Interaction of pulsar winds with interstellar medium: numerical simulation

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Accepted 2004 November 2. Received 2004 November 2; in original form 2004 May 25

ABSTRACT
A time-dependent numerical simulation of relativistic wind interaction with interstellar medium was performed. The winds are ejected from the magnetosphere of rotation-powered pulsars. The particle flux in the winds is isotropic while the energy flux is strongly anisotropic. Pure hydrodynamical winds and magnetized winds with magnetization in the range $3 \times 10^{-3}$–$10^{-1}$ were modelled. The numerical solutions reproduce the most spectacular features observed in the central part of the plerions: toroidal structure and jet-like features. A comparison of the solutions with the observations confirms that magnetization of the wind from a pulsar in the Crab nebula is of the order of $3 \times 10^{-3}$. The plasma flow in the nebula is variable. The variability has different time-scales. Non-periodic 2–3 yr scale variations are superimposed on the quasi-periodical variations with period $\sim 40$ yr. An increase of the wind’s magnetization results in a decrease of the size of the synchrotron nebula. The size of the Vela nebula can be reproduced provided that the wind from Vela is more highly magnetized than the wind from Crab. Nevertheless, the magnetization of the wind from the Vela pulsar is most likely below 0.1.

Key words: MHD – shock waves – pulsars: general – ISM: individual: Crab nebula – jets and outflows – supernova remnants.

1 INTRODUCTION

Rotation powered pulsars (RPS) lose rotational energy in the form of a relatively dense, cold, relativistic wind of plasma, composed of free electron–positron pairs with an embedded, wound up magnetic field (Michel 1991). Particles move relativistically together with the magnetic field frozen into the wind. Therefore, they do not emit synchrotron radiation, which could bring us direct information about the winds. Thus, the very existence of the pulsar winds would remain controversial for us up to now, unless they interact with the interstellar medium (ISM).

The pulsar winds are terminated by a shock wave at the interaction with ISM. The shock redistributes the energy of the particles, accelerating some of them by up to $10^{15}$ eV, and randomizes their pitch angle. They start to emit synchrotron and inverse Compton radiation in the post-shock region, forming a plerion (or synchrotron nebula).

Analysis of the Crab nebula observations in a wide range of wavelengths from optics to very high energy gamma-rays (Kennel & Coroniti 1984; De Jager & Harding 1992; Atoyan & Aharonian 1996) leads to the conclusion that the wind in the preshock region is almost unmagnetized. All the energy flux is concentrated in the kinetic energy of the particles, having Lorentz factor of the order $10^6$. The ratio of the Poynting flux over the kinetic energy flux (magnetization parameter $\sigma$) is estimated to be of the order of $\sigma = 3 \times 10^{-3}$. This conclusion contradicts the pulsar magnetosphere models and presents us with one of the most serious problems in astrophysics.

Theoretical models of the plasma production and acceleration in the pulsar magnetosphere give highly magnetized wind at the light cylinder of pulsars. Magnetization is estimated at of the order $10^4$ (for Crab) (Michel 1991). An average bulk motion Lorentz factor of this wind is of the order of 100 or less (Daugherty & Harding 1982; Hibshman & Arons 2001). This means that practically all the energy flux in the wind which just leaves the pulsar magnetosphere is concentrated in the Poynting flux.

It is difficult to avoid the conclusion that on the way from the light cylinder to the terminating shock wave the Poynting flux is transformed into the flux of kinetic energy of the particles due to an unknown mechanism. This mechanism has remained an enigma for almost 20 years. During this time the problem has taken on a special significance for relativistic astrophysics (Blandford 2002).

None of the theoretical attacks on this problem have resulted in an unambiguous solution. Mechanisms for the transformation of the Poynting flux into the kinetic energy flux have been suggested for non-dissipative (ideal magnetohydrodynamic, hereafter MHD) winds (Begelman & Li 1994; Contopoulos & Kazanas 2002; Vlahakis 2004) as well as for dissipative ones (Coroniti 1990;
Melatos & Melrose 1996; Lyubarsky & Kirk 2001). A mechanism for the annihilation of the magnetic field in the equatorial current sheet (Coroniti 1990) due to dissipative processes was one of the most promising among them. The fullest and most detailed realization of this idea is performed in the work by (Lyubarsky & Kirk 2001). However, this work only confirmed that the magnetic reconnection is unable to provide transformation of the Poynting flux into the plasma kinetic energy for the wind from the Crab pulsar. Therefore, the idea that the pulsar winds are actually highly magnetized in the pre-shock region remains rather popular.

The conclusion for a low magnetization of the pulsar winds is based on the interpretation of the Crab nebula observations under rather crude assumption about plasma dynamics. It is assumed that winds flow radially in the post-shock region (Kennel & Coroniti 1984). The presence of a relatively strong magnetic field can essentially modify the dynamics of the flow in the region downstream from the shock. In particular, magnetic instabilities can result into fast magnetic field energy releases (Lyubarsky 1992; Chiueh, Li & Begelman 1998; Lyubarsky 2003), producing an observable picture. Therefore, it is difficult to make a reliable, unambiguous conclusion about the magnetization of the pulsar winds remaining only in the framework of the Kennel & Coroniti (1984) theory.

To solve the problem with the pulsar wind acceleration it is crucially important to have new observational information about the characteristics of the pulsar winds. Detection of direct gamma-ray signals from the pulsar winds would be the most important step in this direction (Bogovalov & Aharonian 2000; Kirk, Ball & Skjæraasen 1999). However, even the traditional channel of information about pulsar winds could give us new data. In the last few years telescopes on the Hubble, Chandra and XMM–Newton observatories bring us new observational data about the central part of the plerions. Observations in the X-ray (Brinkmann, Aschenbach & Langmeier 1985; Weisskopf et al. 2000) and in the optical (Hester et al. 1995) discovered remarkable torus and jet-like structures in the central part of the Crab nebula.

The same structures were found by Chandra around the Vela pulsar (Helfand, Gotthelf & Halpern 2001; Pavlov et al. 2000, 2001). It appears that, along with rather close similarity between the Crab and Vela plerions, they also have differences. The bolometric Crab nebula luminosity is equal to dozens of percentages of the pulsar rotational losses. In the Crab nebula the X-ray torus consists of two parts: the inner ring with a small radius and the outer diffuse ring with larger radius. The Vela nebula is rather compact and have low luminosity compared with the rotational losses of the pulsar, \( \sim 5 \times 10^{-3} \) (Helfand et al. 2001). The torus in the Vela nebula consists of two rings as well. The radius of the rings is almost the same. No outer diffuse torus was observed in the Vela nebula in the X-ray. However, some signature of the outer torus has been found in the radio (Dodson et al. 2003). Attempts to find the plerionic structure around Vela in the optical are still not successful (Mignani et al. 2003).

The toroidal and jet-like structures were found in several other plerions: around PSR 1509-58 (Kaspi et al. 2001; Gaensler et al. 2002), and in the supernova remnants G0.9+1 (Gaensler, Pivovarof & Garmire 2001) and G54.1+0.3 (Lu & Wang 2001). Apparently, the formation of these structures is a rather general phenomenon for plerions.

The integral characteristics of the Crab nebula are well described by the shock model (Rees & Gunn 1974; Kennel & Coroniti 1984). This theory explains the spectra and luminosity of the Crab nebula in the energetic band from the optical up to TeV gamma-rays pretty well (Atoyan & Aharonian 1996; De Jager & Harding 1992). However, the physics of the interaction of the wind with the interstellar medium is essentially simplified in this theory. The wind is considered as an isotropic. Therefore, this theory in its original form is not able to explain the observable structures of the Crab nebula.

There were several attempts to explain the formation of the toroidal and jet-like structures in plerions (Brinkmann et al. 1985; Begelman & Li 1992; Radhakrishnan & Deshpande 2001; Shibata et al. 2003). The authors of the last paper concluded that the simplest assumption that the pulsar wind is concentrated close to the equatorial plane does not allow to reproduce the X-ray observations of the Crab nebula.

The most natural mechanism for the formation of the toroidal structure have been proposed by Bogovalov & Khangoulia (2002a,b). The key point is that the energy flux in the pulsar winds must be anisotropic. This results in the formation of the toroidal structure in the post-shock region. Simplified calculations of the surface brightness in the framework of this approach reproduced the structure observed in the central part of the Crab nebula pretty well (Bogovalov & Khangoulia 2002b).

Lyubarsky (2002) suggested that the plerionic jets are the result of the collimation of the post-shock flow by the toroidal magnetic field. Our analysis (Khangoulia & Bogovalov 2003) of the post-shock flow with the magnetic field confirmed that the magnetic field can essentially affect the dynamics of the plasma near the rotational axis, resulting in its collimation. Therefore, an accurate modeling of the plasma flow in the plerions should indeed take into account the magnetic field.

The main objective of this work is to study the process of interaction of the anisotropic magnetized relativistic winds with the ISM. In this paper we plan to get information about the dynamics of the post-shock flow under different magnetization parameters and finally to derive the characteristics of the pulsar winds which are relevant to the mechanism of the energy transformation.

2 BASIC CHARACTERISTICS OF THE PULSAR WINDS IN THE PRE-SHOCK REGION

The energy flux density in the wind is a sum of the electromagnetic energy flux \( (c/4\pi)E \times B \) and kinetic energy flux \( \gamma^2 mc^2 n v \) densities, where \( \gamma \) is the Lorentz factor of the bulk motion of the wind, \( n \) and \( v \) are the plasma density in the comoving coordinate system and the velocity, respectively, and \( E \) and \( B \) are the electric and magnetic fields, respectively. Relativistic winds from pulsars flow radially. Their magnetic collimation is infinitesimally small (Beskin, Kuznetsova & Rafikov 1998; Bogovalov 2001). The toroidal magnetic field \( B_\phi \) generated at the rotation of the pulsar is proportional to \( \theta \), where \( \theta \) is the polar angle (Bogovalov 1999). Similar distribution of the toroidal magnetic field takes place in the solar wind (Parker 1963). The electric field in the wind is connected with the magnetic field through the ideality condition \( E = -(v/c)B_\phi \) provided that \( r \gg c/\Omega \), where \( \Omega \) is the angular velocity of the pulsar. Therefore, the electric field is proportional to \( \sin \theta \) as well. Thus, the Poynting flux appears proportional to \( \sin^2 \theta \).

The ratio of the Poynting flux to the kinetic energy flux is close to \( 10^4 \) at the light cylinder of the Crab pulsar (Coroniti 1990). Practically all the energy flux is concentrated in the electromagnetic field. According to Kennel & Coroniti (1984), the situation at the shock is reversed. The ratio of the Poynting flux to the kinetic energy flux becomes equal to \( 3 \times 10^{-3} \) right upstream from the shock (Kennel & Coroniti 1984). The kinetic energy flux here is a sum of the initial kinetic energy flux plus the initial Poynting flux. Therefore, the Lorentz factor of the wind takes the form (Bogovalov &
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Aharonian 2000; Bogovalov & Khangoulian 2002a)

\[ \gamma = \gamma_0 + \gamma_m \sin^2 \theta, \]

where \( \gamma_0 \approx 200 \) is the initial Lorentz factor of the wind near the light cylinder. According to (Daugherty & Harding 1982), \( \gamma_0 \approx 200 \) or even less (Hibschman & Arons 2001). \( \gamma_n = (\Omega / v R_0) \eta \) \[ B_0^2 / (4\pi n_0 m_c^2) \gamma_0 \] \( \approx 10^6 \)–\( 10^7 \) in conventional models of the pulsar winds. Here \( n_0 \) and \( B_0 \) are the initial density and the magnetic field at the light cylinder, respectively. We do not expect any remarkable dependence of \( n_0 \) or \( B_0 \) on \( \theta \). An anisotropy of \( n_0 \) and \( B_0 \) in the limits of one order of magnitude is not important, because the Lorentz factor changes by 4 orders of magnitude with \( \theta \). Therefore, the anisotropy of the particle flux is neglected. We assume that \( \gamma_n \) is constant and \( \gamma \) is proportional to \( \sin^2 \theta \). Hereafter the wind that has isotropic particle flux and anisotropic energy flux we will denote as the anisotropic wind.

In this work we are interested in the study of the dependence of the post-shock flow on the magnetization parameter. Mathematically the problem is formulated as follows. The central source (pulsar) ejects a nanisotropy of the isotropic wind.

Equations of relativistic MHD flows were investigated numerically by several authors (van Putten 1993; Komissarov 1999; van Putten 2002). Astrophysical relativistic MHD winds were simulated successfully for large Lorentz factors and magnetization parameters by Bogovalov (1997, 2001). Numerically, the interaction of the pulsar winds with ISM has been modelled by Komissarov & Lyubarsky (2003) and Bucciantini et al. (2003).

Our method is based on a Godunov-type scheme for Eulerian relativistic MHD, which is similar to the method used by Komissarov (1999). The key point of any numerical scheme is the calculation of the fluxes of conservative variables at the interfaces of the cells. In the original paper by Godunov (1959) the fluxes are calculated by means of the exact solution of a Riemann problem. Even in the case of gas dynamics it takes an iterative process and is very time-consuming. The situation becomes much more complicated for the MHD equations (Ryu & Jones 1995), so its inclusion in computing algorithm is not expedient.

A number of algorithms involving an approximate solution of the Riemann problem were suggested both for gas dynamics (Osher & Solomon 1982; Roe 1986) and MHD (Brio & Wu 1988). The most popular approach for the approximate solution of the Riemann problem introduced by Roe (Roe 1986) considers it in a linearized approximation. We use the same approach. Details of the realization of this approach in our code are given in (Kodoba, Kuznetsov & Ustyugova 2002).

3.2 Initial conditions

We take a spherically symmetric hydrodynamical Kennel & Coroniti (1984) solution as the initial state in all cases. A cold, relativistic,
unmagnetized wind with a specified Lorentz factor and density is
terminated by the ISM with the formation of the shock front. The
position of this front at \( r_{\text{sh}} \) specifies the pressure at large distances in
the nebula \( P_{\text{ext}} \). During the simulation the ratio \( \alpha = \gamma \rho_m / \sqrt{\gamma_0} \), which
we denote hereafter as the anisotropy parameter, increases smoothly
in time up to a specified level. The same procedure was carried out
for \( \sigma \).

3.3 Boundary conditions
The direct numerical simulation of plasma flow takes place only in
plerion (region \( ABCD \) in Fig. 1). This region is limited by a termi-
nating shock from the inner part and by the contact discontinuity
from the outer part, where the plerion is in contact with the shocked
ISM material. The location of these boundaries is unknown a priori.

The dynamics of the relativistic wind (region \( OAB \)) and of the
ISM are not simulated, and their influence on plasma flow in ple-
rior is taken into account by an effective fashion, determining the
boundaries of the settlement region. For the calculation of the posi-
tion and speed of the termination shock \( AB \) an approximate solution
of a problem about the decay of a discontinuity between the ultra-
relativistic wind and the plasma on the interior boundary of plerion
is used.

This problem is reduced to the Riemann problem about the de-
cay of a discontinuity between the ultrarelativistic wind (subscript
‘0’) and the state behind shock (subscript ‘1’). The velocity of the
plasma in both states has two components; the magnetic field has
one component, perpendicular to velocity vector. The terminating
shock decays into the shock wave propagating to the left, shock
wave propagating to the right, shock wave and rarefaction wave.
Additionally, a tangential discontinuity divides the plasma in the
region \( 0 \) and \( 1 \) (Fig. 2). The problem about the decay of the discon-
 tinuity is solved in acoustic approximation, which is based on the
following two assumptions:

(i) It is assumed that the shock wave propagating to the left is
almost steady and variation of MHD variables behind a shock wave
(in region ‘2’) can be described with an adequate accuracy by lin-
earized Hugoniot relationships (regarding basic configuration, when
the shock is in steady state).

(ii) The variation of MHD variables for the shock wave or rarefac-
tion wave propagating to the right (in region ‘3’) can be described
with adequate accuracy by linearized (regarding state ‘1’) RMHD
equations.

For the outer boundary (contact discontinuity), two types of the
boundary conditions were used. In the first type (type I) the pressure
\( P_{\text{ext}} \) was specified as the outer pressure and was defined from the
initial, spherically symmetric solution. Thus, the total pressure in
the flow on the outer sides is prescribed to the external pressure, \( \rho
u^2 + P = P_{\text{ext}} \).

In the second boundary condition (type II), the pressure of the ex-
ternal medium is considered equal to zero and the external pressure
is defined by the inertial pressure of the shell of the external medium.
This pressure is proportional to the value \( \rho \alpha^2 \) at the boundary of the
simulation box.

These two types of conditions on the outer boundary correspond
to different types of confinement taking place in different plerions.
The first one corresponds to the outer pressure confinement taking
place, for example, in the Vela nebula. The second one corresponds
to the inertial confinement taking place in the Crab nebula.

Comparisons of simulations for these two boundary conditions
have shown that the flow in the central part appears essentially iden-
tical for both boundary conditions. Therefore, below we present the
results only for the boundary condition of type I.

3.4 Mesh
The mesh in the region \( ABCD \) is formed by the beams going out
from the centre. They are limited by the terminating shock \( AB \), and
by the contact discontinuity \( CD \). The mesh in the computational
domain is shown in Fig. 3.

The beams are distributed uniformly along the polar angle
\( \theta \leq \pi/2 \). The number of the beams is 90. The nodes along the beam
are distributed according to the equation \( R_{ij} = q R_{0,j} \), where \( j \) is
the beam number and \( i \) is the node number. The number of the nodes
on a beam is 70. The factor \( q \) is calculated from the condition that
the node \( (0,j) \) is located on the terminating shock wave, and the
node \( (N, j) \) is located on the contact discontinuity. It is evident
that the factor \( q \) varies with the beam number. The location of the
terminating shock front and contact discontinuity varies with time.
Therefore, the location of the mesh nodes varies with time as well.
Thus the mesh is variable and curvilinear.

4 RESULTS
The results of simulation are presented in the sequence which al-
 lows us to demonstrate the role of the anisotropy of the energy flux
and magnetic field in the wind separately and all together. Specific
eamples of the simulations are performed for the conventional pa-
 rameters of Crab wind and for a range of the winds with higher
magnetization.

4.1 Hydrodynamical limit
The result of interaction of the unmagnetized pulsar wind with the
uniform interstellar medium is presented in Fig. 4. The energy flux
in the wind is anisotropic with \( \alpha = 1000 \). Type I boundary conditions
were specified for this case. We have fixed the pressure of the ISM
in order to satisfy the condition that the shock front radius must be

\[ \rho \alpha^2 + P = P_{\text{ext}} \]
equal to the radius of the inner ring, observed by *Chandra*, which is 0.11 pc.

The pulsar is located in the lower left-hand corner. The simulation was performed in one quarter of the total space. The wind propagates from the pulsar in the space shown in white. The shock front is not spherical as in the case of an isotropic wind. It takes a form of a tore. The ratio of the distance to the shock front at the equator over the similar distance at the axis is $\sqrt{\gamma_m/\gamma_0}$ (Bogovalov & Khangoulian 2002a). According to Fig. 4(a), disc-like flow is formed near the equator. This flow forms a huge vertex in the plerion at larger distance. The flow is directed to the centre at the axis of rotation. However, the velocity of this flow is very low.

One new important feature of the flow is the formation of the secondary shock. The formation of this shock is inevitable (Bogovalov & Khangoulian 2002a). The wind interacts with the interstellar medium, forming the oblique shock wave. At high latitudes (small polar angles) the inclination angle between the shock front and the flow line is so small that the wind remains supersonic even after passage of the shock front. Finally, all the flow is decelerated to the subsonic velocities. The passage from supersonic to the subsonic flow is always accompanied by the formation of a shock front. Therefore, a secondary shock front must exist in the post-shock region (Bogovalov & Khangoulian 2002a). This shock has been found by Komissarov & Lyubarsky (2003, 2004) and by Del Zanna, Amato & Bucciantini (2004) as well. The structure of the flow in the region of the secondary shock is shown in Fig. 5 in more detail.

The velocity of plasma down-flow from the shock is higher than the velocity in the rest of the plerion above the disc-like flow. Therefore, the Kelvin–Helmholtz instability is developed at the interface between these parts of the flow. One more property of the post-shock flow which has a direct relation to observation is the velocity distribution in the post-shock region. In the Kennel & Coroniti (1984) pure radial flow, the velocity of the plasma behind the shock is close to $c/3$ and goes to zero with distance. In the post-shock flow produced by the anisotropic wind, the velocity of the plasma is close to $c/3$ just down the shock. However, in contrast to the radial outflow, the velocity of the plasma reaches $0.5c$ in the region $2–6r_{sh}$ at the equatorial plane. This agrees well with observations.

In the pure hydrodynamical case, all the wind with anisotropic energy distribution is compressed into the disc-like outflow along the equator in the post-shock region. This happens already at a rather low anisotropy level, of the order of 10. There is no outflow along the axis. This means that in the hydrodynamical limit no jet-like flow is possible. Therefore, the role of the magnetic field looks especially interesting.
4.2 Isotropic magnetized wind

The toroidal magnetic field in the pulsar wind depends on the polar angle \( \theta \), as \( B_\psi = B_\theta \sin(\theta) \). This distribution is valid for the pulsar winds at large distances from the pulsar, provided that the ideality cognition was not violated (Bogovalov 1999). For simplicity, we assume that the same distribution remains valid after the acceleration of the wind. The normalization factor \( B_\theta \) is defined by the magnetization of the wind, which does not depend in this case on the latitude, with the exception of a narrow region near the rotational axis \( \Delta \theta = \sqrt{\gamma_0/\gamma_1} \).

The results of a simulation shown in Fig. 6 were obtained for \( \sigma = 10^{-2}, 10^{-3} \) and \( 10^{-4} \) to demonstrate dependence of the post-shock flow dynamics on the magnetization. It is seen that the magnetization of the wind leads to a rather remarkable collimation of the post-shock flow to the axis of the pulsar rotation. Surprisingly, the magnetization of the wind provides collimation of all the flow. This behaviour of the relativistic plasma downstream from the shock is in strong contrast with the behaviour of the wind in the preshock region. Therefore this feature of the flow deserves a more detailed discussion.

The magnetic collimation of the relativistic winds is weak in the pre-shock region because the magnetic force which collimates the flow to the rotational axis is compensated by the electric force which decollimates the plasma. The collimation process is controlled by the gradient of the electromagnetic pressure \( B_\psi^2 - E^2 = B_\psi^2/\gamma^2 \). The reduction of the collimation force takes place while the velocity of plasma is close to the light velocity, \( \gamma \gg 1 \).

The shock completely changes the situation. The tangential component of the electric field is conserved at the passage through the shock front. However, the toroidal magnetic field increases because the velocity decreases. As a result, after the shock front \( \gamma \approx 1 \) (provided that \( \sigma \ll 1 \)) and the collimating electromagnetic pressure increases by several orders of magnitude. For example, if the Lorentz factor of the wind is of the order of \( \gamma \approx 10^6 \), then the collimating force increases on 12 orders of magnitude in the post-shock region. This explains why the magnetic collimation becomes so efficient in the post-shock flow.

It is interesting that, qualitatively, the post-shock flow almost does not depend on \( \sigma \). The reduction of \( \sigma \) results only in the increase of the radius where the collimation of plasma takes place. The explanation of this phenomena follows from the analysis performed by Khangoulian & Bogovalov (2003). The post-shock flow occurs in the subsonic regime [we assume that the magnetic field is weak and wind is isotropic, as in the Kennel & Coroniti (1984) model]. The density of plasma is almost constant, while the velocity falls with distance as \( r^{-2} \). The frozen-in condition means that the ratio \( B_\psi/\rho \) is proportional to the distance from the rotational axis \( r \). Therefore, the toroidal magnetic field increases linearly with \( r \). The magnetic field pressure increases as \( r^2 \) in this case. Plasma pressure does not depend on the distance from the rotational axis because the density of plasma is constant in first approximation. According to naive expectations, the magnetic field should modify the dynamics of the plasma flow when the magnetic field pressure becomes of the order of magnitude of the plasma gas pressure. But this is not correct.

The ratio of the magnetic field over the gas pressure, \( \beta = B^2/(8\pi P) \), achieves 25 per cent at the point of maxima (where plasma turns from the equator and starts to flow along the rotational axis) for \( \sigma = 0.01 \) and reduces with \( \sigma \) to the level of 1 per cent for \( \sigma = 10^{-4} \). This dependence has a simple explanation.

The curvature radius \( R_c \) of the flow lines in the poloidal plane is defined by a simple equation (Bogovalov 2001):

\[
\frac{\mathbf{u} \mathbf{u}^2}{R_c} = (q \mathbf{E} + \mathbf{J} \times \mathbf{B} - \nabla p) \cdot \mathbf{e},
\]

where \( \mathbf{e} \) is the unit vector perpendicular to the flow line, \( q \) is the electric charge density in the plasma and \( J \) is the electric current density. It follows from this equation that \( R_c \) is defined not only by the force affecting the plasma but by the velocity of the plasma as well. The less the velocity, the smaller \( R_c \). In the post-shock flow the velocity decreases with distance as \( r^{-2} \). Therefore, even a small toroidal magnetic field is able essentially to affect the flow lines. This phenomena has been quantitatively considered by (Khangoulian & Bogovalov 2003).

It is convenient to describe the flow lines by a mass flux function \( \psi \). The solution gives \( \psi \) in the post-shock region (Khangoulian & Bogovalov 2003)

\[
\psi = \psi_0[1 - \cos(\theta) + \sigma h(r)(1 - \cos^2 \theta) \cos \theta],
\]

where \( \psi_0 \) is the total mass flux. The function \( h(r) \) depends on \( r \) as follows

\[
h(r) \sim \left(\frac{r}{r_{sh}}\right)^6.
\]

The affect of the magnetic field becomes remarkable when the product \( \sigma h(r) \) is of the order of 1 (see equation 9). We define the collimation radius \( R_{coll} \) as the radius where \( \beta \) achieves maxima. It is easy to estimate how \( R_{coll} \) scales with \( \sigma \). It follows from (10) that

\[
R_{coll} \sim r_{sh}(\sigma)^{-\frac{1}{2}}.
\]

This dependence agrees well with the results of simulations. Theoretical dependence (equation 11) is shown in Fig. 7 in comparison with the points obtained numerically. There is a pretty good agreement between the analytical estimates and the numerical data. The same can be done for the dependence of \( \beta \) on \( \sigma \). The magnetic field pressure is proportional to the product of \( (r/R_{sh})^2 \). Taking into account equation (11) we obtain that

\[
\beta \sim \sigma^{-2/3}.
\]

The dependence of the maximal value of \( \beta \) at the equator on \( \sigma \), following from equation (12), is presented in Fig. 8, together with the values obtained in the numerical solutions. Again, there is a good agreement between them. This comparison demonstrates the validity of the numerical code developed for the simulation of the relativistic plasma dynamics in the post-shock region.
Lyubarsky (2003). They considered the wind with variability in latitude and longitude of the Crab parameters have been performed recently, by Komissarov & Lyubarsky (2003). They considered the wind with variability in latitude and longitude of the Crab parameters. This is the most representative case for the wind with low anisotropy and magnetization of the wind on the example of Crab nebula.

It is interesting to consider the combined effect of the energy flux and the post-shock flow by its anisotropy and magnetization. After comparing the results of the simulations for \( \sigma = 0.001 \), Circles show the values of \( R_{\text{coll}} \) obtained numerically.

**Figure 7.** \( R_{\text{coll}} \) versus \( \sigma \). The solid line is the law defined by equation (11). The curve is normalized on the value \( R_{\text{coll}} \) obtained in the numerical simulations for \( \sigma = 0.001 \). Circles show the values of \( R_{\text{coll}} \) obtained numerically.

**Figure 8.** \( \beta \) versus \( \sigma \). The solid line is the law defined by equation (12). The curve is normalized on the value \( \beta \) obtained in the numerical simulations for \( \sigma = 0.001 \). Circles show the points obtained in the numerical simulations.

4.3 Anisotropic wind with a low magnetization

It is interesting to consider the combined effect of the energy flux anisotropy and magnetization of the wind on the example of Crab nebula. This is the most representative case for the wind with low anisotropy and magnetization. The first numerical calculations of the post-shock flow for the Crab parameters have been performed recently, by Komissarov & Lyubarsky (2003). They considered the wind with variability in latitude \( \sigma \). Such a wind has been taken to reproduce the toroidal structure and the jets simultaneously. However, it looks more reasonable to consider the case with a uniform magnetization first.

We suggest considering the wind with characteristics which are reliably known. There are no doubts that the wind is magnetized and has anisotropic energy flux in the form (equation 1). No reliable information about the distribution of magnetization is available. In this situation, it is important to consider the simplest case of constant magnetization of the wind. After comparing the results of the simulations with the observations, we can conclude what modifications should be made to get as close an agreement as possible. Therefore, we consider the simplest case of the wind with the conventional value of \( \sigma = 3 \times 10^{-3} \), which is kept constant on all field lines, with the exception of the narrow region around the rotational axis where the toroidal magnetic field goes to zero.

The conventional anisotropy level in the wind should be close to \( 3 \times 10^4 \) because the typical Lorentz factor of electrons at the light cylinder is expected to be of the order of 100, and because the Lorentz factor of the electrons at the shock is \( 3 \times 10^6 \). Nevertheless, we assume that \( \alpha = 10^3 \). This change in the parameter is not significant for us because the difference between the flows with large, different values of \( \alpha \) will take place in a narrow region \( \theta \sim \sqrt{\sigma} \) (see equation 1). The radius of the shock front at the axis will be \( r_{\text{sh}} = r_{\text{sh}}(1/\sqrt{\alpha}) \), where the radius of the shock at the equator \( r_{\text{sh}} \) does not depend on \( \alpha \). The rest of the flow does not depend on \( \alpha \) either. A reduction of \( \sigma \) is convenient from the technical point of view. This helps us to avoid modelling the flow in a very small region near the rotational axis and close to the pulsar, which is actually not so interesting to us.

The post-shock flow, with flow lines and the distribution of \( \beta \), is shown in Fig. 9. The volume synchotron emissivity for the same flow is shown in Fig. 10. This flow is formed \( \sim 900 \) yr after the beginning of the interaction between the flow and the ISM. The calculation was performed with the time-step 1/30 yr. The most remarkable feature of the flow is the formation of jets in the post-shock region. Although the disc-like flow at the equator is formed just behind the shock front, as in the pure hydrodynamical case, the initial magnetic field is finally amplified to the level where it is able to collimate the flow. Thus, the wind anisotropy and magnetic field provide the formation of the disc-like flow along the equatorial plane (which corresponds to the tore) and to the jet-like flow along the rotational axis, even at the conventional uniform magnetization of the wind.

The radius of the collimation of the post-shock flow certainly differs from the radius for the isotropic wind. This occurs because the velocity of the disc-like flow decreases with distance more slowly than in the case of isotropic wind.

**Figure 9.** Flow lines with distribution of \( \beta \) in the post-shock region for the wind with parameters typical for the wind from Crab pulsar, \( \sigma = 3 \times 10^{-3} \), anisotropy level \( 10^3 \) for the age of the plerion 900 yr past the beginning of interaction of the pulsar wind with ISM. Arbitrary time moment.
We found that the jet-like flows are already formed in the post-shock region at $\sigma = 3 \times 10^{-3}$. This conclusion slightly differs from the conclusion of Komissarov & Lyubarsky (2003). They claim that it is necessary that $\sigma$ is at least 3 times higher in order to provide the collimation of the flow. In their simulations jets are not produced at small $\sigma$.

The jet-like flow is filled by the plasma from disc-like flow. This flow is decelerated and directed towards the rotational axis by the tension of the toroidal magnetic field lines. Near the axis this flow is redirected again along the axis. Thus, the jet-like flow is formed only at the distance comparable with $r_{\phi}$. This disagrees with observations of the Crab nebula, which show that the jets are formed at rather small distance from the pulsar compared with the location of the shock front at the rotational axis. The location of the visible launching point of the jets could be placed closer to the pulsar if it is assumed that $\sigma$ is higher at the rotational axis than at the equator. Apparently, the toroidal magnetic field could be taken as strong enough to provide collimation of the part of the flow along the rotational axis right down the shock front. Future numerical experiments will show us if it is really possible.

We do not calculate accurately the expected brightness distribution of the synchrotron radiation from the Nebula, because this is rather complicated work which is not the objective of this paper. We will perform such a calculation later. For a crude estimate we calculated the distribution of the total synchrotron emissivity as the product

$$I = B^2 \epsilon^2 n,$$

where $\epsilon = p/n$ is the average energy of particles and $n$ is density of plasma. In this estimate we do not take into account synchrotron cooling of electrons, which is especially important for the calculation of the spatial distribution of X-rays. However, in any case this distribution is useful because it gives upper limits on the size of the X-ray nebula.

We see that the toroidal structure is reproduced naturally. It extends to a distance $\approx 8r_{\phi}$ along the equator. This is larger than the observed extension $\sim (3-4) r_{\phi}$, but this discrepancy could be removed if one takes synchrotron cooling into account.

Figure 10. Volume synchrotron emissivity of the plerion excited by the Crab-like wind, with arbitrary units and a logarithmic scale. The distribution is taken for the same time-moment as Fig. 9.

Usually, the brightness of the jet-like flow is rather low. This is a general problem, which arises in all calculations of the spatial brightness of the plerions (Bogovalov & Khangoulian 2002a; Komissarov & Lyubarsky 2003). To overcome this problem (Komissarov & Lyubarsky 2003) assumed very large poloidal magnetic field in the jet-like flow. The pressure of this field is as large as 30 per cent of the gas pressure. There are no ideas on how to produce so high a poloidal magnetic field in the jets. Our rather crude estimates of the synchrotron brightness show, nevertheless, that the jets could be visible on the background of the tore. However, again, for final conclusion it is necessary to perform calculations of the brightness distribution taking into account the synchrotron cooling of the electrons.

Bogovalov & Khangoulian (2002a) suggested another mechanism. It is clear that the jet-like flow is collimated by the toroidal magnetic field. This flow must be unstable in relation to the kink instability. This instability is well observed in the case of Vela (Pavlov 2003). Some part of the magnetic field energy is transformed into the kinetic energy of the particles. Therefore, it is natural to assume that an additional acceleration of particles takes place in the jetted flow. The fact that the X-ray jets extend to a larger distance than the tore is also in favour of an additional acceleration of particles in the jets. These particles can brighten the jets to the observed level (Bogovalov & Khangoulian 2002a,b).

4.4 Variability of plerions

The very first observations of the Crab nebula in the optical range (Lampland 1921) demonstrated intrinsic time-variability in the plerion. Spectacular films obtained by HST and Chandra demonstrate the formation and evolution of moving ‘wisp’ in the very central part of the tore. Several mechanisms have been proposed to interpret this phenomena. Begelman (1999) suggested that the wave-like features reflect a Kelvin–Helmholtz instability between the equatorial and higher latitude flows. This is what we really observe in the pure hydrodynamical case. Hester (1998) suggested that the wisp features are compressions comoving with the flow, created by thermal instability driven by synchrotron cooling. In the sequence of work by Arons (2002), Hoshino et al. (1992), Gallant & Arons (1994) and Spitkovsky & Arons (2004) argued that the ‘wisps’ are the result of the modulation of the post-shock flow with Larmor radius of ions present in the pulsar winds. This interesting idea demands the acceleration of ions to the Lorentz factors of the order of $3 \times 10^6$ in the pulsar magnetosphere. Unfortunately, no mechanism is known that provides this acceleration.

The numerical simulations show that the temporal variability of the tore structure arises naturally at the MHD interaction of the anisotropic magnetized winds with ISM. We note that the hydrodynamic flow with the anisotropic distribution of the energy flux shown in Fig. 4 is steady. We do not discuss here the trivial expansion of the plerion itself, nor the small-scale Kelvin–Helmholtz instability of the flow near the tangential discontinuity that is formed at the interface of the fast flow of plasma which has just crossed the shock and slow flow of the plerionic plasma (see Begelman 1999). The flow of the magnetized plasma with isotropic energy flux shown in Fig. 6 is steady in the post-shock region as well. The combination of two factors (the magnetic field and the anisotropy of the energy flux in the pulsar wind) result in a global variability of the flow in the post-shock region. Variation of $\beta$ with time at the point located at $r = 2.6r_{\phi}$ in the equatorial plane is shown in Fig. 11. It is easily seen that the variation is quasi-periodic, with a period $\sim 40$ yr (Fig. 12).
These large-scale variations of the post-shock flow are accompanied by the variations in the position of the terminating shock.

A sequence of snapshots in different phases of the 40-yr period are shown in Fig. 13. Short time-scale variations with characteristic time $\sim 2$–3 yr occurs on the background of the 40-yr quasi-periodical variations (see Fig. 12). They are visible in Fig. 13 as a process of fragmentations of the disc-like flow in small clouds, which is especially clear in the central left-hand panel of Fig. 13 (seventh line). In this panel the small-scale bright features at the distances of 2, 4 and 6 $r_{sh}$ which move out from the terminating shock are easily visible. This phenomena apparently reproduces the process of the ‘wisps’ formation. Similar variations were observed by Chandra during the past several years (Shibata et al. 2003). The estimated period of variations is $\sim 3$ yr. This agrees well with the theoretical value.

In the case of the Crab nebula the numerical experiments allow us to reproduce the most spectacular features in the central part of the plerion. However, this object is still the only example of a successful application of the shock model to plerions. Application of this theory to other plerions, in particular for Vela, has been unsuccessful up to now. Low X-ray luminosity of the Vela plerion and the small size speak in favour of relatively high magnetization of the wind from the Vela pulsar, $5 \times 10^{-2} < \sigma < 0.5$ (Sefako & de Jager 2003). Therefore, the plasma dynamics in the post-shock region for $\sigma > 3 \times 10^{-2}$ is of particular interest.

### 4.5 Anisotropic winds with $\sigma = 0.03$ and 0.1

The post shock plasma flow for $\sigma = 0.03$ (10 times higher than for Crab) is shown in Fig. 14. This flow is non-stationary, as in the case of Crab. The flow in the arbitrary time-moment is shown. The magnetic field in the wind is only 3.3 times higher than in the case of Crab. However, the emissivity distribution changed essentially. The radius of the tore is now comparable with the radius of the shock front.

An increase of $\sigma$ leads to reduction of the plerionic radius. The post-shock flow of the wind with $\sigma = 0.1$ is shown in Fig. 15. It is reasonable to expect that the luminosity of the nebula reduces with $\sigma$ as well.

The plasma flow in the post-shock region is not a steady state. The flow varies with time quasi-periodically. An increase in the magnetic field reduces the period of oscillations and improves their quality. The variation of $\beta$ with time is shown in Fig. 16 for $\sigma = 0.03$. The period of oscillation reduces from 40 yr for Crab to $\sim 4$ yr. This means that the period somehow scales with the magnetization. In addition, the oscillations become more close to harmonic ones.
The numerical simulations show that MHD interaction of the pulsar winds with ISM result in the formation of all of the most spectacular structures in the central part of the plerions. However, one important feature is not reproduced. The X-ray tore in Vela consists of two concentric rings of equal radius. The crude estimate of the synchrotron emissivity shows no signature of the division of the tore into two rings. It is still too early to make the final conclusion that the appearance of these rings are impossible in the simple model under the consideration, because it is necessary to perform calculations of the surface brightness taking into account the evolution of the electron spectrum due to synchrotron cooling and adiabatic losses. However, it is important to take into account that these rings can easily be reproduced at a small modification of the wind characteristics.

The structure of two concentric rings could naturally arise if providing the relative brightening of plasma down the secondary shock fronts is located symmetrically at some distance from equatorial plane. This can be achieved in several ways. For example,

Komissarov & Lyubarsky (2003) assumed that the toroidal magnetic field in the wind goes to zero at the equator. This way the generation of synchrotron radiation right at the equator is suppressed. Another reason for the brightening of the plasma down-flow from the secondary shock could be the more efficient acceleration of particles in comparison with the acceleration at the equator only. A more detailed study is necessary to clarify this question.

5 Discussion

Numerical simulations show that the concentration of the energy flux at the equator, as it is predicted from the theory of the magnetized astrophysical winds, leads to formation of a disc-like flow in the post-shock region and secondary shock fronts located symmetrically in relation to the equatorial plane. This property of the pulsar winds provides the formation of the toroidal structure in the central part of the plerions.

A weak magnetization of the wind results in the formation of the jet-like flow along the rotational axis in the post-shock region. There is no threshold for this process, unlike in the conclusions of Komissarov & Lyubarsky (2003). Any small magnetization finally leads to the collimation of the flow. The reduction of $\sigma$ only leads to the increase of the collimation radius. It is important that the observed structures are very sensitive to the wind magnetization. This opens an interesting way to define the wind magnetization from the observed structure in the central part of plerions. In the case of the Crab nebula, the conventional magnetization $\sigma = 3 \times 10^{-3}$ leads to a reasonable distribution of the flow, which agrees with observations. Actually, the observed structure of the Crab nebula gives us a new independent argument that the wind from the Crab pulsar has a low magnetization in the pre-shock zone. The magnetization of the wind from Vela is certainly higher. Accurate calculations are necessary to estimate $\sigma$ for the wind from Vela. At the present time it is clear that $\sigma \geq 0.01$. However, in any case, the wind from Vela pulsar has a low magnetization as well.

The energy flux concentration at the equator and the magnetization are inseparable properties of the pulsar winds. The success in interpreting the spatial structure of the plerions in frameworks of the shock model proves that we are on the correct path. To obtain a quantitative agreement of the theoretical predictions with
observations it is necessary to specify more appropriate characteristics of the pulsar winds.

Apparently, the most difficult problem will be connected with characteristics of the jet-like flow. It follows from our work that the plerionic jets are formed too far from the shock front. The simplest way to form jets just down the shock front at the axis is to assume that the magnetization of the wind grows with latitude. If it is really so, this would be important information for the theoreticians developing a mechanism for the pulsar wind acceleration.

**ACKNOWLEDGMENTS**

The work is performed under support of collaborative INTAS-ESA grant N 120-99 and RFBR grants N 03-02-17089 and N 03-02-16548.

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