To determine the 8 coefficients, substitute equation (18) and (19) into equation (27) and collect the terms in accordance with equation (28) to get

$$K_{xx} = \frac{1}{C} \lambda\omega \left[\frac{\partial \bar{F}_r}{\partial \epsilon} \cos^2 \phi + \frac{\partial \bar{F}_t}{\partial \epsilon} \cos \phi \sin \phi - \frac{\bar{F}_y}{\epsilon} \sin \phi \right] \quad (29)$$

$$\omega B_{xx} = \frac{1}{C} \lambda\omega \left[\frac{\partial \bar{F}_r}{\partial \left(\frac{\dot{\epsilon}}{\omega}\right)} \cos^2 \phi + \frac{\partial \bar{F}_t}{\partial \left(\frac{\dot{\epsilon}}{\omega}\right)} \cos \phi \sin \phi - \frac{\sin \phi}{\epsilon} \left(\frac{\partial \bar{F}_r}{\partial \left(\frac{\dot{\phi}}{\omega}\right)} \cos \phi + \frac{\partial \bar{F}_t}{\partial \left(\frac{\dot{\phi}}{\omega}\right)} \sin \phi \right) \right] \quad (30)$$

$$K_{xy} = \frac{1}{C} \lambda\omega \left[\frac{\partial \bar{F}_t}{\partial \epsilon} \sin^2 \phi + \frac{\partial \bar{F}_r}{\partial \epsilon} \cos \phi \sin \phi + \frac{\bar{F}_y}{\epsilon} \cos \phi \right] \quad (31)$$

$$\omega B_{xy} = \frac{1}{C} \lambda\omega \left[\frac{\partial \bar{F}_t}{\partial \left(\frac{\dot{\epsilon}}{\omega}\right)} \sin^2 \phi + \frac{\partial \bar{F}_r}{\partial \left(\frac{\dot{\epsilon}}{\omega}\right)} \cos \phi \sin \phi + \frac{\cos \phi}{\epsilon} \left(\frac{\partial \bar{F}_r}{\partial \left(\frac{\dot{\phi}}{\omega}\right)} \cos \phi + \frac{\partial \bar{F}_t}{\partial \left(\frac{\dot{\phi}}{\omega}\right)} \sin \phi \right) \right] \quad (32)$$

$$K_{yx} = \frac{1}{C} \lambda\omega \left[- \frac{\partial \bar{F}_t}{\partial \epsilon} \cos^2 \phi + \frac{\partial \bar{F}_r}{\partial \epsilon} \cos \phi \sin \phi + \frac{\bar{F}_x}{\epsilon} \sin \phi \right] \quad (33)$$

$$\omega B_{yx} = \frac{1}{C} \lambda\omega \left[- \frac{\partial \bar{F}_t}{\partial \left(\frac{\dot{\epsilon}}{\omega}\right)} \cos^2 \phi + \frac{\partial \bar{F}_r}{\partial \left(\frac{\dot{\epsilon}}{\omega}\right)} \cos \phi \sin \phi - \frac{\sin \phi}{\epsilon} \left(\frac{\partial \bar{F}_r}{\partial \left(\frac{\dot{\phi}}{\omega}\right)} \sin \phi - \frac{\partial \bar{F}_t}{\partial \left(\frac{\dot{\phi}}{\omega}\right)} \cos \phi \right) \right] \quad (34)$$

$$K_{yy} = \frac{1}{C} \lambda\omega \left[\frac{\partial \bar{F}_r}{\partial \epsilon} \sin^2 \phi - \frac{\partial \bar{F}_t}{\partial \epsilon} \cos \phi \sin \phi - \frac{\bar{F}_x}{\epsilon} \cos \phi \right] \quad (35)$$

$$\omega B_{yy} = \frac{1}{C} \lambda\omega \left[\frac{\partial \bar{F}_r}{\partial \left(\frac{\dot{\epsilon}}{\omega}\right)} \sin^2 \phi - \frac{\partial \bar{F}_t}{\partial \left(\frac{\dot{\epsilon}}{\omega}\right)} \cos \phi \sin \phi + \frac{\cos \phi}{\epsilon} \left(\frac{\partial \bar{F}_r}{\partial \left(\frac{\dot{\phi}}{\omega}\right)} \sin \phi - \frac{\partial \bar{F}_t}{\partial \left(\frac{\dot{\phi}}{\omega}\right)} \cos \phi \right) \right] \quad (36)$$

where, in the coordinate system selected,

$$\bar{F}_x = \frac{-W}{\mu NLD} \left(\frac{R}{C} \right)^2 \quad (37)$$

$$\bar{F}_y = 0 \quad (38)$$

and all forces and derivatives are calculated for the given steady-state position, defined by (ϵ_0, ϕ_0).

DISCUSSION

David P. Fleming[4]

The authors have chosen a logical model for their analysis, since the infinite groove theory has provided good stability results for gas-lubricated herringbone-grooved bearings [15].[5]

We at Lewis have obtained considerable stability data on water-lubricated herringbone-grooved bearings [16]. It would be enlightening to see a comparison with the results of the authors' analysis. This is not possible from their published data, however, because (1) our geometric parameters differed somewhat from those used by the authors, and (2) all of our results are for an unloaded bearing, that is, zero eccentricity. We did compare our experimental data with the predictions of a laminar small eccentricity analysis [16], and found that the analysis considerably overestimated the value of the dimensionless critical mass $\bar{M}_c$. The disparity became greater as the bearing clearance decreased. There seemed to be no effect of Reynolds number, however. The highest Reynolds number in our work was 700.

Also with regard to stability, the authors' published values for $\bar{M}_c$ for laminar conditions and no pressurization seem to be low by a factor of about 2, compared with our stability analysis and with published data for gas bearings at low compressibility numbers [17]. This is in spite of the authors' data being for finite eccentricity ratios, where stability is usually greater than at zero eccentricity.

With regard to the torque data that is presented, there is no information in the paper as to how this was calculated. Could the authors describe the method used?

Additional References

15 Cunningham, R. E., Fleming, D. P., and Anderson, W. J., "Experimental Stability Studies of the Herringbone-Grooved Gas-Lubricated Journal Bearing," Journal of Lubrication Technology, Trans. ASME, Series F, Vol. 91, Jan. 1969, pp. 52–59.

16 Schuller, F. T., Fleming, D. P., and Anderson, W. J., "Experiments on the Stability of Water Lubricated Herringbone-Groove Journal Bearings, Part I—Theoretical Considerations and Clearance Effects" and "Part II—Effects of Configuration and Groove to Ridge Clearance Ratio," NASA Technical Notes D-4883 and D-5264, 1968 and 1969.

17 "Design of Gas Bearings," Vol. 1, *Design Notes*, Mechanical Technology, Inc., 1966.

R. A. Burton[6]

One can scarcely argue with a derivation wherein the ground rules are clearly stated and everything proceeds by well documented steps from a series of definitions and assumptions. By these standards the authors have done a most admirable job. The only thing I quibble about is the casual way in which they have chosen not to "consider effects of fluid convective inertia." Evidence has been provided[7] that such effects can, at least sometimes, be serious in spiral groove configurations. The reason is that the pressure jump at a step is primarily determined by the velocity change due to the convergence or divergence, while the other pressure effects are determined by friction forces integrated over the land or groove region. If the number of grooves becomes large, the latter pressure fluctuations become small relative to the inertial jump at each step. The primary controlling factor is the ratio of land or groove width to clearance. In the paper by R. A. Burton[7] this was approximately 70:1, based on minimum film thickness for concentric operation. Hence, for a bearing of 1/2-in. bore and 0.001-in. minimum film thickness, this would occur for 22 grooves (a realistic number). For eccentric operation, local minimum film thickness was reduced to the point where land-width/clearance was 140, and inertial effects were still predominant. Thus one would expect the same to prevail in a bearing with, say, 10 grooves.

It would be wrong for us to extrapolate far beyond land-width/clearance ratios investigated experimentally, but it would also be wrong for us not to sound a warning. The inertial ef-

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[5] Numbers in brackets designate Additional References at end of discussion.

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[7] Burton, R. A., "An Experimental Study of Turbulent Flow in a Spiral Groove Configuration," Journal of Lubrication Technology, Series F, Vol. 90, No. 2, Apr. 1968, pp. 339–443.

fects observed in the paper by the discusser[7] led to "negative *stiffness*" and to *negative pressures near the open end of the* grooving. Consequently one should not be surprised to find performance of spiral-groove turbulent-film bearings to be quite different from that predicted in the present study. One should *recognize that for land-width/clearance ratios similar to those* studied the calculated optima will not even be an approximation of the true optimal working conditions—when working conditions do, indeed, exist.

Authors' Closure

The authors thank Mr. Fleming for his comments. We are very pleased to see that he and his associates at NASA-Lewis Labs are preforming experimental studies on helical-grooved bearings with incompressible lubricants since experimental data of this sort are sorely needed, both for their own sake and for purposes of verifying analyses. We are sorry that it was not possible to make a direct comparison of their results with our data, but we hope that comparisons can be made in the future. In this regard, we would expect that our stability data for $\epsilon \leq 0.2$ would pertain to unloaded bearings and that the load and attitude angle curves could be extrapolated to $\epsilon = 0$.

The fact that our published values for critical mass $\bar{M}_c$ are low compared with those in reference [17] may be attributed to the fact that our data are for a configuration type 3 bearing of overall $L/D = 0.833$ which, in effect, consists of two configuration 2 bearings each with $L/D = 0.416$. The data in reference [17], on the other hand, are for a configuration 1 bearing of $L/D = 1.0$. From a comparison of Fig. 5 with Fig. 7, one can see that the dimensionless load capacity (normalized against projected area LD) of a configuration 2 bearing of $L/D = 0.5$ is less than $^1/_2$ that of a configuration 1 bearing of $L/D = 1.0$. This strong dependence of load (and stiffness) on L/D ratio for helical-grooved bearings is responsible for the comparatively low values of $\bar{M}_c$ in Table 3.

A description of how the torque data presented in our paper were calculated is given in the paper by the authors.[8] In *essence, torque data were calculated by integration of the wall* shear stress existing at the surface of the smooth member of the helical-grooved bearing. The analytical determination of this wall shear stress *included* the effects of pressure flows induced *locally* within *each* groove and ridge region. Shear stress was neglected in regions of subambient pressure under the assumption that such regions would be cavitated.

The authors appreciate Dr. Burton's kind remarks and agree with him that we were remiss in not pointing out the potential significance of neglecting convective inertial effects. In the paper[8] we did, ourselves, note that convective inertial effects are quite likely to be important in helical-grooved seals and stated that, *in our opinion*, such effects were the *principal* cause of the discrepancy between experimental data on screw seals operating in the turbulent regime and existing analyses. In support of this view we cited Dr. Burton's experimental measurements on *spiral grooved configurations*.

The discrepancies between experimental sealing coefficients and those predicted by the authors' analyses were on the order of 30 to 40 per cent at $N_{Re} = 5000$, experimental values of sealing *coefficients being higher than predicted values*.

In the case of helical-grooved journal bearings, it is difficult to predict what overall influence convective inertial effects would have on load capacity. If the effect is a locally deleterious one, *as regards the local pressure profiles over groove-ridge pairs, then* one could argue that this would lead to an overall improvement in integrated performance, since this locally deleterious effect would be worse in the region of maximum clearance than in the region of *minimum clearance*. Moreover, the fact that convective inertial effects appear to improve the ability of helical-grooved seals to pump to higher pressure would seem to indicate that these effects would help bearing performance. On the other hand, the negative stiffness effects observed by Dr. Burton would definitely not be helpful. One would have to agree then that experimental studies, of the sort being conducted by the researchers at NASA-Lewis Labs, are needed to establish the limits of validity of the present analyses.

[8] Vohr, J. H., and Chow, C. Y., "Theoretical Analysis of Spiral-Grooved Screw Seal for Turbulent Operation," JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series F, Vol. 91, No. 4, pp 675–686, Oct. 1969, pp. 675–686.