

DISCUSSION

of the paper at small deflections, a fact the authors could not establish owing to lack of knowledge of material properties. However, at large deflections, even the strain-hardening solution considerably underestimates the load. In computing P from equation (2) the value of $\alpha = 5$ was used, this having been measured during the test. This measurement further confirmed the constant magnitude of s . The selected strain-hardening modulus will tend to overestimate the stress at high strains and the computed strain-hardening solution would therefore be expected to represent a maximum enhancement of P as a result of strain hardening. The writer would welcome any comments the authors may have on this discrepancy.

Authors' Closure

The authors wish to thank Mr. Redwood for his comments on the strain hardening problem and for his experimental verification of formula (14) when deflections are small. As will be recalled, it was noted in the paper that this equation was an underestimate of the true load and that the discrepancy could be attributed to strain hardening which had been neglected in obtaining the equation. However, as Mr. Redwood has shown here, a seemingly adequate approximation to the strain hardening solution itself underestimates the load when the deflections are no longer small.

This discrepancy is not surprising since a basic assumption made in this work does not appear to be valid for deflections of the order of the tube radius. In the experimental work quoted by the authors it was observed that, when the deflections became sufficiently large, the top and bottom hinges began to recede from the loading plates. Such behavior has not been included in the above work. When this is done two additional sources of load increase present themselves:

1 Of probably smaller importance is the additional curvature change at the top and bottom hinges. This in itself would probably not affect the total load appreciably.

2 One should consider that, in any experiment, friction is present. In this case, as the hinges recede from the loading plates, sliding occurs between the tube and the plates. Thus frictional forces act on the tube at the points of contact, being directed outward from the vertical axis of symmetry. These forces initiate further changes in curvature throughout the regions between contact points in addition to creating a couple which must be considered in gross equilibrium. This couple depends upon the distance that the hinges have receded and apparently is enough to increase the load necessary to maintain plastic deformation.

It is not known, of course, whether the curve shown in Fig. 1 of this discussion was obtained using a lubricant between the tube and the plates. If not, it is possible that, by doing so, a better agreement can be found between the experimental and analytical results. Conversely, the same goal may be attained by considering the above effects in the analytical work.

Funicular Polyhedra¹

M. L. WIEDMANN.² The author has presented and outlined a solution for an interesting problem which also has application in certain types of modern link structures. As noted, solutions for nets with more than a few meshes would imply the use of electronic computing machines. It would appear that the method may be extended to nets in currents and to certain structures with wind or special loadings.

¹ By L. W. McKeenan, published in the June, 1963, issue of the JOURNAL OF APPLIED MECHANICS, vol. 30, TRANS. ASME, vol. 85, Series E, pp. 219-224.

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The Momentum Integral Approximation for Compressible Magnetogasdynamics Boundary-Layer Flow¹

M. K. JOVANOVIĆ² and **D. R. HAWORTH**² In the writers' opinion the solution presented to the particular problem of this paper represents a definite contribution to the general subject of boundary-layer flows in the presence of a magnetic field. Despite the imposed restrictions which are clearly stated, the results of this paper indicate qualitatively and to some extent quantitatively the effect of a transverse magnetic field on the coefficients of viscous and total drag for compressible boundary-layer flow with a zero pressure-gradient field.

There are two rather insignificant points which we consider require clarification: Under the discussion of the velocity profile, the authors state that for very small values of the interaction parameter, the velocity profiles may be assumed to be "similar." Referring to their definition of the interaction parameter,

$$Q = \frac{\sigma B^2 L}{\rho u}$$

it appears that Q would increase as the vicinity of the wall is approached because $u \rightarrow 0$. This inconsistency is avoided provided L in the definition of Q represents a characteristic length perpendicular to the direction of flow; i.e., if $L \equiv y$.

Also it was stated that "for constant Mach number the magnetic field must increase the boundary-layer thickness causing the velocity gradient at the wall to be decreased." From Fig. 2 it follows that the coefficient of viscous drag, C_f , is decreased by applying a magnetic field, while maintaining the same Mach number in the main flow; i.e., $C_f < C_{f, \text{no m}}$. As a consequence of equations (18) and (24) it follows that the velocity gradient at the wall must be smaller with the magnetic field applied than with no magnetic field. Owing to the decrease of the velocity gradient at the wall and with the assumption of a linear velocity profile, the thickness of the boundary layer must increase. Therefore, the statement of the paper seems to put the "effect" before the "cause."

Unfortunately, with the assumption of similar velocity profiles, which is true only for the case of zero interaction parameter, the results of the authors do not indicate the true shape of the velocity profiles. Such profiles, however, can be deduced from the analysis of incompressible flow given by Rossow.³ Along this line it might be of interest to mention that R. D. Hugelman is working on a similar problem in his doctoral dissertation at Oklahoma State University. His case applies to incompressible flow, and in the analysis a velocity profile of the following form was assumed:

$$\frac{u}{u_\infty} = a \left(\frac{y}{\delta} \right) + b \left(\frac{y}{\delta} \right)^2 + c \left(\frac{y}{\delta} \right)^3 + d \left(\frac{y}{\delta} \right)^4 + e \left(\frac{y}{\delta} \right)^5$$

The coefficients $a, b, c, d,$ and e are functions of x (the coordinate measured along the plate from the leading edge), and the remaining symbols are the same as in the paper under discussion. In this way similar velocity profiles did not have to be assumed; and using an approximate solution of the momentum equation, velocity profiles in the boundary layer were predicted in good agreement with the results of Rossow.³

W. T. SNYDER.⁴ The authors apply the well-known momentum integral method to the determination of the drag of an adiabatic

¹ By J. J. Kauzlarich and A. B. Cambel, published in the June, 1963, issue of the JOURNAL OF APPLIED MECHANICS, vol. 30, TRANS. ASME, vol. 85, Series E, pp. 269-274.

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³ Reference [3] of the paper.

⁴ Thermal Sciences Department, State University of New York at Stony Brook, N. Y.

flat plate immersed in an electrically conducting fluid in the presence of a uniform transverse magnetic field. Although the authors make no specific mention of the electric field, one must infer from the equations that they are considering the short-circuit case. The assumption of Prandtl number unity leads to a solution of the energy equation in the form $T = T(u)$ as is the case for the flat-plate compressible boundary layer in the absence of a magnetic field. The authors state that the assumption of constant properties is implied in the derivation of equation (4). The only assumption necessary, in addition to the $Pr = 1$ assumption, is $C_p = \text{const}$.

The most serious assumption in the analysis is that of similarity of the velocity profiles. It is known^{5,6} that for similarity solutions to exist, the magnetic field must vary in the longitudinal direction as $B \sim x^{-1/2}$. When a similarity solution does not exist, a series solution is necessary which utilizes as an expansion parameter an appropriate function of the longitudinal coordinate. It is therefore not possible to assess quantitatively the validity of the present analysis without the more exact analysis as a basis of comparison. The qualitative trends appear to be valid.

Authors' Closure

It is a pleasure to acknowledge the contributions of Messrs. Jovanovic, Haworth, and Snyder in review of the paper. Only one point needs correction, and that concerns the dimensionless

⁵ W. B. Bush, "Incompressible Flat Plate Boundary-Layer Flow With an Applied Magnetic Field," *Journal of the Aero-Space Sciences*, vol. 27, 1960, pp. 49-58.

⁶ P. S. Lykoudis, "On a Class of Compressible Laminar Boundary Layers With Pressure Gradient for an Electrically Conducting Fluid in the Presence of a Magnetic Field," Proceedings of the 9th International Astronautical Federation, Springer-Verlag, Vienna, 1959, vol. 1, pp. 168-180.

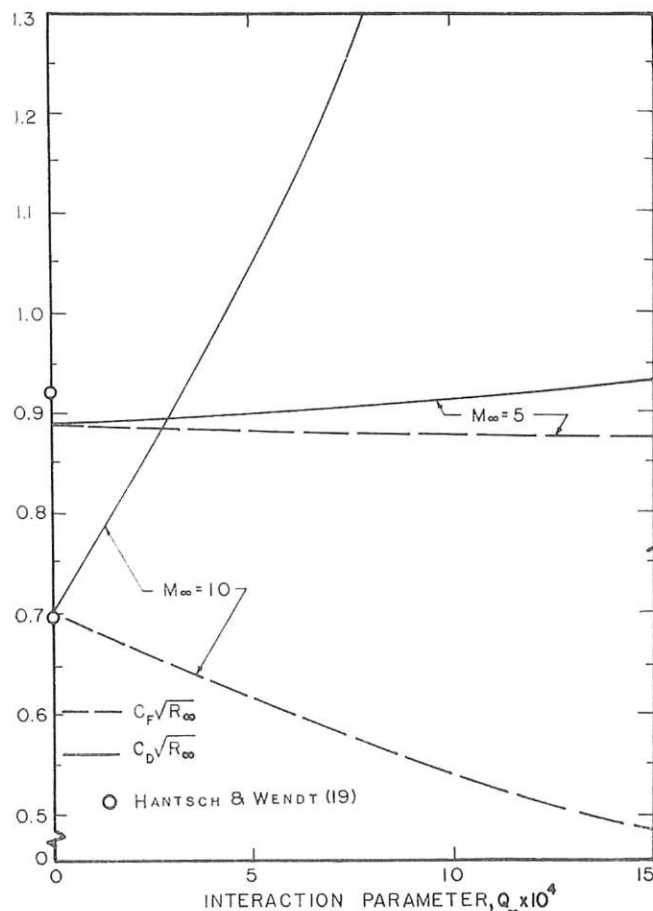


Fig. 1 Drag parameters versus interaction parameter for compressible MHD flow

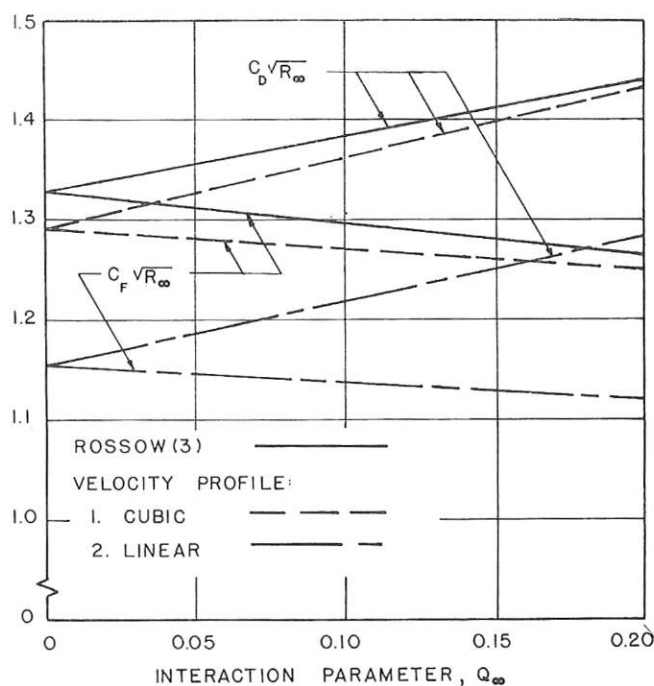


Fig. 2 Drag parameters versus interaction parameter for incompressible MHD flow, and comparison with an exact solution

numbers which are all evaluated at the free stream except for a reference electrical conductivity.

The results of the paper are presented in a more useful form in Figs. 1 and 2 of this Closure. A calculation for the drag coefficient at zero interaction parameter by Hantsch and Wendt⁷ is plotted in Fig. 1, and gives an idea of the improvement of the non-magnetic solution with Mach number. Fig. 2 includes a plot of the solution based on a cubic velocity profile assumption. Assuming a cubic velocity profile only changes the numerical coefficient in equations (35) and (36) from $2/\sqrt{6}$ to 0.913, but does not affect the results plotted in Fig. 3 of the paper. Fig. 2 shows an improved comparison with the exact solution by Rossow⁸ for the assumption of a cubic velocity profile as expected according to experience with zero interaction parameter solutions.

⁷ Reference [19] of the paper.

⁸ Reference [3] of the paper.

The Unsteady Forces Due to Propeller-Appendage Interactions¹

J. P. BRESLIN.² I am grateful for this second opportunity to discuss this paper, having been unable to do so when it was previously given at the Fourth Symposium on Naval Hydrodynamics in Washington, D. C., August, 1962. If this version is different in any essential way from that given last August, such differences have not been made apparent.

The authors hold that this study extends the work which I did in 1956³ in several important respects by representing the effect of

¹ By O. Pinkus, J. R. Lurye, and S. Karp, published in the June, 1963, issue of the *JOURNAL OF APPLIED MECHANICS*, vol. 30, Trans. ASME, vol. 85, Series E, pp. 279-287.

² Director, Davidson Laboratory, Stevens Institute of Technology, Hoboken, N. J.

³ J. B. Breslin, "The Unsteady Pressure Field Near a Ship Propeller and the Nature of the Vibratory Forces Produced on an Adjacent Surface," Davidson Laboratory Report 609, June, 1956.