On the Question of Charge Symmetry in D-D Reactions*

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The question of charge symmetry in D-D reactions is discussed in light of the recently discovered virtual excited states of the alpha particle. We point out their effects on the asymmetries in the angular distributions as well as the provision of a mechanism for p-wave enhancement. Relation with three-nucleon isobars is also discussed.

§ 1. Introduction

Apart from their thermonuclear implications, the exothermic deuteron-deuteron reactions

\[ D + D \rightarrow p + H^3, \]  
\[ D + D \rightarrow n + He^3 \]

are also of interest from the viewpoints of nuclear structure and of general quantum mechanical symmetry principles. Of the latter, the most obvious fact is that the Bose statistics of the initial \( T = 0 \) system requires that there be no interference between the even and odd angular momentum partial waves; and that among the initial singlet, triplet and quintet spin states only the triplet state can couple with the odd partial waves. An immediate consequence of this is that the angular distribution of the reaction products must be symmetric about 90° in the center of mass system. Another fact which follows from overall angular momentum and parity conservation (neglecting the small admixture of weak interaction current) is that the ground states of the three-body nuclei have positive parity. Reaction (1a) also provides the most accurate test to-date of the reciprocity theorem in strong interactions. These observations do not depend on the reaction mechanism or on what goes on in the intermediate state.

In addition, recent measurements of reactions (1a) and (1b) have aroused some controversy with regard to the question of charge symmetry in the three-nucleon isobars. These measurements showed that the zero-to-ninety-degree asymmetry for the angular distribution of the final neutrons is strikingly larger than that of the protons. One might argue that this difference could arise from the difference in the mass radii of He\(^3\) and H\(^3\). Such a difference in the corresponding electromagnetic form factors of these nuclei has been observed in

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electron scattering experiments, for which explanations in terms of states of full and mixed symmetries have been reasonably successful. However, an excessively large difference in the mass radii would appear unlikely for several reasons; in particular, it would make the value of the axial vector coupling constants in the $^{3}$He beta decay inconsistent with values found from other nuclear beta decay experiments. Furthermore, it does not seem likely that this difference should be larger than that already implied by the binding energies.

The purpose of this paper is to point out that the question of charge symmetry, unlike many other symmetry properties in the $D-D$ reactions, does depend on the details of reaction mechanism. In particular, the recently discovered virtual excited states of the alpha particle are expected to play an important role. It is essential that the positions and widths of these levels be known before a quantitative statement can be made with regard to the mass radii of the three-body nuclei. Fortunately in reactions (1a) and (1b) we need only concern ourselves with the $T=0$ levels if total isospin is conserved. The known $T=0$ levels of the alpha particle are now a $0^+$ state around 20 MeV and a $2^-$ state around 22 MeV excitation energy. Both levels have a large width. Existence of higher $T=0$ states is not clear at this stage. Werntz has argued that the $0^+$ state may be a "breathing mode" collective excitation. de-Shalit and Walecka have found that the $2^-$ state emerges at the correct energy in a shell model calculation of the splitting of the negative-parity supermultiplets. Within these multiplets, there are two additional levels that bear the same quantum number ($T=0$) occurring around 24 MeV and 25 MeV. These however have been calculated with a bound state approximation which gives a zero width for all states. In reality, they may turn out to have too great a width to exist. Although it is possible that these states may be relevant in the $D-D$ elastic scattering we shall, for lack of further experimental evidence, illustrate our purpose with only the $0^+$ and $2^-$ states in what follows. The partial widths of these levels are expected to be different for decay through the proton and neutron channels, for reasons similar to those that prevent the triton and the $^{4}$He nucleus from being identical. Thus, the final state interactions for (1a) and (1b) in the vicinity of these resonances intensify the asymmetry between the two reaction channels. Furthermore, with respect to the ground state of $^{4}$He, the $D-D$ system has an energy excess of 23.85 MeV and is therefore closer to the negative parity state than to the positive parity state. This provides enhancement for the odd angular momentum cross-section. At low bombarding energies where one can safely keep only the $s$ and $p$ waves, this means a mechanism for $p$-wave enhancement. We shall estimate the enhancement factor by taking two simple Breit-Wigner forms for the $0^+$ and $2^-$ resonances. From the enhancement factors, relations between the widths of the resonances can be found. The three-body wave functions which enter into the calculation will be chosen to be consistent with the binding energies of the trinucleons (which are
empirically different for H$^3$ and He$^3$).

§ 2. Transition matrix

We shall treat the resonances in the $D$-$D$ reactions in a final state interaction formalism. For this purpose, we write the matrix element in the distorted wave Born approximation as

$$M = \langle X_i^- | \delta V_f | X_i^+ \rangle,$$

where the states are defined to be eigenvectors of Hamiltonians $H_i$ and $H_f$ which describe the separated nuclei plus the center-of-mass interactions between them, both in the initial and final states. The transition operator $\delta V_f$ is equal to $H_i - H_f$, where $H$ is the full Hamiltonian. It is a residual nucleon-nucleon interaction.

The $T=0$ initial isospin wave function can be written as

$$T_0 = \sum \left( \frac{1}{2} \frac{1}{2} \mu_i \mu_i | 00 \rangle \left( \frac{1}{2} \frac{1}{2} \mu_f \mu_f | 00 \right) \gamma_1^\dagger \gamma_2^\dagger \gamma_3^\dagger \gamma_4^\dagger, \right. (3)$$

where the subscripts of the spinors $\gamma$ refer to the four particles ($T_i=\frac{1}{2}$) in the $D$-$D$ system. The spin wave functions can be classified into a singlet ($S=0$), a triplet ($S=1$) and a quintet ($S=2$). In terms of the Pauli spin matrices of the four nucleons, these are

$$S_{SM} = \sum_{m_1m_2m_3m_4} \left( 11mm'|SM \right) \left( \frac{1}{2} m_1 m_2 | 1 m \right) \left( \frac{1}{2} m_3 m_4 | 1 m' \right) \rho_1^m \rho_2^m \rho_3^m \rho_4^m \quad (4)$$

with $S=0, 1, 2$ and $M=-S$ to $+S$. The explicit forms of these nine spin functions were fully displayed first by Schiff. The completely symmetrized (with respect to interchange of the two deuterons) initial state wave function then takes the following form:

$$X_i^+ = T_0 \left\{ (S_0 + \sum S_{2M}) [X_i^+ (\rho) + X_i^+ (-\rho)] + \sum S_{1M} [X_i^+ (\rho) - X_i^+ (-\rho)] \right\}, \quad (5)$$

where

$$X_i^+ (\rho) = \varphi_D (r_1 - r_2) \varphi_D (r_3 - r_4) \psi^\dagger (k \cdot R); \quad R = \frac{1}{2} (r_1 + r_2) - \frac{1}{2} (r_3 + r_4). \quad (6)$$

The relative momentum vector between the two deuterons is denoted by $k$. In Eq. (6) $\varphi_D$ is a normalized wave function of a deuteron and $\psi$ that of the relative motion. If the relative wave function $\psi$ is expanded in a partial wave decomposition

$$\psi (k \cdot R) = \sum_{l} \left[ 4\pi (2l + 1) \right]^{1/2} k^{l} \frac{1}{k R} u_l (k R Y^l (k \cdot R / k R), \quad (7)$$

it is seen from (5) that the even angular momentum partial waves are coupled to the singlet and quintet states and the odd ones to the triplet.

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* Notation of the Clebsch-Gordon coefficient is $(j_1 m_1 j_2 m_2 | JM)$ for coupling states $| j_1 m_1 \rangle$ and $| j_2 m_2 \rangle$ to form $| JM \rangle$.\]
For the final state, we couple the three-body wave function with the single nucleon wave function to form a $T=0$ singlet and triplet. We choose the fully symmetric $S$ state for the three-body wave function $(T=J=\frac{1}{2})$ so that the spin-isospin part is
\begin{equation}
R_{123}^{nm,0} = -\frac{1}{\sqrt{6}} \sum_{m_1 m_2} \left( \frac{1}{2} \right) \langle m_1 m_2 | 00 \rangle \rho_1^m \rho_2^m \rho_3^m \rangle \sum_{\mu_1 \mu_2} \left( \frac{1}{2} \right) \langle \mu_1 \mu_2 | 00 \rangle \eta_1^{\nu_1} \eta_2^{\nu_2} \eta_3^{\nu_3} \right) \\
+ \sum_{\mu_1 \mu_2} \left( \frac{1}{2} \right) \langle \mu_1 \mu_2 | 00 \rangle \eta_1^{\nu_1} \eta_2^{\nu_2} \eta_3^{\nu_3} \right) - \sum_{\mu_1 \mu_2} \left( \frac{1}{2} \right) \langle \mu_1 \mu_2 | 00 \rangle \eta_1^{\nu_1} \eta_2^{\nu_2} \eta_3^{\nu_3} \right) \\
\times \left[ \sum_{m_1 m_2} \langle m_1 m_2 | 00 \rangle \rho_1^m \rho_2^m \rho_3^m \rangle + \sum_{m_1 m_2} \langle m_1 m_2 | 00 \rangle \rho_1^m \rho_2^m \rho_3^m \rangle \right], \tag{8}
\end{equation}
where $n, \nu = \pm \frac{1}{2}$. Equation (8) is antisymmetric with respect to interchange of any pair of nucleons. Coupling Eq. (8) with the single nucleon state $(T=J=\frac{1}{2})$, we get
\begin{equation}
X_f^{-} = \sum_{S' M'} \left( \frac{1}{2} \right) \langle S' | S' \rangle \left( \frac{1}{2} \right) \langle M' | M' \rangle \langle \frac{1}{2} \rangle \langle \frac{1}{2} | 00 \rangle \rho_0^{\nu} \eta_0^{\nu} \eta_0^{\nu} X_f^{-} = B_{S' M'} X_f^{-} , \tag{9}
\end{equation}
where $S'=0, 1$ and $M' = -S'$ to $+S'$. The final state spatial wave function is
\begin{equation}
X_f^{-} = \phi_f [r_1 - r_2, r_3 - \frac{1}{2} (r_1 + r_2)] g_p [r_4 - \frac{1}{2} (r_1 + r_2 + r_3)]. \tag{10}
\end{equation}
In Eq. (10), $\phi$ is a symmetric $S$-state spatial wave function for the trinucleon. The relative wave function $g$ describes the scattering of the single nucleon of momentum $p$ by the recoiling nucleus. It can be expanded in a way similar to (7) (choosing initial momentum $k$ as axis),
\begin{equation}
g^{-}(p \cdot x) = 4\pi \sum_{L, S} i^L h_L^{-} (p x) Y_L^* \left( p \cdot k / pk \right) Y_L(k \cdot x / kx). \tag{11}
\end{equation}
The effects due to the virtual excited states of the alpha particle are completely described by the radial factor $h_L$. In particular, we shall assume the nuclear phase shifts for the appropriate partial waves to go through $\pi/2$ at the resonance energies.

It is sufficient for the present calculation to take an interaction (Serber force) of the form
\begin{equation}
\delta V = \frac{1}{2} (1 + P_{a}) V_0 f (r_3 - r_4), \tag{12}
\end{equation}
where $P_{a} = -\frac{1}{2} (1 + \sigma_3 \cdot \sigma_4) (1 + \tau_3 \cdot \tau_4) \eta$ is an exchange operator for the spin and isospin coordinates of 3 and 4. The effective interaction strength $V_0$ does not affect the angular distribution and will be taken to be the same for the channels (1a) and (1b). Substituting Eqs. (5)--(7), (9)--(11) and (12) into Eq. (2), we calculate the transition matrix element and find
\begin{equation}
M_{even} = V_0 \langle B_{S' M'} | \frac{1}{2} (1 + P_{a}) | T \phi S_{SU} \rangle M_0 \tag{13a}
\end{equation}
with $S=0, 2$ and
\begin{equation}
M_{odd} = V_0 \langle B_{S' M'} | \frac{1}{2} (1 + P_{a}) | T \phi S_{SU} \rangle M_0 \tag{13b}
\end{equation}
with $S=1$, where
\[ M_{e,o} = \langle \phi_T[\mathbf{r}_1 - \mathbf{r}_2, \mathbf{r}_3 - \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)] g_p^- [\mathbf{r}_1 - \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3)] | f(\mathbf{r}_2 - \mathbf{r}_4) | \varphi_D(\mathbf{r}_1 - \mathbf{r}_3) \varphi_D(\mathbf{r}_3 - \mathbf{r}_4) \psi_{\text{even, odd}}\{ \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_3 - \mathbf{r}_4) \}. \] (13c)

The distinction between (13a) and (13b) is that only even or odd angular momentum states are kept in an expansion of \( \psi \) as shown in Eq. (7). We have separated the amplitudes of even and odd parities since they do not interfere with each other.

Let us first evaluate the spin-isospin overlaps in Eqs. (13a) and (13b). If we consider reactions (a) and (b) separately, the \( z \) component of the total isospin, i.e. \( \nu \) in Eq. (9), is a good quantum number. By direct computation, it can be easily shown that the only non-vanishing overlaps are

\[ \langle B_{r_0} | \frac{1}{2} [1 - \frac{1}{2} (1 + \sigma_3 \cdot \sigma_4) (1 + \tau_3 \cdot \tau_4)] | T_0 S_{00} \rangle = \frac{1}{2} (-1)^{\nu+1/2}, \] (14a)
\[ \langle B_{r_0} | \frac{1}{2} [1 - \frac{1}{2} (1 + \sigma_3 \cdot \sigma_4) (1 + \tau_3 \cdot \tau_4)] | T_0 S_{1m} \rangle = \frac{1}{2 \sqrt{6}} (-1)^{\nu+1/2} \delta_{m,0}. \] (14b)

With the interaction chosen, there is no contribution due to the initial quintet state and there are no singlet \( J^P \) triplet transitions.

We next come to the spatial overlaps in Eqs. (13a) and (13b). For a general class of wave functions, the three-nucleon bound states are separable,

\[ \phi_T[\mathbf{r}_1 - \mathbf{r}_2, \mathbf{r}_3 - \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)] = \phi_1(\mathbf{r}_1 - \mathbf{r}_2) \phi_2[\mathbf{r}_3 - \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)]. \] (15)

Using Eqs. (7), (11) and (15) and a delta function for \( f \) in Eq. (12), we immediately find

\[ M_{e,o} = \varphi_D(0) \langle \phi_1(\mathbf{r}) | \varphi_D(\mathbf{r}) | g_p^- (\frac{3}{2}\mathbf{x}) \phi_2(\mathbf{x}) | \psi_{e,o}(\mathbf{k} \cdot \mathbf{x}) \rangle \]
\[ = \frac{(4\pi)^{1/2}}{k} \varphi_D(0) \langle \phi_1(\mathbf{r}) | \varphi_D(\mathbf{r}) \rangle \sum_{l=0} P_l(\cos \theta) \int h_l^- (\frac{3}{2} \mathbf{p} \cdot \mathbf{x}) \phi_2(\mathbf{x}) u_l(\mathbf{k}\cdot\mathbf{x}) \mathbf{x} d\mathbf{x}, \] (16)

where

\[ \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \]
\[ \mathbf{x} = \mathbf{r}_3 - \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2). \]

In Eq. (16), it is understood that the even Legendre polynomials go with \( M_e \) and the odd ones with \( M_o \). The argument of the polynomials is cosine of the scattering angle in the center-of-mass system of the colliding deuterons.

Summing over final spin states, the differential cross-section in the center-of-mass system is related to the matrix elements (13a) and (13b) by

\[ \frac{d\sigma}{d\Omega} = \frac{3m^4 p^4}{16\pi^2 k^2} \sum_{j=0}^\infty \sum_{l=0}^\infty \left| \left( \langle J_l | Y_l^j | \alpha \rangle \right) \right|^2 \]
\[ \times \frac{3m^4 V_0^2 p^6}{16\pi^2} \varphi_0^2(0) \langle \phi_1(\mathbf{r}) | \varphi_D(\mathbf{r}) \rangle^2 \]
\[ \times \sum_{j=0}^\infty \sum_{l=0}^\infty (2l+1) P_l(\cos \theta) \int h_l^- (\frac{3}{2} \mathbf{p} \cdot \mathbf{x}) \phi_2(\mathbf{x}) u_l(\mathbf{k}\cdot\mathbf{x}) \mathbf{x} d\mathbf{x}. \] (17)
The expression (17) could differ for the proton and neutron channels in three aspects. The first is purely kinematical, such as the phase space. This can easily be taken into account. The second is that the bound state wave functions for the triton and He$^3$ nucleus may have a different structure. And the third is that the final state interactions (via decay of the excited states of the alpha particle) described by $h_1$ need not be charge symmetric. We shall fix the first two conditions by the binding energies of the three-body nuclei and explore the third possibility.

For simplicity, we consider collisions at low energies (say, $E_n<150$ KeV) where $s$ and $p$ wave would suffice in the partial wave expansion of (17). The cross-section takes an especially simple form in this case, which can be written as

$$\frac{d\sigma}{d\Omega} = \sigma_0(1 + A \cos^2\theta),$$

(18)

where the normalization $\sigma_0$ is equal to the energy-dependent differential cross-section at $\theta=90^\circ$, and is

$$\sigma_0 = \frac{3m^2V_s^2r}{16\pi k^3} \varphi_0^*(0) \langle \phi_1(r) | \varphi_D(r) \rangle^2 D_0$$

(19)

with

$$D_0 = \int h^{-}_1 \left( \frac{3}{2} p x \right) \phi_1(x) u_t^*(kx) x dx.$$  

(20)

The "asymmetry coefficient" $A$ is simply the ratio of the $p$ wave to the $s$ wave cross-section multiplied by the appropriate spin and orbital angular momentum weight factors:

$$A = 18D_1/D_0.$$  

(21)

In terms of Eq. (18), the branching ratio of reaction (1b) to (1a) is

$$\frac{\sigma_n}{\sigma_p} = \left( \frac{\sigma_0}{\sigma_{0p}} \right) \left( \frac{1 + A_n/3}{1 + A_p/3} \right).$$

(22)

§ 3. Excited states of the alpha particle

Evidence of the $T=0$ virtual excited states of the alpha particle for the two low lying levels exists now in high energy electron scattering as well as low energy nuclear reactions.\(^7\)\(^\text{a}\)\(^\text{b}\)\(^\text{c}\) Because these states are highly unbound, it is suitable to treat their effects on the reactions (1) as final state interactions between the product nuclei. This can be done, for instance, with Watson’s approximation.\(^1\)\(^\text{a}\)\(^\text{b}\) We write
\[ h_i^{-}\left(\frac{3}{2}px\right) = \left(\frac{3}{2}px\right)^{-\frac{1}{2}} \exp\left(i\delta_i\right) \sin\delta_i \; x f(x) \]  
\[ (23) \]
for the \(s\) and \(p\) waves of the final state continuum, where the nuclear phase shifts \(\delta_i\) are assumed to be resonant at the appropriate excitation energy\(^{a}\)

\[ \delta_i = \tan^{-1}\left[\frac{\Gamma_i}{E - E_i}\right]. \]  
\[ (24) \]

With Eq. (23), we can rewrite Eq. (20) as

\[ D_i = \left(\frac{9}{4\rho^2}\right)^{\frac{1}{2}} X_i I_i, \]  
\[ (25) \]
where

\[ I_i = \int dx \; x^{-1} \phi_i(x) u_i(kx) |^2 \]  
\[ (25a) \]
and

\[ X_i = \sin^2\delta_i = \frac{\Gamma_i^2/4}{(E - E_i)^2 + \frac{1}{2} \Gamma_i^2}. \]  
\[ (25b) \]

The resonance energy \(E_i\) in Eq. (24) is 20.1 and 22.0 MeV for the \(s\) and \(p\) states, respectively. The partial resonance widths \(\Gamma_i\) for these states are different for the proton and neutron. We can find relations between these four widths from experiment\(^{b}\) using Eqs. (18)-(21). The results thus found are dependent on the knowledge of the three-body wave function and the initial continuum states. Alternatively, if independent parameters of these resonances can be obtained from other experiments, Eqs. (18)-(21) will then enable one to say quantitative things about the bound state wave function. Unfortunately, the situation with regard to the resonance parameters is not clear-cut at present, so we rather prefer our first approach here.

We compute the cross-sections in accordance with Eqs. (18)-(21) using alternatively a Gaussian and Yukawa wave function for the bound state. The Yukawa wave function is

\[ \phi_i(x) = Ne^{-ax^2}/x, \]  
\[ (26) \]
where \(N = (\alpha/2\pi)^{1/2}\) and \(\alpha = [2\mu|E|]^{1/2}\), with \(|E|\) the separation energy determined from the \(Q\) values of the \(\text{He}^3(\gamma, p)D\) and the \(\text{He}^3(\gamma, n)D\) reactions. One might think of a three-body model in which a nucleon is bound to a deuteron. The asymptotic \(s\)-state wave function of the nucleon relative to the deuteron is given by the zeroth order spherical Hankel function of the first kind. The internal wave function of the "deuteron" is designated by \(\phi_i(r)\). Another simple and well-known analytic form of the three-body wave function is the Gaussian

\[ \phi_{\text{G}}(r, r, r) = N_i N_s \exp \left[ -\frac{r^2}{2} (r_{13}^2 + r_{13}^2 + r_{23}^2) \right], \]  
\[ (27) \]

\(^{a}\) Equations (23) and (24) can be found in their more complex forms, for instance, in reference 7. We believe, however, that they have here included the essential features to be stressed.
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so that

\[ \phi_1(r) = N_1 \exp\left(-\frac{\alpha r^2}{4}\right), \quad N_1 = \left(\frac{3r^4}{2\pi}\right)^{3/4} \]

and

\[ \phi_2(x) = N_2 \exp\left(-\gamma x^2\right), \quad N_2 = \left(\frac{2\gamma^3}{\pi}\right)^{3/4}. \]

The size parameters \( \gamma \) can be determined in a number of ways. We choose to normalize it by comparing the momentum representations of \( \phi_2 \) from Eqs. (26) and (27), i.e.

\[ \int \exp(iq \cdot x) N \exp\left(-\alpha x^2\right) d^3x = \int \exp(iq \cdot x) N_2 \exp\left(-\gamma x^2\right) d^3x, \quad (28) \]

where \( |q| = |p - k| \approx |p| \). From Eq. (28), it follows that

\[ \frac{(\alpha \gamma)^{3/2}}{(\alpha^2 + q^2)} = \frac{(2\pi)^{3/4}}{2} \gamma \exp\left(-q^2/4\gamma^2\right). \quad (29) \]

Taking the experimental values for \( |E_p| = 5.493 \text{ MeV} \) and \( |E_n| = 6.258 \text{ MeV} \), we find \( \alpha = 0.449 \text{ F}^{-1} \) and \( \alpha_2 = 0.420 \text{ F}^{-1} \). At threshold, \( q^2 = p^2 = (3m/2)Q \), where the \( Q \) value here refers to reactions (1a) and (1b); and using Eq. (29) we find \( \gamma_p = 0.410 \text{ F}^{-1} \) and \( \gamma_n = 0.380 \text{ F}^{-1} \).

If resonance effects on the initial state are secondary, the continuum wave function \( \psi \) [Eq. (7)] can in principle be calculated with a Coulomb optical-model potential. The optical-model parameters must be chosen to reproduce the D-D elastic scattering data. Data at low incident energies \( (E_p < 150 \text{ KeV}) \), however, are curiously scanty. So as not to introduce arbitrary parameters we shall assume that the elastic scattering in this region is dominated by the Coulomb interaction which, together with the centrifugal barrier, tends to keep the deuterons at a relatively large distance apart. For our actual calculation, a uniformly charged sphere of radius \( R = (1.5 F) (A_1^{1/3} + A_2^{1/3}) \), with \( A_1 = A_2 = 2 \), is employed for the initial state scattering in both channels (1a) and (1b). The potential due to the charged sphere is

\[ V_e = \frac{(e^2/2R_e)(3 - R^2/R_e^3)}{e^2/R} \text{ for } R \leq R_e, \]
\[ e^2/R \text{ for } R \geq R_e. \quad (30) \]

At large distance of separation, the asymptotic solution of \( \psi \) is

\[ \psi \rightarrow \sqrt{\frac{\mu}{k}} \left( 1 + i\eta \right) \exp\left(-\left(i/2\right)\eta \pi\right) \exp(i\mathbf{k} \cdot \mathbf{R}) F(-i\eta, 1, ik\xi), \quad (31) \]

where \( \Gamma \) and \( F \) are the gamma function and the confluent hypergeometric function and \( \eta = \mu k^3/k \), and \( \xi = R - k \cdot R/k \). The asymptotic partial waves \( u_i \) can be expressed in terms of the usual Coulomb functions and phase shifts \( \sigma_i \)

\[ u_i \rightarrow \exp(i\sigma_i) \left( \cos \sigma_i F_1(\eta, kR) + \sin \sigma_i G_1(\eta, kR) \right). \quad (32) \]
The entire solutions of \( u \) are calculated numerically with a UCLA optical-model scattering program. With these solutions of \( u \) and the representation of \( h \) [Eq. (23)], the radial integrals \( I \) [Eq. (25a)] are evaluated numerically with the Yukawa and Gaussian wave functions [Eqs. (26) and (27)]. Once this is done, we can use Eqs. (21) and (25) to relate the ratios of the phase shifts \( X_i/X_o \) to the experimental asymmetry coefficients

\[
A = R \left( \frac{X_i}{X_o} \right),
\]

where

\[
R = \frac{81}{2p^2} \left( I_i/I_o \right).
\]

The values of \( R_n \) and \( R_p \) for the Gaussian and Yukawa wave functions are summarized in Table I for \( E_p(1ab) = 30, 70, 110 \) and 150 KeV. In Fig. 1, we plot the experimental asymmetry coefficients \( A^* \) at these energies against the computed values of \( R \). The slope of the straight lines drawn through these points gives the ratios \( \frac{X_i}{X_o} \). We summarize these values in Table II. These ratios are directly related to the positions and widths of the virtual excited states of the alpha particle. If we take the positions of these levels from other experiments, we have an independent estimate of the widths. Of particular interest is the fact that the resulting widths are different for the proton and the neutron, although the form of the wave function (Gaussian or Yukawa) makes little difference. In addition, the ratio \( X_{in}/X_{ip} \) can be computed from Eqs. (18)–(20). This ratio is directly related to the ratio of the 90° differential cross section for the neutron and for the proton. We write

\[
\frac{d\sigma_n/d\Omega(90°)}{d\sigma_p/d\Omega(90°)} = R_0 \left( \frac{X_{in}}{X_{ip}} \right).
\]

The computed values of \( R_0 \) are also listed in Table II. These values depend little on the energy and are very close to unity. On the other hand the exper-

\[\text{Fig. 1.} \ \ A_n \ \text{and} \ A_p \ \text{vs.} \ R_n \ \text{and} \ R_p \ [\text{see Eq. (33)}] \ \text{for the Gaussian and Yukawa wave function.}\]

\[\text{\[33\]}\]

\[\text{\[34\]}\]
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Table I.

<table>
<thead>
<tr>
<th>$E_D$ (KeV)</th>
<th>30</th>
<th>70</th>
<th>110</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_a$</td>
<td>0.0643</td>
<td>0.0961</td>
<td>0.1277</td>
<td>0.1592</td>
</tr>
<tr>
<td>$R_p$</td>
<td>0.0523</td>
<td>0.0782</td>
<td>0.1040</td>
<td>0.1296</td>
</tr>
<tr>
<td>$R_0$</td>
<td>1.019</td>
<td>1.019</td>
<td>1.019</td>
<td>1.019</td>
</tr>
<tr>
<td>Yukawa</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_a$</td>
<td>0.0637</td>
<td>0.0952</td>
<td>0.1267</td>
<td>0.1580</td>
</tr>
<tr>
<td>$R_p$</td>
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<td>0.0775</td>
<td>0.1031</td>
<td>0.1287</td>
</tr>
<tr>
<td>$R_0$</td>
<td>0.974</td>
<td>0.974</td>
<td>0.973</td>
<td>0.973</td>
</tr>
</tbody>
</table>

$A_n$(reference 1))

| $A_n$      | 0.45±.06 | 0.79±.03 | 1.07±.02 | 1.32±.02 |

$A_p$(reference 1))

| $A_p$      | 0.20±.04 | 0.45±.02 | 0.61±.02 | 0.77±.02 |

$\frac{d\sigma_n/d\Omega(90\degree)}{d\sigma_p/d\Omega(90\degree)}$

| $\frac{d\sigma_n/d\Omega(90\degree)}{d\sigma_p/d\Omega(90\degree)}$ | .940±.010 | .954±.004 | .966±.003 | .975±.003 |

Table II.

<table>
<thead>
<tr>
<th></th>
<th>Gaussian</th>
<th>Yukawa</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{1n}/X_{0n}$</td>
<td>8.25</td>
<td>8.38</td>
</tr>
<tr>
<td>$X_{1p}/X_{0p}$</td>
<td>5.85</td>
<td>5.92</td>
</tr>
<tr>
<td>$X_{0n}/X_{0p}$</td>
<td>1.04−1.08</td>
<td>1.00−1.03</td>
</tr>
<tr>
<td>$X_{1n}/X_{1p}$</td>
<td>1.47−1.53</td>
<td>1.42−1.46</td>
</tr>
</tbody>
</table>

The inclusion of the excited states of the alpha particle in the $D$-$D$ reactions provides a mechanism for $p$-wave enhancement in the cross-section because the $D$-$D$ threshold is closer to the negative than to the positive parity state. It also accounts for a sizeable portion of the gap between the angular asymmetries in the proton and neutron channels. Although charge symmetry appears to hold for the $0^+$ state, it is ostensibly violated in the higher excited state. This suggests that the mechanism for the $0^+$ excited state is intrinsically different from that of the negative parity states. It should be noted that our phenomenological analysis here has not included the possibility that the "entrance channel" may also be dominated by resonances and we have assumed that additional excited states, other than the $0^+$ and $2^-$ states, in the "exit channel", even

§ 4. Summary

The inclusion of the excited states of the alpha particle in the $D$-$D$ reactions provides a mechanism for $p$-wave enhancement in the cross-section because the $D$-$D$ threshold is closer to the negative than to the positive parity state. It also accounts for a sizeable portion of the gap between the angular asymmetries in the proton and neutron channels. Although charge symmetry appears to hold for the $0^+$ state, it is ostensibly violated in the higher excited state. This suggests that the mechanism for the $0^+$ excited state is intrinsically different from that of the negative parity states. It should be noted that our phenomenological analysis here has not included the possibility that the "entrance channel" may also be dominated by resonances and we have assumed that additional excited states, other than the $0^+$ and $2^-$ states, in the "exit channel", even

imental ratios on the left-hand side of Eq. (34) are energy dependent. This means that $X_{0n}/X_{0p}$ is also energy dependent. Of course the widths of the excited states are in general energy dependent. The ratio $X_{0n}/X_{0p}$ is more sensitive to the energy variation than $X_i/X_o$ because the energy denominators of $X_{0n}$ and $X_{0p}$ are essentially the same. From the first three rows of quantities in Table II obtained so far, we can easily find $X_{1n}/X_{1p}$, which gives the relation between the widths of the $2^-$ state for the neutron and the proton.
though they may exist, are unimportant. In this sense, further experimentation may well prove that the results here are only preliminary. We contend, however, that any progress toward an understanding of the three-nucleon system from the $D-D$ reactions must not preclude the role played by the excited states of the alpha particle.

Acknowledgement

The author thanks Professor Leonard Schiff for a reading of the manuscript.

References

2) Further bibliography on the $D-D$ reactions can be found in R. V. Winsor, Los Alamos Scientific Laboratory Report LA-3322-MS (1965).
5) I. Schiff, Phys. Rev. 133 (1964), B802.
7) See W. E. Meyerhof, Rev. Mod. Phys. 37 (1965), 512.
9) A. de-Shalit and J. D. Walecka, Phys. Rev., to be published.
10) L. I. Schiff, Phys. Rev. 51 (1937), 793.

Note Added in Proof:

If other negative-parity levels of the alpha particle than the $2^-$ reachable by the $p$ wave exist, they can be separated only with a spin-orbit interaction. There has been some very recent evidence for a $0^-$ level less than half an MeV away from the $2^-$. Contribution to the $p$-wave enhancement due to this level has thus been included in our $X_i$. One obtains in this case a form similar to, but more complicated than Eq. (25b), which has been included here only for illustrative purpose.