$$K = \overline{K} p_0 LD/C_2 = 1.5 \text{ MN/m}$$

Leakage flow $m = \pi DC\rho V = 1/2\pi\mu D\text{Re}$ from which m = 17.2 g/s.

For the tapered seal

$$\overline{K} = 0.095$$

$$\text{Re}/\text{Re}_0 = 0.438$$

which yield the dimensional quantities

$$K = 9.5 \text{ MN/m}$$

$$\text{m} = 22.1 \text{ g/s}$$

The optimum clearance ratio is $H = 1.82$.

Thus, for this case use of a tapered seal would increase stiffness over five times while increasing leakage less than a third. As previously mentioned, when stiffer seals are used, it may be possible to reduce the seal clearance because of the better control of seal and rotor motions. Thus, tapered seals may actually offer a means to reduce leakage.

Concluding Remarks

An analysis has been presented to calculate the radial stiffness of annular gas path seals. For most operating conditions a tapered seal, whose clearance decreases in the flow direction, has a stiffness much greater than that of a straight seal, with only a modest increase in leakage. With a straight seal, negative stiffness sometimes occurs; use of a tapered seal completely eliminates this undesirable situation.

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DISCUSSION

A. O. Lebeck[1]

The author has made a useful and interesting contribution to the literature on annular gas path seals. The results should be particularly helpful to seal designers faced with minimizing rubbing of such seals.

In a practical seal applications, it would seem possible that the eccentricity ϵ would not be much less than unity as is assumed in the author's analysis. If the seal were rubbing, then $\epsilon \rightarrow 1$. It would appear that the range of greatest interest would be for ϵ between zero and one. For results in this range, the author's assumptions of one dimensional flow may no longer be valid and a two-dimensional treatment, including hydrodynamic effects, may be required. It is possible that the increase in stiffness of the tapered seal compared to the straight seal observed for small ϵ may not follow for larger ϵ. Would the author please comment on these points.

I. Etsion[2]

This paper is a worthwhile contribution not only to the literature on gas path seals but also to that on piston type mechanisms where radial stiffness is desired to prevent hydraulic lock. The fact that a radial force can be generated by hydrostatic pressure differential across an axisymmetric body enclosed within a cylinder has been known for some three decades, e.g. [12]. However, the early works were confined to laminar flow only where a clearance convergence is a precondition for obtaining any radial stiffness. The author's results showing possible negative stiffness for straight seals in the laminar regime reassure the importance of maintaining converging clearance in the direction of pressure drop. In this respect it would be worthwhile to mention the possibility of increasing the stiffness by using a stepped configuration [13].

A comparison of the present results for the laminar regime with those of reference [13], using the author's terminology, demonstrates the potential improvement. A maximum stiffness over leakage ratio is obtained in [13] for a stepped configuration having a clearance ratio $\overline{H} = 1.5$ and a step extending over 0.3 of the total length L. This configuration gives a dimensionless stiffness $\overline{K} = 0.13$. From the author's Fig. 2, an optimum tapered configuration for maximum $\overline{K}$/Re gives values of $\overline{K}$ between 0.08 and 0.09 at values of seal parameter up to 10; hence, about 30 to 38 percent lower stiffness than for an optimum stepped configuration. The reason is probably due to the fact that a stepped configuration has two parameters; namely, clearance ratio and step length to play with, while in a tapered configuration only the clearance ratio can be optimized.

Could the author comment on the performance of stepped configuration as compared to that of a tapered one in the turbulent regime?

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[1] Associate Professor, Department of Mechanical Engineering, University of New Mexico, Albuquerque, N. Mex.

[2] Department of Mechanical Engineering, Technion, Haifa, Israel.

Author's Closure

The discussers' interest is appreciated. Professor Lebeck is certainly correct in stating that a two-dimensional solution would be desirable for large eccentricities, if only to verify the utility of the one-dimensional approach. Hydrodynamic effects may also be significant in some cases, although for moderate eccentricities and a large sealed pressure difference they are overwhelmed by hydrostatic effects.

If the one-dimensional flow assumption is retained for the present, simple physical reasoning suggests that a tapered seal will be stiffer than a straight seal even for $\epsilon = 1$. This is because the fluid exit is completely blocked on the low-clearance side, making the pressure equal to p_0 along the entire seal length on that side. Allowing for two-dimensional flow will change this somewhat, but qualitatively the tapered seal should still be stiffer.

In principle, having two adjustable parameters should produce a stiffer seal. This is borne out in the laminar flow results cited by Dr. Etsion, where the stepped configuration is significantly stiffer than the taper. For an indication of what one might expect from turbulent compressible flow, the results of [4] for an incompressible fluid can be examined. In this work, both tapered and stepped seals were analyzed. If the step length is restricted to reasonable values (not less than 0.2 of the seal length), the tapered seal is actually stiffer than the stepped for conditions of large C_2/L. For the smallest values of C_2/L in [4], the tapered seal shows about 27 percent lower stiffness than the stepped.

For turbulent flow, then, it appears there is less difference between tapered and stepped designs than for laminar flow. Furthermore, which configuration is optimum depends on geometry and operating conditions.