Two new approaches for the stochastic least cost design of water distribution systems

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Abstract The problem of stochastic (i.e. robust) water distribution system (WDS) design is formulated and solved here as an optimisation problem under uncertainty. The objective is to minimise total design costs subject to a target level of system robustness. System robustness is defined as the probability of simultaneously satisfying minimum pressure head constraints at all nodes in the network. The decision variables are the alternative design options available for each pipe in the WDS. The only source of uncertainty analysed is the future water consumption uncertainty. Uncertain nodal demands are assumed to be independent random variables following some pre-specified probability density function (PDF). Two new methods are developed to solve the aforementioned problem. In the Integration method, the stochastic problem formulation is replaced with a deterministic one. After some simplifications, a fast numerical integration method is used to quantify the uncertainties. The optimisation problem is solved using the standard genetic algorithm (GA). The Sampling method solves the stochastic optimisation problem directly by using the newly developed robust chance constrained GA. In this approach, a small number of Latin Hypercube (LH) samples are used to evaluate each solution's fitness. The fitness values obtained this way are then averaged over the chromosome age. Both robust design methods are applied to a New York Tunnels rehabilitation case study. The optimal solutions are identified for different levels of robustness. The best solutions obtained are also compared to the previously identified optimal deterministic solution. The results obtained lead to the following conclusions: (1) Neglecting demand uncertainty in WDS design may lead to serious under-design of such systems; (2) Both methods shown here are capable of identifying (near) optimal robust least cost designs achieving significant computational savings.

Keywords robust design; stochastic optimisation; uncertainty; water distribution network

Water Science and Technology: Water Supply **Vol 4 No 5–6 pp 355–363** © IWA Publishing 2004

Introduction

The water distribution systems (WDS) need to be rehabilitated, i.e. re-designed periodically. Unfortunately, the budgets available are usually quite limited. To make things more difficult, a number of uncertainties exist in the decision making process. One of the most uncertain quantities is the water consumption. While it is possible to estimate the present water demand reasonably well (Obradovic and Lonsdale, 1998), the situation becomes much worse when future demands needs to be predicted. Given the above, the need for considering the design of WDS under uncertain demands within an optimisation framework is obvious.

The least cost design of WDS is a topic that has captured a lot of interest in the past. The problem was initially solved using non-evolutionary optimisation methods (Schaake and Lai, 1969; Alperovits and Shamir, 1977). Later on, after identifying limitations of local search algorithms, a number of authors solved the least cost design problem using evolutionary algorithms, and Genetic Algorithms (GAs) in particular (Dandy *et al.*, 1993; Simpson *et al.*, 1994). After initial successes, the GA-based WDS design was extended to another level by applying and modifying GAs to solve real-life WDS design problems (Halhal *et al.*, 1997; Savic and Walters, 1997).

However, all aforementioned least cost design approaches share one serious limitation: all hydraulic simulation model input variables (e.g. demands) are assumed to be known with

100% certainty. Obviously, this is not true in the case of real-life systems and may lead to under-design of the WDS. This was noted first by Lansey *et al.* (1989) who developed a least-cost design methodology which takes into account uncertainty of both hydraulic simulation model inputs and outputs. The optimal design problem was formulated as a single-objective, chance constrained minimisation problem and solved using the Generalised Reduced Gradient (GRG2) technique.

Later on, Xu and Goulter (1999) developed another approach in which a probabilistic hydraulic model was used for the first time in the WDS design optimisation. The WDS hydraulic model uncertainties were quantified using the analytical technique known as the first-order reliability method (FORM). To calculate the uncertainties, the FORM method requires repetitive calculation of the first-order derivatives and matrix inversions, which is computationally very demanding even in the case of small networks and may lead to a number of numerical problems. The least cost design problem was, again, solved using the GRG2 optimisation method. Being a local search method, GRG2 can easily be trapped in the local minimum (Savic and Walters, 1997). Recently, Tolson *et al.* (2004) overcame the problem associated with the use of GRG2 by using GAs to solve the optimal WDS design problem. However, the authors still used the FORM method to quantify uncertainties which has its own drawbacks.

To overcome the limitations of all the aforementioned WDS design approaches, two robust design methodologies were developed recently (Kapelan *et al.*, 2003; Babayan *et al.*, 2004). Further improvements to these two methodologies are presented here.

This paper is organised as follows: after this introduction a robust least-cost design problem is formulated. This is followed by presentation of the two methodologies used to solve the aforementioned problem. Both methodologies are then tested and compared on a case study. At the end, relevant conclusions are drawn.

Robust least cost design problem

The objective of the robust least cost design model presented here is to minimise total design (i.e. rehabilitation) costs subject to a minimum required level of design robustness. More specifically, the optimisation problem is formulated as follows:

$$Minimise \quad Cost(D_1, D_2, \ldots, D_{Nd}) \tag{1}$$

subject to:

(a) Target robustness constraint:

$$P(H_i \geq H_{i,\min};\ i=1,\ldots,N_n) \geq P_{\min} \tag{2}$$

(b) Mass and energy balance constraints (valid for each demand sample):

$$\sum_{m=1}^{N_i} Q_m - Q_{d,i} = 0 \quad (i=1,\ldots,N_n) \tag{3}$$

$$H_{i,u} - H_{i,d} - \Delta H_i = 0 \quad (i=1,\ldots,N_t) \tag{4}$$

(c) Decision variables constraint:

$$D_i \in D \quad (i=1,\ldots,N_d) \tag{5}$$

where: *Cost* – total design/rehabilitation cost; D_i – value of the *i*-th discrete decision variable (in general, design/rehabilitation option index); D – discrete set of available design/rehabilitation options; P – design robustness defined as probability that heads (H_i) at all networks nodes are simultaneously equal to or above the corresponding minimum requirements for that node ($H_{i,\min}$); $P_{\min}$ – minimum required (i.e. target) level of robustness; Q_m – flows in all

N_i pipes connected to the i-th network node; $Q_{d,i}$ – demand at i-th node; $H_{i,u}$ – head at upstream node of the i-th pipe; $H_{i,d}$ – head at downstream node of the i-th pipe; ΔH_i – difference between i-th pipe's total headloss and pumping head; N_d – number of decision variables; N_l – number of network links; N_n – number of network nodes.

In the model presented here it is assumed that nodal demands are the only source of uncertainty, i.e. it is assumed that all other WDS simulation model inputs are deterministic variables. In addition to this, uncertain nodal demands are assumed to be independent random variables following some probability density function (PDF).

When using GA to solve the optimisation problem (1)–(5), constraints (3)–(4) can be automatically satisfied by linking GA to the deterministic WDS solver, i.e. by using WDS solver to calculate nodal heads for each demand sample. Constraint (5) can also be automatically satisfied by using the appropriate GA coding. Therefore, when using GA, the optimisation problem (1)–(5) can be transformed into the following equivalent problem:

$$Minimise \quad Obj = Cost + Penalty \tag{6}$$

where $Penalty = pr \cdot \max(0, P_{\min} - P)$ and pr is the penalty rate constant.

Solution methodology

The optimisation problem (1)–(5) is solved using two different methods: (1) the Integration method and (2) the Sampling method. Both methods use GA as their search engine. Once the GA run has converged, the best solution identified is re-evaluated using a large number of Monte Carlo samples (100,000 in the case study here).

Integration method

The main idea behind this approach is to replace the stochastic constraint (2) with the following set of deterministic constraints:

$$\xi_{Hi} \geq H_{i,\min} + \alpha\sigma_{Hi} \quad (i = 1, \ldots, N_n) \tag{7}$$

where: ξ_{Hi}, σ_{Hi} – mean value and standard deviation of the head at node i (which depends on vector of demands Q_d in the whole network), α – parameter which determines the level of system robustness. Unfortunately, because of the implicit relationship between demands and heads it is impossible to calculate the standard deviation in (7) analytically. Also, straightforward numerical evaluation requires an unreasonable amount of computational effort, therefore we will use a simplified method of evaluating σ which is based on some assumptions. First of all assume the validity of superposition principle:

$$\begin{aligned} &H_i(\xi_{Q1} + t_1, \xi_{Q2} + t_2, \ldots, \xi_{QM} + t_M) - H_i(\xi_{Q1}, \xi_{Q2}, \ldots, \xi_{QM}) \\ &\quad \approx \sum_{j=1}^{M} (H_i(\xi_{Q1}, \ldots, \xi_{Qj} + t_j, \ldots, \xi_{QM}) - H_i(\xi_{Q1}, \xi_{Q2}, \ldots, \xi_{QM})) \end{aligned} \tag{8}$$

where: ξ_{Qi} – mean value of demand at node i. Note that demand PDF is usually non-zero in some area only (for uniform distribution) or decreases exponentially with distance from the mean value (normal distribution). Hence, all we need is that a superposition principle is satisfied in some area around ξ_Q – about two standard deviations of demand are enough in most cases. Taking into account (8) and assumption about the nodal demands' independence we have the following approximation for nodal head means and standard deviations:

$$\xi_{Hi} \approx H_i(\xi_Q) + \sum_{j=1}^{Nn} \alpha_{ij} \tag{9}$$

$$\sigma_{Hi}^2 \approx \sum_{j=1}^{Nn} \int_{-\infty}^{\infty} (H_i(Q_j) - H_i(\xi_Q) - \alpha_{ij})^2 \eta_j (Q_j)\, dQ_j \tag{10}$$

$$\alpha_{ij} = \int_{-\infty}^{\infty} (H_i(Q_j) - H_i(\xi_Q)) \eta_j (Q_j)\, dQ_j \tag{11}$$

Integrals in (10) and (11) are 1D and could be calculated using conventional numerical formulas. To estimate the standard deviation for all N_n nodes in a network using formulas (10) and (11) one needs to perform $(n-1)N_n+1$ model runs, where n is the odd number of points in the formula for numerical integration. Note that formula (10) allows us to estimate relative contribution of uncertainty in demand in each node to the nodal heads uncertainty. This information can be used to build up the list of "significant" nodes and model the demands in the rest of the network as certain, leading to significant computational time savings. To summarize, the following algorithm is proposed to solve the least cost design problem under uncertainty using GA.

1. Identify the sets of critical and significant nodes. Take some initial configuration of the network (e.g. the existing network configuration). Then, compute the head mean and deviation at each node using formulae (9)–(11). Nodes at which constraint (7) is violated are added to the set of "critical" nodes Ω. Nodes which relative contribution to standard deviation in critical nodes is more than some prescribed level (say, 5%) form the set of "significant" nodes Λ. Note that this step requires $N*(n-1)+1$ state estimate calculations, where N is the number of demand nodes and n is the number of points in the quadrature formula used.
2. Run GA to find the optimal robust design. Use the GA to obtain solution of problem (6) with the penalty term written as:

$$Penalty = pr \cdot \sum_{i=1}^{Nn} \max(0, H_{i,\min} + \alpha\sigma_{Hi} - \xi_{Hi}) \tag{12}$$

From equation (12) it is obvious that, to compute the penalty function value for the current system configuration [D], ξ_{Hi} and σ_{Hi} need to be computed first. This can be done using equations (9)–(11). Note that the total number of state estimate calculations (i.e. hydraulic solver runs) necessary to obtain the value of fitness function (10) is $|\Lambda|(n-1)+1$ where $|\Lambda|$ is the number of significant nodes and n is the number of quadrature formula points. Finally, note that as the GA run progresses, the sets of critical nodes and significant nodes may have to be updated periodically. This can be done using the procedure described in step 1.

Sampling method

A modified standard GA (Goldberg, 1989) capable of performing efficient search under uncertainty is presented here. The GA is named the robust chance constrained GA (rccGA) and is based on: (a) the robust GA developed recently by Chan Hilton and Culver (2000) and (b) the chance constrained GA developed by Loughlin and Ranjithan (1999). The main idea behind the rccGA is to breed solutions, re-evaluate them during the search process using small number of samples and finally, identify the best solution as the fittest that has survived over a number of generations. The search procedure is as follows:

1. Create the initial GA population at random and initialise the age of each chromosome to zero. Evaluate the fitness of each chromosome by calculating the *Obj* value in equation (6). When calculating the robustness value P, use a small number of Latin Hypercube samples (e.g. 5–20 samples).

2. Sort the chromosomes in term of their fitness values. Identify the best chromosome as the one with the lowest *Obj* value. Keep a separate record of the best chromosome identified so far.
3. Create the next generation population as follows:
   - 3.1. Directly copy better half of the (sorted) current generation population. Increase the age of these chromosomes by one.
   - 3.2. Fill-in the rest of the population by using the standard GA selection, crossover and mutation operators applied to the existing generation population. Set the age of these chromosomes equal to zero.
   - 3.3. Calculate the fitness values for all chromosomes in the new population. When doing so, use equation (6) with the robustness value P equal to the averaged robustness value over each chromosome's age.
4. Sort the chromosomes in terms of their fitness values and identify the fittest chromosome. Update the best chromosome record if the newly identified best chromosome: (a) has a better fitness value than the best chromosome identified so far and (b) is of age equal to or higher than the minimum required age (i.e. has survived for at least the minimum required number of generations).
5. Repeat steps 3–4 until some GA convergence criterion is met. Once the GA has converged, re-evaluate the best solution found using a large number of Monte Carlo samples (100,000 in the case study shown here).

In the methodology presented here, each time the chromosome's fitness value is calculated, a total of N_s sets of random nodal demands are generated. The corresponding nodal heads are then obtained by running the WDS solver for each of the demand samples. Finally, robustness P in (2) is estimated as the fraction (i.e. percentage) of total number of samples Ns for whom the minimum head requirement condition is met simultaneously at all network nodes.

Note that when evaluating the objective value in equation (6), the conventional approach would be to use a Monte Carlo (MC) sampling technique. Rather than doing that, in the approach presented here, the Latin Hypercube (LH) technique is used. In the LH sampling technique (McKay *et al.*, 1979), values of stochastic input variables (nodal demands) are generated in a random yet constrained way. Without going into details of the LH sampling technique, note that the main advantage of the LH over the MC sampling technique is the better random sample stratification leading to more accurate estimation of the empirical PDF tails (important here when evaluating robustness of the design). The drawback associated with the use of LH sampling technique when compared to the MC sampling technique is the increased computational effort required to generate the same number of samples. Fortunately, when the number of samples is small (e.g. less than 100) this difference is not significant.

The robust chance constrained GA presented here is effectively exploiting the fact that GA search process is of stochastic nature with a population of solutions evaluated at each generation. It is a well known fact that GA determines (near) optimal solution by combining highly fit building blocks of population chromosomes (Goldberg, 1989). As the search progresses, the population is likely to have more and more chromosomes containing highly fit building blocks. As a consequence, a relatively large number of indirect evaluations of these building blocks are likely to be found in the population even if a small number of samples are used to evaluate each chromosome's fitness.

Case study

Problem description

The two robust least cost design methodologies presented in this paper are tested and compared on the well known literature problem of New York Tunnels reinforcement

(Schaake and Lai, 1969) (see Figure 1). The original objective of the problem was to identify the least cost network rehabilitation solution which satisfies minimum head requirements at all nodes for a set of fixed nodal demands. The only means of rehabilitation allowed is to duplicate existing pipes with new ones. A total of 16 options are available for each pipe in the network: do not duplicate that pipe or duplicate the pipe with a new pipe having one of 15 available diameters (range 91–518 cm). Therefore, even though New York Tunnels network is fairly small, the optimisation problem is quite large (total number of possible solutions, i.e. network configurations is $16^{21} = 1.9 \times 10^{24}$). The total cost of rehabilitation is calculated using the cost model shown in Murphy *et al.* (1993).

So far, a number of authors have solved the problem as a deterministic one. A review of these approaches can be found in Savic and Walters (1997). The optimal deterministic solution identified previously by Murphy *et al.* (1993) is presented in Table 1. Unlike in the deterministic approaches, it is assumed here that nodal demands are uncertain variables following Gaussian PDF with mean equal to the deterministic demand value and standard deviation equal to 10% of the mean value. The deterministic demands and other network data used here are taken from Murphy *et al.* (1993). The EPANET WDS hydraulic solver (Rossman, 2000) is used to calculate unknown heads and flows for each demand sample.

The objective of the analysis shown here is to compare the Sampling and Integration methods in terms of the optimal cost-robustness trade-off curves obtained. To do so, in the case of the Sampling method, optimisation problem (6) is solved for three different levels of target robustness P_{min} (90%, 95% and 99% using 5, 10 and 20 LH samples respectively). In the case of the Integration method the same optimisation problem is solved for three different, iteratively determined values of α (1.5, 2.0, and 2.8). Note that in each of the six cases mentioned above, multiple GA runs were performed with different random seeds (i.e. different initial random populations). All GA runs were done with the population of 200 and stopped after 500 generations (equivalent to 100,000 fitness evaluations). Solutions shown in the next section represent the best solutions obtained in this process.

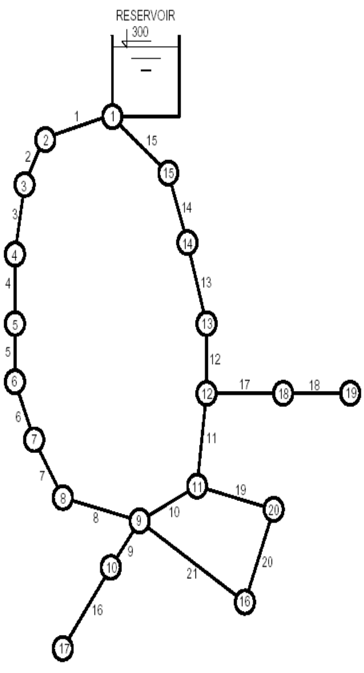

Figure 1 New York Tunnels network

Table 1 New York Tunnels problem solutions

Pipe	Deterministic Solution Murphy *et al.* (1993) D_i (cm)	Stochastic Solution (90% target robustness) Sampling D_i (cm)	 Integration D_i (cm)
1–14	–	–	–
15	305	457	457
16	213	244	244
17	244	274	274
18	213	213	213
19	183	183	274
20	–	–	–
21	183	213	183
Cost ($m)	38.80	47.08	47.91
Robustness *P*	35.3%	91.7%	90.9%

Results and discussion

The best results obtained for the problem formulated in the previous section are shown in Figure 2. The curve "Full Sampling" represents the solutions obtained by solving the optimisation problem (6) using the standard GA with 1000 LH samples for each fitness evaluation. Note that level of robustness of each solution shown in Figure 2 was re-calculated using 100,000 MC samples once the relevant optimisation process was finished. From Figure 2 it can be noted that: (1) Sampling method solutions are in slightly better agreement with the Full sampling solutions than the Integration method solutions and (2) the cost of the solution raises exponentially with the robustness level.

Details of the optimal robust solutions for $P_{min} = 90\%$ are shown in Table 1. The following can be noted from this table: (1) The robust solutions identified using the Sampling and Integration methods are similar (Sampling method solution is only slightly less costly and more robust than the solution identified using the Integration method); It is believed that if the GA was let to run for more than 500 generations, both solutions would have identified the same (optimal) solution; (2) Both stochastic (i.e. robust) solutions have the cost higher than the optimal deterministic solution identified by Murphy *et al*. (1993). This is the price that has to be paid for the increased robustness of the two solutions; (3) Robustness of the deterministic solution identified by Murphy *et al*. (1993) is quite low. This is the consequence of the deterministic optimisation which left no or very little redundancy in the system to cope with demand fluctuations.

Conclusions

The following main conclusions can be drawn from the case study presented here: (1) Neglecting demand uncertainty in WDS design problems may lead to serious under-design

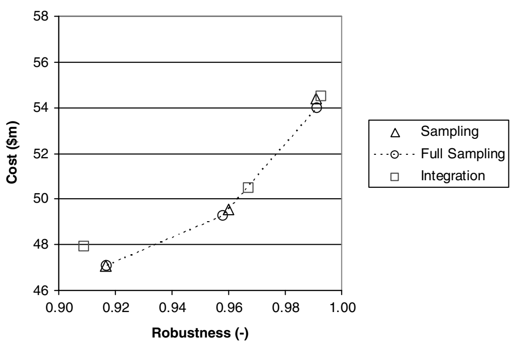

Figure 2 Robustness vs. cost trade-off curve

of such systems; (2) Both robust design methodologies presented here seem to be capable of identifying (near) optimal least cost design solutions under uncertain demands while achieving significant computational savings when compared to the full sampling technique.

The tests performed here to compare the Integration and Sampling robust WDS design methodologies are quite limited. Based on the results obtained it seems that Integration method has the following advantages over the Sampling method: (1) It does not depend on distinctive features of GA and therefore can be used with any other optimisation method; (2) The computational complexity of the method does not increase with the target level of robustness. In the example shown here a constant number of 15 WDS solver calls were required to evaluate chromosome fitness in all GA runs. In the case of the Sampling method, this number varied between 5 (for 90% target robustness) and 20 (for 99% target robustness).

On the other hand, the Integration method seems to have the following disadvantages over the Sampling method: (1) Target robustness level cannot be specified directly. As a consequence, several optimisation runs may have to be performed in an iterative type procedure to obtain the required solution; (2) Integration method needs to periodically update the list of critical nodes during the GA search process; (3) The computational complexity of the Integration method increases with an increased number of critical nodes in the network which is not expected to be the case in the Sampling method.

Acknowledgements

This work was supported by the U.K. Engineering and Physical Sciences Research Council grants GR/R14712/01, GR/S26446/01 and AF/000964.

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