7), has the form

$$w(z) = -1 + \frac{x}{3} \left[ \frac{A_1 + 2A_2}{A_0 + A_1x + A_2x^2} \right], \quad x = 1 + z. \quad (18)$$

[Note that, because $w(z)$ was derived from the Ansatz (7), its domain of validity is $z \leq \text{few}$]. In Figs 5(a), (b) and (c), we show the evolution of the equation of state $w(z)$ with redshift for different values of $\Omega_{m0}$. 

Table 1. The weighted average $\bar{w}$ (equation 16) over specified redshift ranges. The best-fit value and $1\sigma$ and $2\sigma$ deviations from the best-fit are shown.

<table>
<thead>
<tr>
<th>$\Delta z$</th>
<th>$\bar{w}$</th>
<th>$1\sigma$</th>
<th>$2\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 0.414</td>
<td>-0.969</td>
<td>+0.120</td>
<td>+0.198</td>
</tr>
<tr>
<td>0.414 - 1</td>
<td>-0.108</td>
<td>+0.230</td>
<td>+0.242</td>
</tr>
<tr>
<td>1 - 1.756</td>
<td>0.069</td>
<td>+0.100</td>
<td>+0.130</td>
</tr>
</tbody>
</table>

$\Omega_{m0} = 0.3$ is assumed.
The evolution of $w(z)$ with redshift for different values of $\Omega_{\text{om}}$. The reconstruction is done using the polynomial fit to DE, equation (7). In each panel, the thick solid line shows the best fit, the light grey contour represents the 1σ confidence level, and the dark grey contour represents the 2σ confidence level around the best fit. The dashed line represents $\Lambda$CDM. No priors are assumed on $w$. However, for larger values of the equation of state does not change significantly with change in the matter density. However, for larger values of $\Omega_{\text{om}}$, a smaller current value of $w_0 = w(z = 0)$ is preferred. In all three cases considered, the present value of the equation of state is $w_0 \lesssim -1.2$ for the best fit, and the equation of state rises steeply from $w \lesssim -1.2$ to $w \simeq 0$ with redshift. In fact, the behaviour of $w$ appears to be extremely different from that in $\Lambda$CDM ($w = -1$). We note here that, for this analysis, the errors on $w$ appear to decrease with redshift. This may appear counter-intuitive, as there are fewer SNe at higher redshifts, but this is merely a construct of the fact that $w$ depends non-linearly on the parameters of the Ansatz (see Appendix A).

Quintessence models satisfy the WEC $\rho + p = \phi^2 \geq 0$ and it would be interesting to see how the imposition of the WEC as a prior on the equation of state will affect the results of our analysis. We therefore perform the same analysis as above with the added constraint $w_0 \geq -1$ (note that this implies $w(z) \geq -1$ for all $z \geq 0$ for our fitting function of $H(z)$ provided $A_1 \geq 0$). The results are shown in Figs 6(a), (b) and (c). We see that in this case the errors are larger and the evolution of the equation of state with redshift follows a much gentler slope. Such an equation of state would be largely consistent with the cosmological constant model. [These results are in broad agreement with an earlier analysis of Saini et al. (2000) in which a smaller SNe data set was used and a different Ansatz for the luminosity distance was applied.]

The 1σ and 2σ limits are shown in each case. The $\chi^2$ per degree of freedom for the best fit for the different cases is given in Table 2. We find that for $0.2 < \Omega_{\text{om}} < 0.4$, the behaviour of the equation of state does not change significantly with change in the matter density. However, for larger values of $\Omega_{\text{om}}$, a smaller current value of $w_0 = w(z = 0)$ is preferred. In all three cases considered, the present value of the equation of state is $w_0 \lesssim -1.2$ for the best fit, and the equation of state rises steeply from $w \lesssim -1.2$ to $w \simeq 0$ with redshift. In fact, the behaviour of $w$ appears to be extremely different from that in $\Lambda$CDM ($w = -1$). We note here that, for this analysis, the errors on $w$ appear to decrease with redshift. This may appear counter-intuitive, as there are fewer SNe at higher redshifts, but this is merely a construct of the fact that $w$ depends non-linearly on the parameters of the Ansatz (see Appendix A).

Quintessence models satisfy the WEC $\rho + p = \phi^2 \geq 0$ and it would be interesting to see how the imposition of the WEC as a prior on the equation of state will affect the results of our analysis. We therefore perform the same analysis as above with the added constraint $w_0 \geq -1$ (note that this implies $w(z) \geq -1$ for all $z \geq 0$ for our fitting function of $H(z)$ provided $A_1 \geq 0$). The results are shown in Figs 6(a), (b) and (c). We see that in this case the errors are larger and the evolution of the equation of state with redshift follows a much gentler slope. Such an equation of state would be largely consistent with the cosmological constant model. [These results are in broad agreement with an earlier analysis of Saini et al. (2000) in which a smaller SNe data set was used and a different Ansatz for the luminosity distance was applied.]

The evolution of $w(z)$ with redshift for different values of $\Omega_{\text{om}}$, using the prior $w(z) \geq -1$, $z \geq 0$. The reconstruction is done using the polynomial fit to DE, equation (7). In each panel, the thick solid line shows the best fit, the light grey contour represents the 1σ confidence level, and the dark grey contour represents the 2σ confidence level around the best fit. The hatched region is forbidden by the prior $w(z) \geq -1$. The dashed line represents $\Lambda$CDM. The $\chi^2$ per degree of freedom for each case is given in Table 2.

### Table 2. $\chi^2$ per degree of freedom for best-fitting and $\Lambda$CDM models

<table>
<thead>
<tr>
<th>$\Omega_{\text{om}}$</th>
<th>$w_0$</th>
<th>$\chi^2_{\text{min}}$</th>
<th>$\chi^2_{1\sigma}$</th>
<th>$\chi^2_{2\sigma}$</th>
<th>$\chi^2_{p}$</th>
<th>$\chi^2_{1\sigma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>-1.093</td>
<td>1.0077</td>
<td>1.0213</td>
<td>1.0442</td>
<td>1.0359</td>
<td>1.1242</td>
</tr>
<tr>
<td>0.20</td>
<td>-1.198</td>
<td>1.0071</td>
<td>1.0207</td>
<td>1.0436</td>
<td>1.0384</td>
<td>1.0663</td>
</tr>
<tr>
<td>0.30</td>
<td>-1.334</td>
<td>1.0056</td>
<td>1.0192</td>
<td>1.0421</td>
<td>1.0409</td>
<td>1.0417</td>
</tr>
<tr>
<td>0.40</td>
<td>-1.470</td>
<td>1.0043</td>
<td>1.0179</td>
<td>1.0408</td>
<td>1.0578</td>
<td>1.0638</td>
</tr>
<tr>
<td>0.50</td>
<td>-1.606</td>
<td>1.0038</td>
<td>1.0174</td>
<td>1.0403</td>
<td>1.0912</td>
<td>1.1168</td>
</tr>
</tbody>
</table>

*Figures 5.* The evolution of $w(z)$ with redshift for different values of $\Omega_{\text{om}}$. The reconstruction is done using the polynomial fit to DE, equation (7). In each panel, the thick solid line shows the best fit, the light grey contour represents the 1σ confidence level, and the dark grey contour represents the 2σ confidence level around the best fit. The hatched region is forbidden by the prior $w(z) \geq -1$. The dashed line represents $\Lambda$CDM. The $\chi^2$ per degree of freedom for each case is given in Table 2.
In Table 2, we show how the $\chi^2$ for the best-fitting evolving DE models compare with that for $\Lambda$CDM. We find that $\chi^2_{\Lambda\text{CDM}} > \chi^2_{\text{best fit}}$ always. For $\Omega_{m0} = 0.3$, the value of $\chi^2_{\Lambda\text{CDM}}$ is just within 2$\sigma$ of the best-fitting $\chi^2$, but for $\Omega_{m0} = 0.2$, or $\Omega_{m0} = 0.4$, $\chi^2_{\Lambda\text{CDM}}$ is outside the 2$\sigma$ limits of the best fit. It is also noteworthy that when the prior $w(z) \geq -1$ ($z \geq 0$) is used, the best-fitting model has a slowly evolving equation of state with $w_0 = -1$ and the $\chi^2$ for the best fit becomes smaller for a smaller value of the matter density. When no priors are assumed on $w$, the trend reverses, and better fits are obtained for larger values of $\Omega_{m0}$. From this it appears that at least at $1\sigma$ the evolving DE model is favoured over $\Lambda$CDM, and it does as well, if not better at the 2$\sigma$ level, depending upon the value of the present-day matter density.

### 2.1.3 Using Priors on $\Omega_{m0}$

Instead of assuming an exact value for $\Omega_{m0}$, which is somewhat optimistic given the present observational scenario, we may use the 2dF prior on $\Omega_{m0}$ and calculate $w(z)$ as a function of $(\Omega_{m0}, A_1, A_2)$. It should be noted here that the 2dF error bars on $\Omega_{m0}$ have been calculated using 2dF data in conjunction with CMB, and this calculation assumes a $\Lambda$CDM model, therefore this prior should be used more as a benchmark for the value of $\Omega_{m0}$ rather than as an absolute when considering evolving DE models. The resultant ‘marginalised’ $w$ is shown as a function of the redshift in Fig. 7(a).

The nature of the equation of state for the analysis with the added prior $w(z) \geq -1$ ($z \geq 0$) is shown in Fig. 7(b). We find that the general nature of evolution of the equation of state is not changed by adding this extra information on the matter density. If no priors are assumed on the equation of state to begin with, $w(z)$ still rises sharply from $w_0 \lesssim -1$ up to $w \simeq 0$ at maximum redshift and the analysis appears to favour a fast-evolving equation of state of DE over the standard $\Lambda$CDM model. If a prior $w \geq -1$ is assumed, then the marginalized equation of state is more consistent with the cosmological constant. From this we see that marginalization over $\Omega_{m0}$ does not lead to any significant change in our results. In the subsequent sections, we will show our results for $\Omega_{m0} = 0.3$.

From the above analysis, we find, therefore, that our results change significantly depending upon whether or not the prior $w \geq -1$ is imposed. We saw earlier that in the absence of any prior on $w(z)$, the best-fitting equation of state rose from $w \lesssim -1$ at $z = 0$ to $w \simeq 0$ at $z \sim 1$. By imposing a prior on $w(z)$ is therefore to make the best-fitting $w(z)$ grow much more slowly with $w = -1$ being preferred at $z = 0$. Our results show that the reconstructed equation of state with the prior $w \geq -1$ is in good agreement with a cosmological constant at the 68 per cent CL. However, if no prior is imposed, then the steeply evolving DE models are favoured over the cosmological constant at $1\sigma$, and are at least as likely as the cosmological constant at the 2$\sigma$ level.

### 2.1.4 Age and deceleration parameter of the Universe

We may also use this Ansatz to calculate other quantities of interest, such as the age of the Universe, $t(z)$, and the deceleration parameter, $q(z)$:

\[
t(z) = \int_{1+z}^{\infty} \frac{dx}{xH(x^2)},
\]

(19)\[q(z) = -\frac{\dot{a}}{aH^2} \equiv \frac{H^2}{H^2}x - 1.
\]

(20)

where $x = 1 + z$.

In Fig. 8 we plot the evolution of the age of the Universe with redshift. We find that the best-fitting age of the Universe today is $t_0 = 12.8$ Gyr if the Hubble parameter is taken to be $H_0 = 72$ km s$^{-1}$ Mpc$^{-1}$, which is slightly lower than the age of a $\Lambda$CDM universe, $t_0 = 13.4$ Gyr (both values are for $\Omega_{m0} = 0.3$). At the 2$\sigma$ level, the age of the Universe today would vary between $11.2 \lesssim t_0 \lesssim 13.6$ Gyr.

Fig. 10, later, shows the evolution of the deceleration parameter with redshift. We find that the behaviour of the deceleration parameter for the best-fitting universe is quite different from that in $\Lambda$CDM cosmology. Thus, the current value of $q_0 \simeq -0.9$ is significantly lower than $q_0 \simeq -0.55$ for $\Lambda$CDM (assuming $\Omega_{m0} = 0.3$). Furthermore, the rise of $q(z)$ with redshift is much steeper in the case of the best-fitting model, with the result that the Universe begins to accelerate at a comparatively lower redshift $z \simeq 0.3$ (compared with...
z ≃ 0.7 for ΛCDM and the matter dominated regime (q ≃ 1/2) is reached by z ≃ 1.

2.1.5 Using priors on age of the Universe

Important consistency checks on our best-fitting universe may be provided by observations of the age of the Universe. Unfortunately, estimates of the age of the Universe from different methods can produce widely varying results one reason for which is that estimates of the Hubble parameter itself can vary significantly. For instance, the HST key project yields $H_0 = 72 \pm 8\,\text{km\,s}^{-1}\text{Mpc}^{-1}$, while studies of the Sunyaev–Zel’dovich effect in galaxy clusters give a significantly lower value $H_0 = 60 \pm 10\,\text{km\,s}^{-1}\text{Mpc}^{-1}$ (Krauss 2001). Estimates of the ages of the oldest globular clusters suggest $t_0 = 12.5 \pm 2.5\,\text{Gyr}$, at the 95 per cent confidence level (Krauss 2001; Krauss & Chaboyer 2001; Gnedin, Lahav & Rees 2001; Hansen et al. 2002; Gratton et al. 2003; De Marchi et al. 2004) and this age estimate is consistent with several other measurements including observations of eclipsing spectroscopic binaries (Thompson et al. 2001; Chaboyer & Krauss 2002), results from radioactive dating of a metal-poor star (Cayrel et al. 2001) and WMAP data (Spergel et al. 2003) (see also Alcaniz, Jain & Dev 2002). The results from the WMAP experiment suggest $t_0 = 13.4 \pm 0.3\,\text{Gyr}$ with a Hubble parameter $H_0 = 72 \pm 5\,\text{km\,s}^{-1}\text{Mpc}^{-1}$, for ΛCDM cosmology (which satisfies the WEC). Adding SDSS and SNe Ia data to WMAP, Tegmark et al. (2004) find an age of $t_0 = 14.1_{-0.9}^{+1.0}\,\text{Gyr}$ for a slightly closed ΛCDM universe with $H_0 = 66_{-2}^{+3}\,\text{km\,s}^{-1}\text{Mpc}^{-1}$. Although these results cannot be carried over to evolving DE models including those implied by our best-fitting reconstruction (which violate the WEC), they provide an indication of the range within which the age of the Universe might vary. Keeping in mind these various results, we use two different priors on the Hubble parameter: $H_0 = 72 \pm 8\,\text{km\,s}^{-1}\text{Mpc}^{-1}$ (1σ bound from HST; Freedman et al. 2001), and $H_0 = 66 \pm 6\,\text{km\,s}^{-1}\text{Mpc}^{-1}$ (approximate bound from WMAP, SDSS, SNe Ia; Tegmark et al. 2004). For each case, we choose three different Gaussian priors on the present age of the Universe: $t_0 = 13 \pm 1, 14 \pm 1$ and $15 \pm 1\,\text{Gyr}$, respectively, and perform the reconstruction for a $\Omega_{\text{m}} = 0.3$ universe. The results are shown in Fig. 9. We find that, for a Hubble parameter of $H_0 = 72 \pm 8\,\text{km\,s}^{-1}\text{Mpc}^{-1}$, and with an additional prior on the age of the Universe, $t_0 = 13 \pm 1\,\text{Gyr}$, the best fit remains nearly the same, showing a rapid evolution of the equation of state from $w \sim 0$ at $z \sim 1$ to $w \sim -1.2$ at $z = 0$, and the errors become narrower. As the age is increased, the best-fitting equation of state evolves more slowly, and the $\chi^2_{\text{data}}$ also increases (see Table 3). For the prior $H_0 = 66 \pm 6\,\text{km\,s}^{-1}\text{Mpc}^{-1}$, we find that the lowest $\chi^2_{\text{data}}$ is obtained for the age prior of $t_0 = 14 \pm 1\,\text{Gyr}$, which once again matches our best fit. It should be noted that the errors are smaller in all cases, even though the $\chi^2$ may be larger.

We must remember that the addition of a new prior which is consistent with the underlying data set would lead to a natural reduction in errors. However, the addition of a prior inconsistent with the data set would lead to a shift of the likelihood maximum as well as a reduction in errors, and the results would then fail to reflect the actual information present in the data set. That is this is happening here for the higher values of age can be seen from the fact that although the errors are reduced, the $\chi^2_{\text{data}}$ is actually larger. Therefore priors from other observations should be added prudently to ensure that they do not lead to incorrect representation of the data. Because there is as yet no clear model-independent consensus on the age of the Universe, the results we obtain in this section should be interpreted with a degree of caution.

Fig. 10 shows the evolution of the deceleration parameter with redshift. We find that the behaviour of the deceleration parameter for the best-fitting universe is quite different from that in ΛCDM cosmology. Thus, the current value of $q_0 \approx -0.9$ is significantly lower than $q_0 \approx -0.55$ for ΛCDM (assuming $\Omega_{\text{m}} = 0.3$). Furthermore the rise of $q(z)$ with redshift is much steeper in the case of the best-fit model, with the result that the Universe begins to accelerate at a comparatively lower redshift $z \approx 0.3$ (compared with $z \approx 0.7$ for ΛCDM) and the matter dominated regime ($q \approx 1/2$) is reached by $z \approx 1$.

2.2 Robustness of results

Based on the above analysis, it is tempting to conclude that the dominant component of the Universe today is DE with a steeply evolving equation of state which marginally violates the weak energy condition. (Of course, the less radical possibility of weakly time dependent DE satisfying the weak energy condition remains an alternative, too.) However, before any such dramatic claims are made, we need to check if our results are in any fashion a consequence of inherent bias in the statistical analysis itself, or in the sampling of the data. We therefore perform the following simple exercises to satisfy ourselves of the robustness of our results.

2.2.1 Using different subsets of SN data

In an attempt to understand how the nature of the reconstructed equation of state is dependent on the distribution of data, we perform the reconstruction exercise on different samples of data. We have confined ourselves to the case where $\Omega_{\text{m}} = 0.3$ for these exercises. First, we may exclude the SCP data points from the 172 SNe primary fit, leading to a subsample of 130 SNe. We call this the HZT sample. Figs 11(a) and 12(a) show the result of performing the analysis on this subsample without any constraints. The $\chi^2$ per degree of freedom for the best fit is $\chi^2_{\text{HZT}} = 0.9707$, which is lower than $\chi^2_{\text{CDM}} = 0.9939$ for this sample. In this case we find that, though the error bars are slightly larger, overall the DE density behaves in
Is there SN evidence for DE metamorphosis?

Figure 9. The evolution of $w(z)$ with redshift for $\Omega_{m0} = 0.3$ using a Gaussian prior on the age of the Universe today: $t_0 = 13 \pm 1$ Gyr (panels a and d), $t_0 = 14 \pm 1$ Gyr (panels b and e), and $t_0 = 15 \pm 1$ Gyr (panels c and f). The reconstruction is done using the polynomial fit to DE, equation (7). The top panel is obtained using a Gaussian prior $H_0 = 72 \pm 8$ km s$^{-1}$ Mpc$^{-1}$, while for the bottom panel, $H_0 = 66 \pm 6$ km s$^{-1}$ Mpc$^{-1}$. No priors are assumed on $w(z)$. In each panel, the thick solid line shows the best fit, the light grey contour represents the 1$\sigma$ confidence level, and the dark grey contour represents the 2$\sigma$ confidence level around the best fit. The dashed line represents $\Lambda$CDM. The $\chi^2$ per degree of freedom for each case is given in Table 3.

Table 3. $\chi^2$ per degree of freedom for the Ansatz (7) which best fits the SNe data after different age priors are imposed. $w_0$ is the present value of the equation of state of DE in best-fitting models.

<table>
<thead>
<tr>
<th>$H_0$ km s$^{-1}$ Mpc$^{-1}$</th>
<th>$t_0$ Gyr</th>
<th>$w_0$</th>
<th>$\chi^2_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>72 $\pm$ 8</td>
<td>13 $\pm$ 1</td>
<td>1.271</td>
<td>1.0062</td>
</tr>
<tr>
<td>72 $\pm$ 8</td>
<td>14 $\pm$ 1</td>
<td>1.099</td>
<td>1.0197</td>
</tr>
<tr>
<td>72 $\pm$ 8</td>
<td>15 $\pm$ 1</td>
<td>0.904</td>
<td>1.0407</td>
</tr>
<tr>
<td>66 $\pm$ 6</td>
<td>13 $\pm$ 1</td>
<td>1.553</td>
<td>1.0145</td>
</tr>
<tr>
<td>66 $\pm$ 6</td>
<td>14 $\pm$ 1</td>
<td>1.324</td>
<td>1.0057</td>
</tr>
<tr>
<td>66 $\pm$ 6</td>
<td>15 $\pm$ 1</td>
<td>1.153</td>
<td>1.0139</td>
</tr>
</tbody>
</table>

The crucial question of course is whether the reconstructed equation of state of DE depends upon the Ansatz which is used in the exercise, i.e. whether the behaviour of the equation of state merely...
reflects a bias in the Ansatz itself. In this section we show how the Ansatz performs in recovering DE models whose equation of state is known, from simulated data. This Ansatz was demonstrated to work extremely well when simulations of SNAP data were used (Alam et al. 2003). However, simulation of SNAP-like data is an optimistic exercise, since data of this quality is unlikely to be available in the near future. We now demonstrate the accuracy with which the Ansatz can recover the fiducial background cosmological model if data is simulated using present-day observational standards. In Figs 14(a), (b) and (c), we show how well the Ansatz recovers the equation of state for three fiducial models (assuming \( \Omega_m = 0.3 \)):

(i) a quiessence DE model with a constant equation of state: \( w = -0.5 \),

(ii) a generalized Chaplygin gas model with \( p = A/\rho^\alpha \) : with \( \alpha = 0.5 \) and the present-day equation of state \( w_0 = -0.9 \), which would

give rise to an effective equation of state

\[
    w(z) = w_0 + \frac{|w_0|}{1 + |w_0|(1 + z)^{(1+w_1)/|w_1|}},
\]

and

(iii) a model with a linearly evolving equation of state: \( w(z) = w_0 + w_1 z \), with \( w_0 = -1.2 \), \( w_1 = 1 \).

(For DE models with \( w = -1, -2/3, -1/3 \) the Ansatz is exact, therefore we don’t show the results for these cases.)

We find that in all three cases, the fiducial model lies within the 68 per cent confidence limits around the best-fit \( w(z) \). Based on this result, we claim that within the 1σ error bars, the reconstructed equation of state represents the true properties of DE when we use real data.

2.2.3 Using other Ansatz

It is also important to check whether the results of our reconstruction can be replicated using other Ansatz such as fits to the luminosity distance or the equation of state. Many different fits have been suggested in the literature (see for example Huterer & Turner 1999; Saini et al. 2000; Weller & Albrecht 2002; Gerke & Efstathiou 2002). Here we choose the fit suggested in Linder (2003) in which the equation of state of DE is expanded as

\[
    w(z) = w_0 + \frac{w_1 z}{1 + z},
\]

The luminosity distance can therefore be expressed as

\[
    d_L(z) = \frac{c}{H_0} \int_0^{1 + z} \frac{dx}{\sqrt{\Omega_m x^2 + \Omega_X}},
\]

where \( \Omega_X = \left(1 - \Omega_m\right) x^{3(1+w_0+w_1)} \exp[3w_1(1/x - 1)] \).

We find that for this fit, the errors in the equation of state get larger with redshift, however this fit too demonstrates that the equation of state of DE increases rapidly with redshift (Fig. 15a) when no priors are assumed on the equation of state (EOS). The \( \chi^2 \) per degree of freedom at the best fit is \( \chi^2_{\text{df}} = 1.0298 \). When the prior \( w(z) \geq -1 (z \geq 0) \) is invoked, the best-fitting EOS remains very close to the ΛCDM model (Fig. 15b). Therefore, from this Ansatz, we may make the statement that at low redshifts, the equation of state of
Is there SN evidence for DE metamorphosis?

Figure 12. The evolution of $w(z)$ with redshift for $\Omega_m = 0.3$, using (a) HZT data, and (b) SCP data. The reconstruction is done using the polynomial fit to DE, equation (7). No priors are assumed on $w(z)$. In both panels, the thick solid line shows the best fit, the light grey contour represents the $1\sigma$ confidence level, and the dark grey contour represents the $2\sigma$ confidence level around the best fit. The horizontal dashed line represents $\Lambda$CDM.

Figure 13. The evolution of $w(z)$ with redshift for $\Omega_m = 0.3$, with the prior $w(z) \geq -1$, $z \geq 0$, using (a) HZT data, and (b) SCP data. The reconstruction is done using the polynomial fit to DE, equation (7). In both panels, the thick solid line shows the best fit, the light grey contour represents the $1\sigma$ confidence level, and the dark grey contour represents the $2\sigma$ confidence level around the best fit. The hatched region is forbidden by the prior $w(z) \geq -1$. The horizontal dashed line represents $\Lambda$CDM.

Figure 14. Reconstructed equation of state, $w(z)$, for simulated data corresponding to three fiducial DE models: (a) quiescence with $w = -0.5$, (b) generalised Chaplygin gas: $p = A/\rho^\alpha$, with $\alpha = 0.5$, $w_0 = -0.9$, and (c) $w = w_0 + w_1 \, z$ with $w_0 = -1.2$, $w_1 = 1.0$. $\Omega_m = 0.3$ is assumed and the reconstruction is done using the polynomial fit to DE, equation (7). In each panel, the thick solid line is the best fit, the dashed line represents the exact model value, and the light grey contour represents the $1\sigma$ confidence level around the best fit.
DE shows the same signs of rising steeply with redshift if no priors are assumed on the equation of state, thus supporting our earlier results. The large errors in the equation of state at redshifts of \( z \gtrsim 0.5 \) however make it difficult to make any definitive statements about the behaviour of DE at high redshifts.

A limitation of the fit (22) is that it is unable to describe very rapid variations in the equation of state. An Ansatz which accommodates this possibility has been suggested in Bassett et al. (2002)

\[
w(z) = w_i + \frac{w_f - w_i}{1 + \exp[(z - z_t)/\Delta]},
\]

where \( w_i \) is the initial equation of state at high redshifts, \( z_t \) is a transition redshift at which the equation of state falls to \( w(z_t) = (w_i + w_f)/2 \) and \( \Delta \) describes the rate of change of \( w(z) \).

The resulting luminosity distance has the form:

\[
d_L(z) = \frac{c}{H_0} \int_1^{1+z} \frac{dx}{\sqrt{\Omega_m x^2 + \Omega_X}},
\]

where \( \Omega_X = (1 - \Omega_m) \exp[3 \int_0^{z_t-1} (1 + w(z)) dz/(1 + z)] \).

The results for the analysis using this fit to the equation of state are shown in Fig. 15. We find that when the reconstruction is done without any priors on the equation of state (Fig. 15a), the best fit is remarkably close to the result for Ansatz (7) (Fig. 5b). The \( \chi^2 \) per degree of freedom at the minimum is \( \chi^2_{\text{min}} = 1.0175 \) for this fit. The errors in this case are somewhat larger, especially at high redshift.

If we constrain \( w(z) \geq -1 \), then as before, the evolution of the equation of state is much slower (Fig. 15b). So the reconstruction using this Ansatz appears to confirm our earlier results.

The above exercises lead us to conclude that our results are neither dependent on the nature of the statistical analysis nor on the manner in which the SNe data is sampled. It therefore appears that DE with a steeply evolving equation of state provides a compelling alternative to a cosmological constant if data are analysed in a prior-free manner and the weak energy condition is not imposed by hand.

### 2.3 Reconstructing DE using a new SN sample

As this paper was nearing completion, a new data set consisting of 23 Type Ia SNe was released by the HZT team (Barris et al. 2003). It is clearly important to check whether or not these new data points corroborate the findings reported in the previous sections. Accordingly, we use a subset of 200 Type Ia SNe with \( A_V \leq 0.5 \)
from the 230 SNe sample of Tonry et al. (2003), and 22 SNe with
$AV \leq 0.5$ from the new sample to obtain a best fit for our
Ansatz with $\Omega_{\text{tot}} = 0.3$. We then plot the magnitude deviation of our best-
fitting universe from an empty universe with ($\Omega_{\text{tot}}, \Omega_{\Lambda}) = (0.0, 0.0)$ in order to illustrate how well our model fits the data (Fig. 17).
For clarity, we plot the median values of the data points. We obtain
medians in redshift bins by requiring that each bin has a width of
at least 0.25 in $\log z$ and contain at least 20 SNe. For comparison,
we also plot an $\Lambda$CDM ($\Omega_{\text{tot}}, \Omega_{\Lambda}) = (0.3, 0.7)$ model, as well
as open cold dark matter (OCDM) and standard cold dark matter
(SCDM) models. From Fig. 17 we see that our DE reconstruction is
a much better fit to SNe beyond $z \sim 0.8$ than $\Lambda$CDM. At low
redshifts ($z \sim 0.1$) the agreement between data and the two models is rather marginal. We now add 22 of the new SNe (rejecting one
with $AV > 0.5$) to our existing data set of 172 SNe and perform
DE reconstruction on this new data set of 194 SNe, assuming $\Omega_{\text{tot}} = 0.3$ and no other priors. The resultant Fig. 18 is similar to Fig. 5(b),
with slightly smaller errors and has a best-fitting $\chi^2_{\text{red}} = 1.015$. The
above exercises point to the robustness of results reported in previous
sections, and indicate that evolving DE agrees well with the full data
set containing 194 Type Ia SNe.

3 THE ACCELERATING UNIVERSE
AND THE ENERGY CONDITIONS

The energy conditions

(i) strong energy condition: $\rho + 3p \geq 0$ (SEC),
(ii) weak energy condition: $\rho \geq 0, \rho + p \geq 0$ (WEC)

play a vitally important role in our understanding of the accelerating
universe, both in the context of inflation and DE. We therefore
consider it worthwhile to review certain key developments which
deepened our understanding of these issues.

In an expanding FRW universe the SEC implies that the Universe
decelerates while the WEC forbids the pressure from becoming too
negative. Additionally, in the 1960s and early 1970s it was noted
that energy conditions play a crucial role in the formulation of the
singularity theorems in cosmology. Indeed, one of the necessary
conditions for the existence of an initial/final singularity in big bang
cosmology is that matter satisfies both the SEC and WEC (Hawking
& Ellis 1973).

By the late 1970s it became clear that not all forms of matter
satisfy the energy conditions. Perhaps the best example of a form
of matter which satisfied the weak energy condition but violated the
strong one is the cosmological constant, introduced into cosmology by
Einstein in 1917. In addition, the vacuum expectation value of
the energy momentum tensor, $\langle T_{\mu\nu}\rangle_{\text{vac}}$, which describes quantum ef-
fects (particle production and vacuum polarization) in an expanding
universe could, in certain cases, violate both WEC and SEC (Birrell
& Davies 1982; Grib, Mamaev & Mostepanenko 1980). For certain
space-times, such as de Sitter space, the vacuum energy momentum
tensor generates a cosmological constant since $\langle T_{\mu\nu}\rangle_{\text{vac}} = \Lambda g_{\mu\nu}$. Thus
by the late 1970s it was well known that neither of the energy
conditions could be held as being sacrosanct.\footnote{The importance of quantum effects to DE model building has been empha-
sized in Sahni & Habib (1998) and Parker & Raval (1999a,b).}

The 1980s, as we all know, led to great advances in the devel-
opment of the inflationary paradigm. The inflaton field mimics the
behaviour of a cosmological constant over sufficiently small in-
tervals of time and therefore violates the SEC. Early DE models
were based on inflaton-type scalars which coupled minimally to
gravity (quintessence). Quintessence violates the SEC but respects
the WEC. Precisely because of the latter property, not \textit{any}
experimentally obtained $d_L(z)$ is compatible with quintessence, as
emphasized in Sahni & Starobinsky (2000). [The same observation
holds for $H(z)$, as the latter can be derived from $d_L(z)$ using equa-
tion (3).] Clearly if observations do indicate that the WEC is vi-
olated by DE then more general (WEC-violating) models for DE
should be seriously considered. One example of WEC-violating
DE is provided by scalar–tensor gravity. Scalar–tensor models con-
tain at least two functions of the scalar field (dilaton) describ-
ing DE. As shown in Boisseau et al. (2000), these two functions,
namely, the scalar field potential and its coupling to the Ricci
scalar $R$, are sufficiently general to explain any $H(z)$ obtained from observations.

The WEC can also be effectively violated in DE models constructed in Braneworld cosmology. It has recently been shown that such models, with $w_{\text{eff}} < -1$, are in excellent agreement with SN data (Alam & Sahni 2002). As the field equations in these models are derived from a higher dimensional Lagrangian, the unusually rapid acceleration of the four-dimensional universe arises because of the full five-dimensional theory and not because of matter which continues to satisfy the energy conditions and whose density remained finite and well behaved at all times (Sahni & Shtanov 2003). This behaviour is in contrast to phantom, which assumes a conventional ‘perfect fluid’ form for the energy-momentum tensor and therefore contains pathological features such as an energy density which diverges in the future and a sound speed which is faster than that of light (Starobinsky 2000; Caldwell 2002; Sahni & Shtanov 2003).

The fact that the observed luminosity distance (derived from SN observations) is better fitted by DE violating the WEC than either quintessence or a cosmological constant was first noticed by Caldwell (2002). Caldwell called this ‘phantom energy’ and showed that larger values of $\Omega_{\text{DE}}$ ($\Omega_{\text{DE}} \geq 0.2$) implied increasingly more negative values for the equation of state ($w \lesssim -1$) of phantom.6 Caldwell’s results have since been confirmed by larger and better-quality SNe data sets – for instance Knop et al. (2003) find that, in the absence of priors being placed on $\Omega_{\text{DE}}$, the DE equation of state has a 99 per cent confidence limit of being $< -1!$ Both Caldwell (2002) and Knop et al. (2003), however, work under the assumption that the equation of state of DE is unevolving, so that $w = \text{constant}.$

In this paper we have shown that suspending the WEC prior and allowing the DE equation of state to evolve brings out dramatically new properties of DE. Thus the DE model which best fits the SNe observations has an equation of state which rapidly evolves from $w(z) \lesssim -1$ at present ($z = 0$) to $w(z) \simeq 0$ at $z \sim 1$. DE therefore appears to have properties which interolate between those of dark matter (dust) at early times and those of a ‘phantom’ ($w \lesssim -1$) at late times.

4 CONCLUSIONS AND DISCUSSION

This paper reports the model independent reconstruction of the cosmic equation of state of DE in which no priors are imposed on $w(z)$. In the literature the imposition of various priors frequently precedes the analysis of observational data sets. Such a procedure is well founded and entirely justified when priors are dictated by complementary information such as orthogonal observations coming from different data sets. However, on occasion the use of priors is justified on ‘theoretical grounds’ and in this case one must be careful so as not to prejudice nature. (Compelling theoretical reasons might well reflect our own particular conditioning or set of prejudices!) In the case of the analysis of Type Ia SN data, the priors most frequently used have been $w = \text{constant}$ and $w \geq -1$. Both confine DE to within a narrow class of models. Moreover, as shown in Maor et al. (2002), the imposition of such priors on the cosmic equation of state can, on occasion, lead to gross misrepresentations of reality.

In this paper we do not impose any priors on $w(z)$ and reconstruct the equation of state of DE in a model independent manner. In this case our best fit $w(z)$ evolves from $w \lesssim -1$ at $z = 0$, to $w \simeq 0$ at $0.8 \lesssim z \lesssim 1.75$ (the upper limit is set by observations). This result is robust to changes in the value of $\Omega_{\text{DE}}$ and remains in place within the broad interval $0.1 \leq \Omega_{\text{DE}} \leq 0.5$. Our reconstruction clearly favours a model of DE whose equation of state metamorphoses from $w = 0$ in the past to $w \simeq -1$ today. An excellent example of a model which has this property is the Chaplygin gas (Kamenshchik et al. 2001). However, in this model DE does not violate the weak energy condition (if it was not already violated initially). Our results also lend support to the DE models discussed in Bassett et al. (2002), Corasaniti et al. (2003) in which the DE equation of state shows a late-time phase transition. An interesting example of an evolving DE model in which $w(0) < -1$ at present whereas $w(z) > -1$ at earlier times is provided by the Braneworld models (called BRANE1) examined in Sahni & Shtanov (2003) which have been shown to agree very well with current SN observations (Alam & Sahni 2002).

It is also conceivable that the observed rapid growth in the EOS might characterize ‘unified’ models of dark matter (DM) and DE (DE). We end this paper with a small speculation on this last possibility. Because the nature of both DM and DE is currently unknown, it may be that a mechanism exists which converts DM (with $w = 0$) into DE (with $w \simeq -1$) in regions with sufficiently high density contrast $\delta \rho/\rho \gg 1$. (This would happen if, for instance, the rate of conversion of DM into DE depended upon $(\delta \rho/\rho)^x$, $x \gg 1$, etc.) As the conversion of DM to DE is confined to high peaks of the density field this process will not occur uniformly in the entire Universe but will be restricted to regions occupying a small filling fraction $\langle FF \rangle$ [$\langle FF \rangle \geq 1$ for regions with $\delta \rho/\rho \gg 1$; see for instance Sheth et al. (2003) and references therein]. This process could commence as early as $z \sim 10–20$ when the first peaks in a CDM model collapse. Because DE does not cluster and because $\rho_{\text{DM}}/\rho_{\text{DE}}$ grows rapidly as the Universe expands, DE from high density regions ($\langle FF \rangle \geq 1$) will spread at the speed of light, percolating through the entire Universe ($\langle FF \rangle \sim 1$) by $z \sim 1$. As the creation of DE is tagged to the formation of structure, this model may not encounter the ‘coincidence problem’ which plagues other scenarios of DE including quintessence. (However, this model might have problems in producing a sufficiently homogeneous and isotropic distribution of DE on the largest scales.) The concrete mathematical framework for a phenomenological model of this kind will be worked out in a companion paper.

In summary, evolving DE models have been shown to satisfy SNe observations just as well (if not better) than the cosmological constant. Our best-fitting equation of state, in the absence of any priors, evolves from $w(z) \lesssim -1$ at $z = 0$ to $w(z) \simeq 0$ at $z \sim 1$. Indeed, Fig. 17 shows that our best fit EOS is better able to account for the relative brightness of SNe at $z \geq 0.8$ than LCDM. However, the evolution in $w(z)$ is much weaker if the prior $w(z) \geq -1 (z \geq 0)$ is imposed. Due to the paucity of SNe data beyond $z = 1.2$ (to date, there is only a single data point beyond $z = 1.2$, SN1999bb at $z = 1.75$) it is not clear whether $w(z) \simeq 0$ is a stable asymptotic value for the reconstructed DE equation of state at high redshifts.7 New SN data at $z \geq 1$ from ongoing as well as planned surveys (SNAP), combined with data from other cosmology experiments (CMB, LSS, S-Z survey’s, lensing, etc.), are bound to provide important insights.

6 Note, however, that we will not consider the theoretical model of phantom matter based on a ghost scalar field proposed in this paper because, as is well known, it is unstable with respect to particle creation (particle + antiparticle of the ghost scalar field plus particle + antiparticle of all usual matter fields) and to the loss of spatial homogeneity at both quantum and non-linear classical levels.

7 An alternative explanation for the relative brightness of SNe at these redshifts, say, by gravitational lensing (Barber et al. 2000) could clearly alter the high-$z$ properties of our best fit.
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APPENDIX A: PROPAGATION OF ERRORS

We have seen that the error bars on w(z) for the analysis using Ansatz (7) are non-monotonic with redshift. Low-redshift behaviour of the equation of state affects the luminosity distance at all higher redshifts. High-redshift behaviour effects fewer such
distances. This leads to an expectation that high-$z$ behaviour of the
equation of state should be poorly constrained as opposed to the
low-$z$ behaviour. This seems to contradict the behaviour seen in our
figures. To investigate if this could be explained by our specific
method of error analysis we describe the Fisher matrix error bars
below and show that they are almost identical to what we obtain in
our method.

In an analysis which uses an Ansatz with $n$ parameters $p_i$, the
Fisher information matrix is defined to be

$$F_{ij} = \left\langle \frac{\partial^2 L}{\partial p_i \partial p_j} \right\rangle,$$

(A1)

where $L = -\log L$, $L$ being the likelihood. For an unbiased es-
imator, the errors on the parameters will follow the Cramér–Rao
inequality: $\Delta p_i \geq 1/\sqrt{F_{ii}}$.

Because the likelihood function is approximately Gaussian near the
maximum likelihood (ML) point, the covariance matrix for a
maximum likelihood estimator is given by

$$(C^{-1})_{ij} \equiv \frac{\partial^2 L}{\partial p_i \partial p_j}.$$

(A2)

The Fisher information matrix is therefore simply the expectation
value of the inverse of the covariance matrix at the ML-point.

Given the covariance matrix, the error on any cosmological quan-
tity $Q(p_i)$ is given by:

$$\sigma_Q^2 = \sum_{i=1}^{n} \left( \frac{\partial Q}{\partial p_i} \right)^2 C_{ii} + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left( \frac{\partial Q}{\partial p_i} \right) \left( \frac{\partial Q}{\partial p_j} \right) C_{ij}.$$

(A3)

Thus the nature of the errors on a quantity will depend essentially
on the manner in which it is related to the parameters of the system.

Figure A1. The deviation of $H^2/H_0^2$ from corresponding lCDM values
over redshift for the Ansatz (7). The thick solid line shows the best fit and
the light grey contour represents the $1\sigma$ confidence level around the best fit.
The dashed horizontal line denotes lCDM. $\Omega_{0m} = 0.3$ is assumed.

We now consider how errors propagate for different cosmological
quantities for the polynomial fit to DE which we have used for most
of the results in this paper:

$$H^2/H_0^2 = \Omega_{0m}(1 + x)^3 + A_0 + A_1 x + A_2 x^2, \quad x = 1 + z,$$

(A4)

where $A_1 = 1 - \Omega_{0m} - A_1 - A_2$. If $\Omega_{0m}$ is held constant then the
parameters of the system are $(A_1, A_2)$.

We obtain the covariance matrix in $(A_1, A_2)$ from the ML analysis,
and then using equation (A3), calculate the errors on cosmological
quantities of interest. For example, the errors on the quantity $\Delta H^2 = (H^2 - H_{\text{CDM}}^2)/H_0^2$ are given by:

$$\sigma_{\Delta H^2}^2 = (x - 1)^2 \left[ C_{11} + 2(x + 1)C_{12} + (x + 1)^2 C_{22} \right].$$

(A5)

Although the term $C_{12}$ is negative we find that $\sigma_{\Delta H^2}^2$ still in-
creases with redshift. This is shown in Fig. A1. The errors shown are
approximately similar to those obtained in Fig. 3.

The corresponding errors on the equation of state can be calcu-
lated using equations (18) and (A3), and has the somewhat more
complicated expression:

$$\sigma_w^2(x) = \frac{x^2 \left[ f_1 C_{11} + 2 f_1 f_2 C_{12} + f_2 C_{22} \right]}{9(1 - \Omega_{0m} + A_1 (x - 1) + A_2 (x^2 - 1))^2},$$

(A6)

where

$$f_1 = 1 - \Omega_{0m} - A_2 (x - 1)^2,$$

$$f_2 = 2x(1 - \Omega_{0m}) + A_1 (x - 1)^2,$$

and $A_1$, $A_2$ are the mean values of the parameters. Although in
this case it is difficult to predict the behaviour of error bars, after
substituting the numerical values we obtain the error bars that are
shown in Fig. A2. This figure can be compared to Fig. 5(b), having
almost identical errors.

Figure A2. The variation of the equation of state of DE $w(z)$ over redshift
for the Ansatz (7). The thick solid line shows the best fit and the light grey
contour represents the $1\sigma$ confidence level around the best fit. The dashed
horizontal line denotes lCDM. $\Omega_{0m} = 0.3$ is assumed.

This shows that the nature of our error bars is not an artifact of
our specific method of error analysis. However, as shown in Fig. 15,
a two-parameter expansion in $w(z)$ shows monotonically deterio-
rating errors in $w(z)$ with the redshift, while the expansion in $H_2(z)$
shows errors that improve with redshift (Fig. 5b). This indicates that
the nature of error bars might be affected by which quantity is be-
ing approximated. In the limit of infinite terms in the expansion of
various quantities all the methods should produce identical result.
The practical need for truncating these expansions make these ap-
proximations slightly different from each other. More specifically,
we require setting of priors

$$f(z) = \sum_{n=0}^{\infty} a_n z^n,$$

(A7)

$$a_n = 0; \quad (n > N_p),$$

(A8)

where $f(z)$ could be $H(z)$, $w(z)$ or any other physical quantity
and $N_p$ is the chosen number of parameters. The non-linear priors

in the above equation make different finite expansions inequivalent. Because we do not know for certain if the underlying model for the accelerating expansion involves an energy component with negative pressure in a FRW setting, we are forced to choose one of the alternatives for approximations. We hope that with increasingly high quality data the effect of such truncations will eventually disappear.

APPENDIX B: RECONSTRUCTION OF OTHER DARK ENERGY MODELS

We have seen in Fig. 1 that the Ansatz (7) works well for several physically motivated models of quintessence, Chaplygin gas and SUGRA. In this section we take this exercise further and see how well it can reconstruct some of the other fits to DE known in literature. In Figs B1(a) and (b), we show results for simulations using $\Omega_{m0} = 0.3$ and two different fits to the equation of state of DE, as follows.

(i) The fit suggested in Linder (2003): $w(z) = w_0 + w_1 z/(1 + z)$. For this we consider three sets of values in order of increasing evolution of $w(z)$: (a) $w_0 = -0.8, w_1 = 0.5$, (b) $w_0 = -1.0, w_1 = 1.0$, and (c) $w_0 = -1.2, w_1 = 2.0$.

(ii) The non-perturbative $w(z)$ suggested in Corasaniti & Copeland (2003) and Corasaniti et al. (2003), which has the parameters $w_0^Q$ (the DE equation of state today), $w_m^Q$ (the DE equation of state at the matter dominated epoch), $z_c$ (the redshift where equation of state changes from $w_m^Q$ to $w_0^Q$), and $\Delta$ (the width of transition). For the simulation we again use three sets of values in order of increasing growth rate of $w(z)$: (a) $w_0^Q = -0.8, w_m^Q = -0.5, z_c = 1.5, \Delta = 0.1$, (b) $w_0^Q = -1.0, w_m^Q = -0.2, z_c = 0.6, \Delta = 0.07$, and (c) $w_0^Q = -1.2, w_m^Q = 0.1, z_c = 0.11, \Delta = 0.03$.

We find that in both cases, the Ansatz recovers the measured quantity to within 0.5 per cent accuracy in the redshift range important for SNe observations. Thus we find that even for fits for which the Ansatz does not return exact values, it can recover cosmological quantities to a high degree of accuracy.

Figure B1. The deviation $\Delta \log(d_L H(z))/\log(d_L H_0)$ between actual value and that calculated using the Ansatz (7) over redshift for different values of parameters for (a) the Linder fit, and (b) the Corasaniti fit for equation of state of DE, with $\Omega_{m0} = 0.3$. In panel (a), the solid line shows the deviation for the Linder fit with $w_0 = -0.8, w_1 = 0.5$, the dotted line for $w_0 = -1.0, w_1 = 1.0$, and the dot-dashed line for $w_0 = -1.2, w_1 = 2.0$. In panel (b), the solid line represents the Corasaniti fit with $w_0^Q = -0.8, w_m^Q = -0.5, z_c = 1.5, \Delta = 0.1$, the dotted line shows $w_0^Q = -1, w_m^Q = -0.2, z_c = 0.6, \Delta = 0.07$, and the dot-dashed line, $w_0^Q = -1.2, w_m^Q = 0.1, z_c = 0.11, \Delta = 0.03$. The dashed horizontal line in both panels represents zero deviation from model values, which is true for $\Lambda$CDM, and $w = -1/3, w = -2/3$ quiessence models.

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