DISCUSSION

F. A. McClintock[2]

The rules governing finite plastic deformation under multiaxial stress are perhaps better established than the authors indicate. The appropriate choice of a definition of finite strain depends on the physical mechanism of the deformation. For instance, a definition useful in rubber elasticity [19][3] is different from one useful in plasticity because rubber "remembers" its original dimensions, as shown by its return to shape on release of load. For predicting the strain hardening of metals, Hill [20], pp. 26 and 30, has pointed out that a correlation based on an equivalent strain defined in terms of current coordinates,

$$\int d\bar{\epsilon}^p = \int \sqrt{\frac{2}{9}\left[(d\epsilon_x - d\epsilon_y)^2 + (d\epsilon_y - d\epsilon_z)^2 + (d\epsilon_z - d\epsilon_x)^2\right] + \left[d\gamma_{xy}^2 + d\gamma_{yz}^2 + d\gamma_{zx}^2\right]/3},$$

leads, for tension or compression, to the familiar logarithmic strain for tension and compression, and for torsion to $\gamma/\sqrt{3}$, where γ is the tangent of the shear angle. He also shows that for the Mises yield criterion and the associated flow rule (so that the plastic potential, from which the stress-strain relations are derived, is similar in shape to the yield locus), use of this equivalent strain amounts to assuming that the strain hardening depends on the plastic work.

The maximum shear stress yield criterion is less satisfactory, not so much because of the fit with an experimentally determined yield locus, but rather because the stress-strain relations derived from it, according to the associated flow rule, indicate that deformation should consist of pure shear on a single plane. Since this does not, in fact, occur in the tensile test, there is an ambiguity in defining an equivalent shear strain. However, taking the equivalent shear strain increment in the tensile test to be the sum of the principal shears in two perpendicular planes in the tensile test, gives through Mohr's circles in those planes, assigning half the axial strain to each,

$$d\bar{\gamma} = 2d\epsilon.$$

This equivalent strain increment also gives the work when multiplied by the maximum shear stress. With this definition of shear strain, the curve for compression in Fig. 3 would be much foreshortened and more similar to that in Fig. 5.

It must be acknowledged that neither of these theories takes into account the development of anisotropy due to the development of preferred orientation, but the good agreement which the authors found between tension and compression tests, where the anisotropy would be expected to be different, shows that this effect is not likely to be too serious. The derivation of the hyperbolic sine law is based on finding the compression in a shear test (the inverse of what is desired here). It also involves the assumption that the principal axes of stress must coincide with the principal axes of total strain at all stages. This would be true in the case of a material in which the current state of stress determined the current state of strain, but in plastic deformation, the current state of stress only sets the current strain increment. The hyperbolic sine expression is not appropriate for rubber elasticity either, because in it, use was made of the so-called natural strain, which is based on the concept of integrated strain increments.

The apparent crossing of the curves at finite strains is not necessarily physically unreasonable, for it could simply mean that at these very large strains, the effects of anisotropy and preferred orientation could cause different amounts of strain hardening in two cases. In any event, changing the scale or the definition of strain will not eliminate the intersection when one of the curves is horizontal; it will only defer it. Thus the explanation for the difference between the cutting and the compression test must be due to other factors.

While the foregoing comment may be of considerable importance in other cases, in the present experiments the strain hardening is relatively unimportant. Therefore, these remarks should in no way detract from the authors' main point, which is a very ingenious and simple experimental technique for getting more information on both the cutting process itself and the stress-strain behavior of materials at very large strains.

[2] Professor of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Assoc. Mem. ASME.

[3] Numbers in brackets designate Additional References at end of this discussion.

Additional References

19 L. R. G. Treloar, "The Physics of Rubber Elasticity," Clarendon Press, Oxford, 1949.

20 R. Hill, "The Mathematical Theory of Plasticity," Clarendon Press, Oxford, 1950.

Authors' Closure

The authors wish to thank Professor McClintock for his thoughtful discussion. It is certainly true, as he points out, that the assumption of isotropic hardening together with the flow (incremental) theory of plasticity leads to a comparison of compression and cutting which is not affected by finite strain. In contrast, the approach used in the paper is essentially that of the deformation (total strain) theory of plasticity in which the maximum shear strain is used to compare different states of deformation. With this latter approach it is necessary to allow for finite strains and it may be shown, following Jaeger [5], that the maximum shear strain in compression is sinh $(\frac{3}{2}\epsilon)$.

Although the incremental theories of plasticity have usually been considered more realistic than the total strain theories, there is little data which compare the two types of prediction for very large strains. Perhaps the simplest type of comparison to make is that of torsion and compression. Tests by Shaw [2] on a steel and unpublished data of the authors on aluminum show excellent correlation when the finite strain expression is used. Possibly, this agreement arises from factors other than finite strain, but it illustrates that the same correction should be equally applicable to cutting.