Crushing of a Tube Between Rigid Plates[1]

R. G. REDWOOD.[2] The authors' formula (14) is identical to that derived in an unpublished report.[3] In this report, tests are described on several tubes of mild steel and aluminum alloy with ratios of thickness to diameter in the range 0.008 to 0.054. Like the authors' results, in every case the experimental loads lie above the predicted curve.

A strain-hardening solution cannot be used to predict behavior because it will depend upon the curvature at the yield hinges. However, such a solution is derived in the following text and is compared with experimental values by employing values of the curvature measured during a test.

Consider a material whose nominal stress-strain relationship conforms to rigid-linear strain-hardening behavior. If the region of plasticity is considered as a length ds subjected to a uniform change in curvature $1/r$, then the work done in creating this curvature change is

$$U = \frac{M_0}{r}\left(1 + \frac{E'h}{6\sigma_0 r}\right) ds$$

where

σ_0 = initial yield stress
E' = strain hardening modulus
$M_0 = \frac{1}{4}\sigma_0 L h^2$
L = length of yield hinge

When the deflection of the loading plate is δ, the angle through which the hinges at the extremities of the horizontal diameter have rotated (2β from the authors' Fig. 2) is $2\sin^{-1}(\delta/d)$, and the distance between the two loading points on either the top or the bottom of the tube is $d\sin^{-1}(\delta/d)$. If the regions at the top and bottom between the loading points are assumed to remain flat (experimental evidence suggests that this is approximately so until deflections become large), the curvatures of these regions has changed from $2/d$ to zero. The energy absorbed is therefore

$$U_1 = 4M_0 \sin^{-1}(\delta/d)\left(1 + \frac{E'h}{3\sigma_0 d}\right)$$

If the bending at the ends of the horizontal diameter takes place uniformly over a small length s, the curvature change is approximately $(2/s)\sin^{-1}(\delta/d)$, and the energy absorbed at these two positions may be written

$$U_2 = 4M_0 \sin^{-1}(\delta/d)\left(1 + \frac{E'h}{3\sigma_0 s}\cdot\sin^{-1}(\delta/d)\right)$$

The total energy absorbed when the deflection is δ is therefore

$$U = U_1 + U_2$$
$$= 8M_0 \sin^{-1}(\delta/d)\left(1 + \frac{E'h}{6\sigma_0}\left\{\frac{1}{d} + \frac{1}{s}\cdot\sin^{-1}(\delta/d)\right\}\right)$$

[1] By John A. DeRuntz, Jr., and P. G. Hodge, Jr., published in the September, 1963, issue of the JOURNAL OF APPLIED MECHANICS, vol. 30, TRANS. ASME, vol. 85, Series E, pp. 391–395.

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[3] R. H. Burton and J. M. Craig, "An Investigation into the Energy Absorbing Properties of Metal Tubes Loaded in the Transverse Direction," B.Sc.(Eng.) Report, University of Bristol, Bristol, England, 1963.

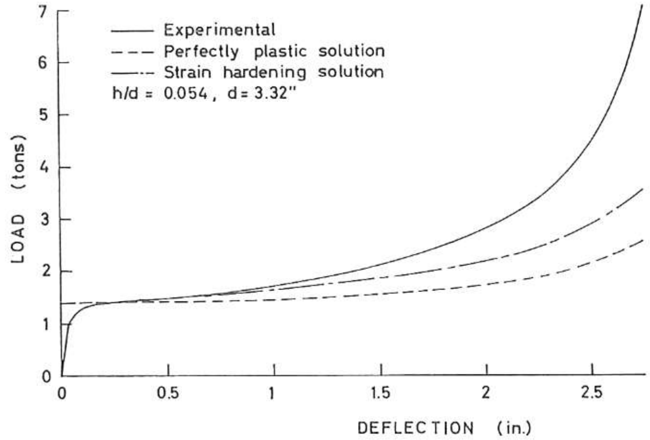

Fig. 1

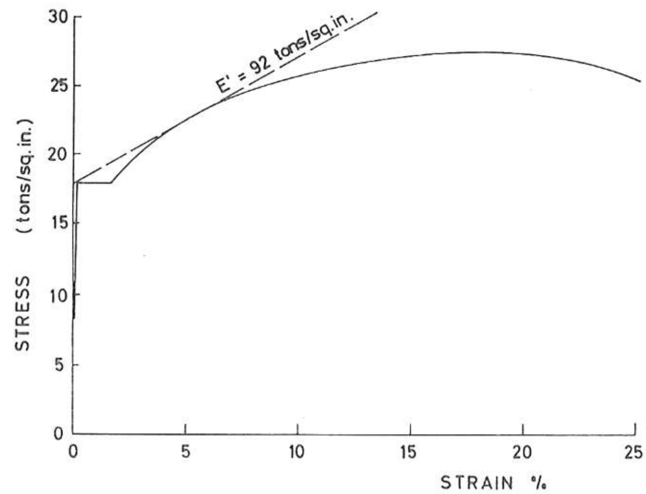

Fig. 2

The load corresponding to this deflection is

$$P = \frac{dU}{d\delta}$$
$$= \frac{8M_0}{d[1-(\delta/d)^2]^{1/2}}\left(1 + \frac{E'h}{6\sigma_0}\left\{\frac{1}{d} + \frac{2}{s}\cdot\sin^{-1}(\delta/d)\right\}\right)$$
$$+ 4M_0\frac{E'h}{3\sigma_0}(\sin^{-1}(\delta/d))^2\frac{d}{d\delta}\left(\frac{1}{s}\right) \quad (1)$$

When $E' = 0$ this reduces to the authors' formula (14). This expression can be simplified since the term $E'h/6\sigma_0 d$ is small compared with unity and may therefore be omitted. Furthermore, experimental work[3] indicates that s does not vary with δ. Writing $s = \alpha h$, (1) may be rewritten approximately as

$$P = \frac{8M_0}{d[1-(\delta/d)^2]^{1/2}}\left(1 + \frac{E'}{3\sigma_0\alpha}\cdot\sin^{-1}(\delta/d)\right) \quad (2)$$

The results of a test carried out on an annealed mild-steel tube with $h/d = 0.054$ are shown in Fig. 1. The material properties, based on a tension test, were σ_0 = 18 tons/sq in. and E' = 92 tons/sq in., Fig. 2. The excellent agreement between measured and predicted yield loads confirms the validity of equation (14) of the paper at small deflections, a fact the authors could not establish owing to lack of knowledge of material properties. However, at large deflections, even the strain-hardening solution considerably underestimates the load. In computing P from equation (2) the value of $\alpha = 5$ was used, this having been measured during the test. This measurement further confirmed the constant magnitude of s. The selected strain-hardening modulus will tend to overestimate the stress at high strains and the computed strain-hardening solution would therefore be expected to represent a maximum enhancement of P as a result of strain hardening. The writer would welcome any comments the authors may have on this discrepancy.

Authors' Closure

The authors wish to thank Mr. Redwood for his comments on the strain hardening problem and for his experimental verification of formula (14) when deflections are small. As will be recalled, it was noted in the paper that this equation was an underestimate of the true load and that the discrepancy could be attributed to strain hardening which had been neglected in obtaining the equation. However, as Mr. Redwood has shown here, a seemingly adequate approximation to the strain hardening solution itself underestimates the load when the deflections are no longer small.

This discrepancy is not surprising since a basic assumption made in this work does not appear to be valid for deflections of the order of the tube radius. In the experimental work quoted by the authors it was observed that, when the deflections became sufficiently large, the top and bottom hinges began to recede from the loading plates. Such behavior has not been included in the above work. When this is done two additional sources of load increase present themselves:

1 Of probably smaller importance is the additional curvature change at the top and bottom hinges. This in itself would probably not affect the total load appreciably.

2 One should consider that, in any experiment, friction is present. In this case, as the hinges recede from the loading plates, sliding occurs between the tube and the plates. Thus frictional forces act on the tube at the points of contact, being directed outward from the vertical axis of symmetry. These forces initiate further changes in curvature throughout the regions between contact points in addition to creating a couple which must be considered in gross equilibrium. This couple depends upon the distance that the hinges have receded and apparently is enough to increase the load necessary to maintain plastic deformation.

It is not known, of course, whether the curve shown in Fig. 1 of this discussion was obtained using a lubricant between the tube and the plates. If not, it is possible that, by doing so, a better agreement can be found between the experimental and analytical results. Conversely, the same goal may be attained by considering the above effects in the analytical work.

Funicular Polyhedra[1]

M. L. WIEDMANN.[2] The author has presented and outlined a solution for an interesting problem which also has application in certain types of modern link structures. As noted, solutions for nets with more than a few meshes would imply the use of electronic computing machines. It would appear that the method may be extended to nets in currents and to certain structures with wind or special loadings.

[1] By L. W. McKeehan, published in the June, 1963, issue of the Journal of Applied Mechanics, vol. 30, Trans. ASME, vol. 85, Series E, pp. 219–224.

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The Momentum Integral Approximation for Compressible Magnetogasdynamic Boundary-Layer Flow[1]

M. K. JOVANOVIC[2] and D. R. HAWORTH.[2] In the writers' opinion the solution presented to the particular problem of this paper represents a definite contribution to the general subject of boundary-layer flows in the presence of a magnetic field. Despite the imposed restrictions which are clearly stated, the results of this paper indicate qualitatively and to some extent quantitatively the effect of a transverse magnetic field on the coefficients of viscous and total drag for compressible boundary-layer flow with a zero pressure-gradient field.

There are two rather insignificant points which we consider require clarification: Under the discussion of the velocity profile, the authors state that for very small values of the interaction parameter, the velocity profiles may be assumed to be "similar." Referring to their definition of the interaction parameter,

$$Q = \frac{\sigma B^2 L}{\rho u}$$

it appears that Q would increase as the vicinity of the wall is approached because $u \rightarrow 0$. This inconsistency is avoided provided L in the definition of Q represents a characteristic length perpendicular to the direction of flow; i.e., if $L \equiv y$.

Also it was stated that "for constant Mach number the magnetic field must increase the boundary-layer thickness causing the velocity gradient at the wall to be decreased." From Fig. 2 it follows that the coefficient of viscous drag, C_f, is decreased by applying a magnetic field, while maintaining the same Mach number in the main flow; i.e., $C_f < C_{f/nm}$. As a consequence of equations (18) and (24) it follows that the velocity gradient at the wall must be smaller with the magnetic field applied than with no magnetic field. Owing to the decrease of the velocity gradient at the wall and with the assumption of a linear velocity profile, the thickness of the boundary layer must increase. Therefore, the statement of the paper seems to put the "effect" before the "cause."

Unfortunately, with the assumption of similar velocity profiles, which is true only for the case of zero interaction parameter, the results of the authors do not indicate the true shape of the velocity profiles. Such profiles, however, can be deduced from the analysis of incompressible flow given by Rossow.[3] Along this line it might be of interest to mention that R. D. Hugelman is working on a similar problem in his doctoral dissertation at Oklahoma State University. His case applies to incompressible flow, and in the analysis a velocity profile of the following form was assumed:

$$\frac{u}{u_\infty} = a\left(\frac{y}{\delta}\right) + b\left(\frac{y}{\delta}\right)^2 + c\left(\frac{y}{\delta}\right)^3 + d\left(\frac{y}{\delta}\right)^4 + e\left(\frac{y}{\delta}\right)^5$$

The coefficients a, b, c, d, and e are functions of x (the coordinate measured along the plate from the leading edge), and the remaining symbols are the same as in the paper under discussion. In this way similar velocity profiles did not have to be assumed; and using an approximate solution of the momentum equation, velocity profiles in the boundary layer were predicted in good agreement with the results of Rossow.[3]

W. T. SNYDER.[4] The authors apply the well-known momentum integral method to the determination of the drag of an adiabatic

[1] By J. J. Kauzlarich and A. B. Cambel, published in the June, 1963, issue of the Journal of Applied Mechanics, vol. 30, Trans. ASME, vol. 85, Series E, pp. 269–274.

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[3] Reference [3] of the paper.

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