The Effect on Micropulsations of a Localized Earth Conductivity Anomaly

W. J. Hughes

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Summary

The effect on the surface magnetic micropulsation fields of a localized conductivity anomaly is studied. It is found that, whereas the largest variations in the horizontal component occur over the anomaly, those of the vertical component occur at a distance to either side, the distance depending on the depth of the anomaly. The frequency most affected is also found to be a function of the depth of the anomaly.

Introduction

The effect on ground based micropulsation measurements of currents induced by inhomogeneities of the Earth’s conductivity has been studied previously (Rankin & Reddy 1972; Hughes 1973). But it is also interesting to ask how much can be inferred about the local conductivity structure of the Earth from such measurements. Signals of micropulsation frequencies penetrate to a depth in the upper crust where many conductivity anomalies do occur. It seems likely, therefore, that micropulsations could prove to be a useful probing tool.

To further the understanding of how a conductivity anomaly can affect the magnetic fields on the surface, a very simple model was studied. The Earth was taken to have a uniform conductivity except for a small cylinder of higher conductivity in which enhanced currents could flow. This is the simplest two-dimensional model which has a localized anomaly. The cylinder of high conductivity can be thought of as a delta function in the two-dimensional conductivity profile. So the solution of this model is then effectively the Green’s function of the general two-dimensional problem.

Previously, several techniques of finding the theoretical response of particular anomalies have been developed. Because, in a two-dimensional problem, Maxwell’s equations split into two independent sets, one of which has the electric field parallel to the strike of the anomaly (E-polarization) and the other of which has the magnetic field in this direction (H-polarization), most mathematical models have been made two-dimensional. Weaver (1963) and Weaver & Thompson (1972) have attempted an analytical solution of a vertical discontinuity for the E-polarization case. While Swift (1970), Jones & Price (1970, 1971) and Jones & Pascoe (1972) have used computational techniques to solve for various geometries. An analogue method using scale models has also been developed by Dosso (1966a,b).

In previous work, Hughes (1973) found the effect on the surface magnetic field of a two-dimensional anomaly in which the conductivity varied periodically in a horizontal direction. The present work is an extension of this. By a Fourier transform of the solution of this model when in its simplest form, the model of a small cylinder of high conductivity in an otherwise uniform Earth is obtained. The effect
of such a structure on the horizontal and vertical components of the magnetic field is then computed. The presence of a real anomaly is normally inferred by anomalies in the normally small vertical magnetic component. As a vertical magnetic field is only induced in the $E$-polarization case, this work is restricted to this configuration.

In the above paper, a study was made of the scale size of the anomaly, and it was shown that scale lengths of the order of the skin depth gave rise to the largest anomalous fields. The advantage of studying a small localized anomaly is that the anomalous fields can be studied away from the anomaly. Thus it can be determined how the effect falls off with distance and how this alters with the depth of the anomaly. For this reason in this paper the anomalous fields are plotted against distance from the anomaly.

The method of obtaining the solution

The first model solved by Hughes (1973) consisted of three horizontally layered slabs, the middle of which had a conductivity which varied as $\sigma = \sigma_0 + \sigma_1 \sin vx$ where $x$ is measured in a horizontal direction. The other two slabs had uniform conductivity. The source field was taken as a uniform, normally incident, plane electromagnetic wave. By giving the two uniform slabs a conductivity of $\sigma_0$ and by making the middle, anomalous slab thin, the expressions for the surface field components given in that paper become much simplified to

$$H_z = \frac{-\omega \mu \sigma_1 \nu \exp \{i(\gamma_0 + \gamma_1)z\} \sin vx}{\gamma_0(\gamma_1 + iv)}$$

$$H_x = 1 + \frac{i \omega \mu \sigma_1 \nu \exp \{i(\gamma_0 + \gamma_1)z\} \cos vx}{\gamma_0(\gamma_1 + iv)}$$

where $t$ is the thickness of the anomalous slab, which is at a depth $z$; $\omega$ is the frequency and $\gamma_n^2 = i\omega \mu \sigma_0 - n^2 v^2$.

If the conductivity in the second slab is split into a normal and an anomalous part, then the surface field components could also be expressed in this manner, as is done by Schmucker (1970). The normal part of the vertical field is zero while the fields have been normalized so that of the horizontal component is unity. The anomalous parts are those containing $\sigma_1$.

By performing a Fourier transform on the anomalous part of the conductivity, we get a ‘delta function’ in conductivity. As the anomalous fields are given by a linear expression, we can similarly perform the transform on these and so obtain the surface response of such a conductivity anomaly.

The integration over $\nu$-space was done numerically and hence is equivalent to a Fourier series expansion, the value of the anomalous fields being summed for various values of $\nu$. The summation was carried out for values of $\nu = n\pi/L$, $n = 1, \ldots, N$. The result of this approximation to the integral was that the inhomogeneity was, in effect, repeated every $2L$. This was made large enough for the inhomogeneities to be so far apart as to not affect each other. Also, features of a scale length less than $2L/N$ could not be modelled. But, it was shown by Hughes (1973) that small scale features are not important.

Presentation of results

The size of the cylinder was made as small as possible to approximate it to a delta function. The values picked for $L$ and $N$ were 300 and 600 km respectively. This enabled the cylinder to be made 1 km wide. It was found that decreasing this any further had no discernible effect on the results. A background conductivity of 0.01 mho m$^{-1}$ was chosen as a typical Earth conductivity. Table 1 gives the skin
Localized conductivity anomaly

Table I

Skin depth in a medium of conductivity 0.01 mho m\(^{-1}\).

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Period (s)</th>
<th>Skin depth (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10(^{-3})</td>
<td>1000</td>
<td>159.3</td>
</tr>
<tr>
<td>3 \times 10(^{-2})</td>
<td>333</td>
<td>87.2</td>
</tr>
<tr>
<td>10(^{-2})</td>
<td>100</td>
<td>50.4</td>
</tr>
<tr>
<td>3 \times 10(^{-1})</td>
<td>33</td>
<td>27.6</td>
</tr>
<tr>
<td>10(^{-1})</td>
<td>10</td>
<td>15.9</td>
</tr>
<tr>
<td>3 \times 10^0</td>
<td>3</td>
<td>8.7</td>
</tr>
<tr>
<td>10^0</td>
<td>1</td>
<td>5.0</td>
</tr>
</tbody>
</table>

depths for periods between 1 and 1000 s in such a conductor. As the size of the parameters \(\sigma\) and \(t\) affect only the magnitude of the anomalous parts, their values can be picked at will. The values taken of \(t = 0.1\) km and \(\sigma_1 = 0.0001\) mho m\(^{-1}\) give an effective conductor of cross-sectional area 0.1 km\(^2\) and conductivity 0.016 mho m\(^{-1}\). This is a rather small increase in conductivity, but the anomalous fields would just be linearly scaled up if the increase was larger.

The results of the calculations are shown in Figs 1–4. Fig. 1 shows the effect of a localized anomaly 5 km below a point on the surface corresponding to 0 km on the abscissa. The four graphs are (a) the amplitude of the vertical magnetic component; (b) the phase of the vertical component; (c) the change in amplitude of the horizontal component; and (d) the phase of the anomalous horizontal component. The values are plotted along a surface traverse of the anomaly. All are given for various periods of incident signal.

Fig. 1(a) shows that the anomalous vertical field is zero immediately above the anomaly, but reaches a maximum about 5 km either side of it. The effect is greatest for a period of 3 s. This is approximately the period which has a skin depth equal to the depth of the anomaly in this region. Fig. 1(b) shows that there is associated with this a 180° phase change in the anomalous vertical field above the anomaly. The combination of these two effects is well known to those who study anomalies using Parkinson arrows or transfer function analysis (Parkinson 1959, 1962; Everett & Hyndman 1967). These arrows always point towards a current concentration which necessitates a sudden reversal in direction as one passes over a current, and they are largest just to the side of a current.

Fig. 1(c) shows the change in amplitude of the horizontal component of the magnetic field. These curves have more than one turning point since the anomalous and normal parts are, in general, not in phase. It will be remembered that the normal part of the field was taken as unity, with zero phase, while the variation of the phase of the anomalous part is shown in Fig. 1(d). The curves in Fig. 1(c) show that most of the change is within 15 km of the anomaly, within which area the phase shows a marked change. Again it will be noticed that the effect is largest in the short period curves.

Fig. 2(a) and (b) show the amplitude of the vertical field and changes in amplitudes of the horizontal field for the same anomaly put at a depth of 20 km. These graphs are very similar to those in Fig. 1, but the effect is now more marked at greater distances from the anomaly. The maxima of the vertical field amplitude now occur at about 20 km and the largest corresponds to a period of 33 s. Plots of phase are not shown as these are also very similar to those in Fig. 1.

By burying the anomaly even deeper, the same trends are seen to continue. Fig. 2(c) and (d) show the fields produced by the same anomaly at a depth of 40 km. The maxima of the vertical field amplitude now occur between 20 and 40 km from the centre of the anomaly, the distance increasing with increasing period. The maximum effect now occurs for a period between 100 and 300 s. The same trends can
Fig. 1. (a) The amplitude of the vertical magnetic component; (b) the phase of the vertical magnetic component; (c) the change in amplitude of the horizontal component; and (d) the phase of the anomalous horizontal component for the anomaly at a depth of 5 km.
Fig. 2. (a) The amplitude of the vertical magnetic component; and (b) the change in amplitude of the horizontal component for the anomaly at a depth of 20 km; (c) and (d) as (a) and (b) but for the anomaly at a depth of 40 km.
FIG. 3. The amplitude of the surface vertical field as a function of the position of the anomaly with respect to the observer. (See text for details.)
Fig. 4. The vertical magnetic field as a function of frequency seen (a) 10 km; and (b) 30 km from above the anomaly; curves show values for various depths of the anomaly.

also be deduced from the graphs of change in horizontal field amplitude. For the deeper anomaly the maxima are broader and the largest maximum occurs at a longer period.

Thus, there are two effects which can help to determine the depth of a localized current; firstly the distance from the anomaly at which the largest vertical field occurs and secondly the period at which this is largest. The first effect is shown more clearly in Fig. 3. This is a contour plot of the amplitude of the vertical field seen at the surface drawn as a function of the position of the anomaly with respect to the observer. The top left-hand corner is the origin. The horizontal axis gives the horizontal distance to the anomaly and the vertical axis gives the depth of the anomaly. The crosses are at 4-km intervals making both axes 40 km long. Thus the value at the bottom right-hand corner is the value seen at the surface caused by an anomaly 40 km away horizontally and at a depth of 40 km. The plots on the left, marked un-normalized, give the value of the vertical field for the same anomaly at all depths.
These show that the anomalous field falls off logarithmically as the anomaly is sunk deeper, the fall off being faster for shorter periods. In the plots on the right, the fields have been normalized so that the same maximum field is seen at each depth. The wedge-shaped structure on which the anomaly must lie for the largest effect to be felt is clearly seen. The wedge becomes sharper at higher frequencies.

Fig. 4 shows how the effect peaks with frequency. Fig. 4(a) shows the amplitude of the vertical field at 10 km from the anomaly as a function of frequency. The curves are for different depths of the anomaly. Fig. 4(b) is this repeated for a distance of 30 km from the anomaly. All the curves have a gentle rise from low frequencies and then a faster fall off at high frequencies. This effect was noted as characteristic by Hughes (1973) and by Hyndman & Cochrane (1971) using experimental data. As most observational data come from a few stations but cover a large frequency band, these curves are closest to those produced from experimental data, and hence are those which should be compared with such data. These curves show how the amplitude peaks at a lower frequency as the anomaly becomes deeper. This is given approximately by the frequency for which the depth to the anomaly is the skin depth in the surrounding medium.

Conclusions

The effect of a localized anomaly on the surface magnetic fields has been studied. The most striking feature is the double peak in the vertical field, the distance between the peaks increasing as the anomaly is placed deeper. The frequency at which the vertical field is largest is also a function of the depth of the anomaly, and so can also be used to help determine the geoelectric structure.

However, the largest horizontal fields occur immediately above the anomaly. Thus the largest anomalous horizontal fields do not coincide with the vertical ones. But most micropulsation properties, such as polarisation characteristics, are calculated from the horizontal components. So the normal method of inferring the presence of an anomaly from the presence of a vertical component can sometimes be misleading. An observatory sited immediately above this anomaly would show a small vertical field, yet the polarisation characteristics would be significantly altered.

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Physics Department,
Imperial College,

References


