flat plate immersed in an electrically conducting fluid in the presence of a uniform transverse magnetic field. Although the authors make no specific mention of the electric field, one must infer from the equations that they are considering the short-circuit case. The assumption of Prandtl number unity leads to a solution of the energy equation in the form $T = T(u)$ as is the case for the flat-plate compressible boundary layer in the absence of a magnetic field. The authors state that the assumption of constant properties is implied in the derivation of equation (4). The only assumption necessary, in addition to the Pr = 1 assumption, is C_p = const.

The most serious assumption in the analysis is that of similarity of the velocity profiles. It is known[5,6] that for similarity solutions to exist, the magnetic field must vary in the longitudinal direction as $B \sim x^{-1/2}$. When a similarity solution does not exist, a series solution is necessary which utilizes as an expansion parameter an appropriate function of the longitudinal coordinate. It is therefore not possible to assess quantitatively the validity of the present analysis without the more exact analysis as a basis of comparison. The qualitative trends appear to be valid.

Authors' Closure

It is a pleasure to acknowledge the contributions of Messrs. Jovanovic, Haworth, and Snyder in review of the paper. Only one point needs correction, and that concerns the dimensionless numbers which are all evaluated at the free stream except for a reference electrical conductivity.

[5] W. B. Bush, "Incompressible Flat Plate Boundary-Layer Flow With an Applied Magnetic Field," *Journal of the Aero-Space Sciences*, vol. 27, 1960, pp. 49–58.

[6] P. S. Lykoudis, "On a Class of Compressible Laminar Boundary Layers With Pressure Gradient for an Electrically Conducting Fluid in the Presence of a Magnetic Field," Proceedings of the 9th International Astronautical Federation, Springer-Verlag, Vienna, 1959, vol. 1, pp. 168–180.

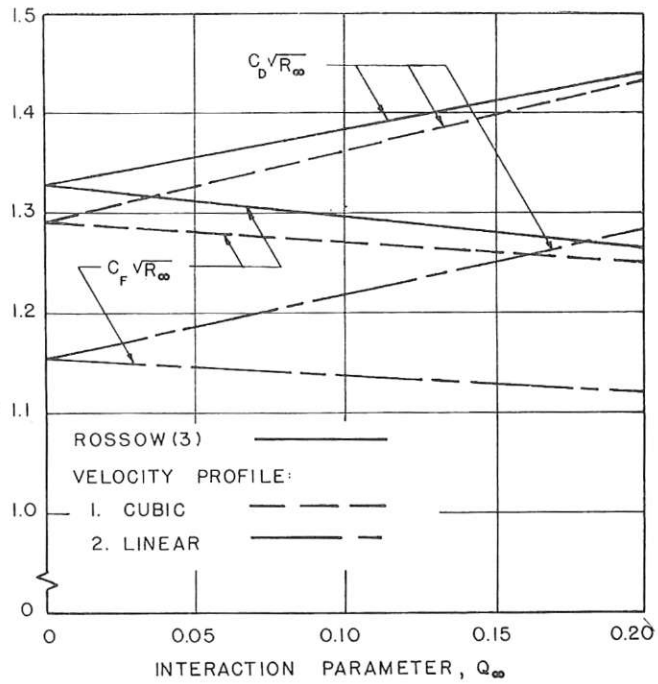

Fig. 2 Drag parameters versus interaction parameter for incompressible MHD flow, and comparison with an exact solution

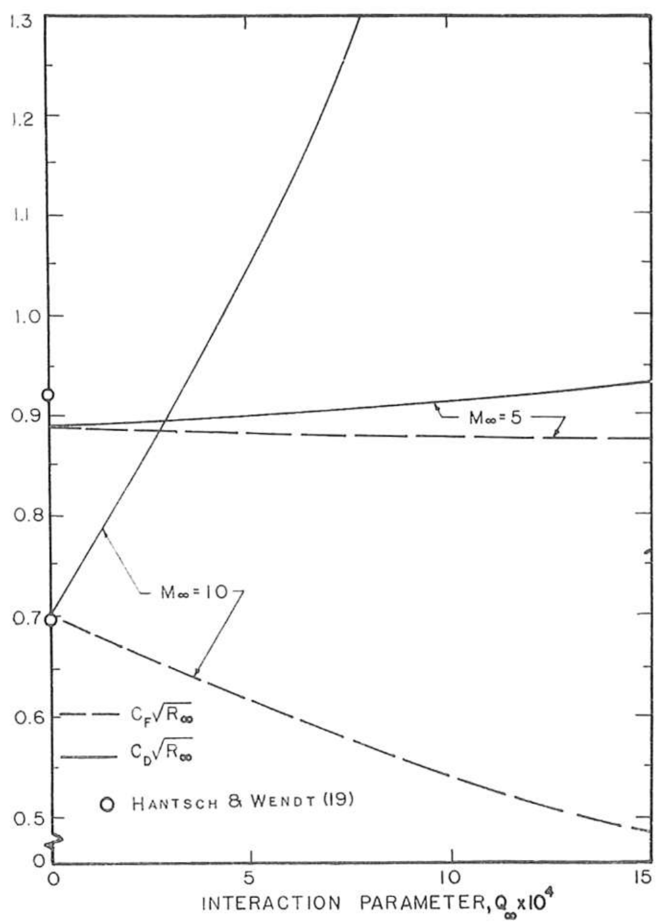

Fig. 1 Drag parameters versus interaction parameter for compressible MHD flow

The results of the paper are presented in a more useful form in Figs. 1 and 2 of this Closure. A calculation for the drag coefficient at zero interaction parameter by Hantsch and Wendt[7] is plotted in Fig. 1, and gives an idea of the improvement of the non-magnetic solution with Mach number. Fig. 2 includes a plot of the solution based on a cubic velocity profile assumption. Assuming a cubic velocity profile only changes the numerical coefficient in equations (35) and (36) from $2/\sqrt{6}$ to 0.913, but does not affect the results plotted in Fig. 3 of the paper. Fig. 2 shows an improved comparison with the exact solution by Rossow[8] for the assumption of a cubic velocity profile as expected according to experience with zero interaction parameter solutions.

[7] Reference [19] of the paper.

[8] Reference [3] of the paper.

The Unsteady Forces Due to Propeller-Appendage Interactions[1]

J. P. BRESLIN.[2] I am grateful for this second opportunity to discuss this paper, having been unable to do so when it was previously given at the Fourth Symposium on Naval Hydrodynamics in Washington, D. C., August, 1962. If this version is different in any essential way from that given last August, such differences have not been made apparent.

The authors hold that this study extends the work which I did in 1956[3] in several important respects by representing the effect of

[1] By O. Pinkus, J. R. Lurye, and S. Karp, published in the June, 1963, issue of the JOURNAL OF APPLIED MECHANICS, vol. 30, Trans. ASME, vol. 85, Series E, pp. 279–287.

[2] Director, Davidson Laboratory, Stevens Institute of Technology, Hoboken, N. J.

[3] J. B. Breslin, "The Unsteady Pressure Field Near a Ship Propeller and the Nature of the Vibratory Forces Produced on an Adjacent Surface, Davidson Laboratory Report 609, June, 1956.

the finite chord and by accounting for the effect of change of lift or thrust of the blade section arising from the angle of attack induced by the flow about the barrier or appendage. These are certainly interesting and important extensions of the very crude representation which I employed. However, in lieu of solving this extended problem, it seems to me that they have retreated from attacking the problem which they posed by resorting to the use of the "substitution vortex" concept. They have also ignored, in the process of simplification, the influence of the flow induced by the vortices shed from the appendage on the force developed by the propeller blade section. One would expect that this interference would be large in the case that the blade passes downstream of the appendage and hence cuts through the street of shed vorticity.

From a practical point of view, it does not seem reasonable to consider the blade section as extending along a line parallel to the y-axis, since this would imply an angle of attack of -27 deg for a ratio of blade speed to free stream speed of 2.0 which has been used herein. For a propeller blade developing lift from angle of attack, the clearance between the trailing edge of the plate and the leading edge of the blade would be the essential parameter in view of the fact that the lift is concentrated near the leading edge. The blade should therefore be placed at the pitch angle formed by the tangential and axial velocity vectors.

One of the more important extensions of this problem, which has not been considered here nor in the stator-rotor problem solved by Kemp and Sears[4] in 1953, is the inclusion of the influence of blade thickness. Study of the fluctuating pressure field about a ship propeller model has shown that the pressure arising from blade loading alone is insufficient to account for the character and the magnitude of measured blade-frequency pressure distributions. It has been found that both the magnitude and the character of the pressure signature can be accounted for by the sum of the effects stemming from blade loading and blade thickness distribution. This may be seen in Fig. 1.

The effect of blade thickness can be included in this two-dimensional problem by calculating the force produced by a single point source (in two dimensions) and by then integrating this result over the distribution of such sources required to form a symmetrical section. As a matter of fact, both the source and vortex disturbances can be calculated at once by recognizing that the complex potential can be expressed by

$$w = \sigma \ln (z - z_0) \tag{1}$$

where

$$\sigma = -m \quad \text{for a source} \tag{2}$$

$$= ic \quad \text{for a vortex} \tag{3}$$

Then the quasi-steady circulation density γ_0 needed to construct the flat appendage is found by inverting the integral equation:

$$\int_{-l}^{l} \frac{\gamma_0(\xi)d\xi}{x - \xi} = Im\left(\frac{\sigma}{x - z_0}\right) \tag{4}$$

where $z_0 = x_0 + iy_0$ = position of disturbing singularity.

The total force on the plate due to a concentrated moving singularity is then

$$F = {}^1/_2F_0 + F_1 \tag{5}$$

$$F_0 = 2\pi\rho U\, IM \left\{\sigma\left[\frac{z_0 - l}{z_0 + l} - 1\right]\right\} \tag{6}$$

$$F_1 = 2\pi\rho \frac{d}{dt} IM\{\sigma[(z_0^2 - l^2)^{1/2} - z_0]\} \tag{7}$$

where IM designates "imaginary part of."

The extension to distributed singularities is straightforward, yielding forms which can be integrated but which are algebraically unwieldy. An IBM 1620 was used to evaluate the result for the case of a section having a thickness ratio of 0.05 sweeping past a plate at $x_0 = 1.2l$. Comparisons are made in terms of thrust by introducing a lift coefficient $C_L = T/{}^1/_2\rho(2c)V^2$ which is taken at a practical value of 0.20. Results are shown in Fig. 2, where it may be observed that the effect of thickness is to produce a force of the order of 35 percent of the blade thrust loading.

As a final remark, it is my opinion that these two-dimensional analyses are of doubtful application to the case of ship propeller-appendage interactions because the flow is highly three-dimensional. It would seem much more worthwhile to re-examine the turbine rotor-stator interaction problem by accounting for the blade thickness. In that case, the flow is highly two-dimensional and the blades have very high thickness ratios.

[4] N. H. Kemp and W. R. Sears, "Aerodynamic Interference Between Moving Blade Rows," *Journal of the Aeronautical Sciences*, vol. 20, 1953, pp. 585–597.

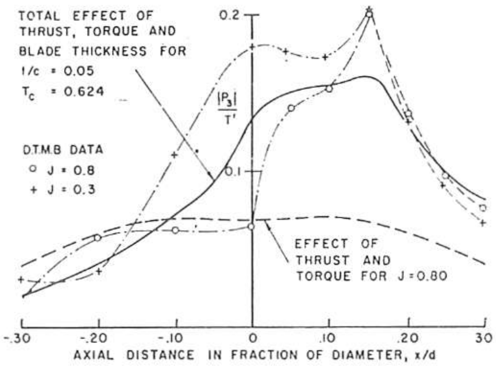

Fig. 1 Variation of blade-frequency pressure amplitude with axial distance from a 3-bladed propeller

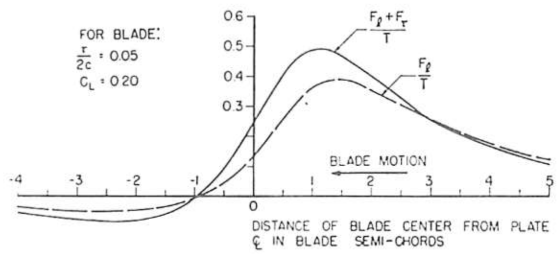

Fig. 2 Forces on appendage due to blade loading and sum due to loading and thickness

Geared Five-Bar Motion, Parts 1 and 2[1]

KURT HAIN.[2] The problems treated in these two investigations are of great significance in the further development of the science of mechanisms; for here we find as yet untapped possibilities of synthesizing complex motions with a relatively small amount of mechanism. Particularly noteworthy, however, is the simplicity of the mathematical results for motion-producing devices of this or equivalent type.

In order to encourage further development along these lines, I should like to cite some further mechanisms, which are based on the five-bar chain and additional gearing.

Fig. 1 shows the five-bar linkage studied by Freudenstein and

[1] By F. Freudenstein and E. J. F. Primrose, Part 1; and E. J. F. Primrose and F. Freudenstein, Part 2, published in the June, 1963, issue of the JOURNAL OF APPLIED MECHANICS, vol. 30, TRANS. ASME, vol. 85, Series E, pp. 161–175.

[2] Research Engineer, Institut für Landtechnische Grundlagenforschung der Forschungsaustalt für Landwirtschaft, Braunschweig, Germany.