Anomalous Interactions of High Energy Muons

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If the cosmic ray data on penetrating showers with large transferred energies produced by high energy muons are accepted, the muon would have anomalous interactions other than the electromagnetic one. The hypothesis of a new neutral (vector) boson $\chi$ with a mass of $\sim 10m_N$ is convenient to explain the penetrating showers and is not inconsistent with the muon-pair and muon-bundle production by energetic cosmic rays muons. The possible existence of a quadratic strong interaction of the intermediate weak boson $W$ is also discussed in connection with these phenomena.

§ 1. Introduction

Despite the confirmation of the existence of two neutrinos $\nu_e$ and $\nu_\mu$, the problem of how to explain the $\mu$-$e$ mass difference has been left unsolved as yet. The main theories which have been proposed in order to explain the origin of the $\mu$-$e$ mass difference are the following three:

(I) The muon is considered as an excited state of the electron due to the electro-magnetic interaction. That is, of two solutions of a self-consistent mass equation due to the spontaneously broken symmetry, the zero mass solution is assigned to the electron and the non-vanishing solution to the muon. The existence of $\nu_\mu(\equiv \nu_\tau)$, however, stands against this theory.

(II) A new neutral (vector) boson $\chi$ is introduced. The boson couples only with the muon and strange baryons such as $A$. $\chi$ is the agent of the $\mu$-$e$ mass difference and the breaking of the $SU(3)$ symmetry for the hadrons. If $\chi$ couples with $\nu_\mu$, the question why the mass of $\nu_\mu$ is very small must be answered. On the other hand, if one assumes no coupling of $\chi$ with $\nu_\mu$, the universality of $\beta$ decay and muon decay is badly destroyed.

(III) On the assumption that the intermediate weak boson has a quadratic strong coupling with the muon and the strange baryons, a unified explanation is made of the $\mu$-$e$ mass difference, the breakdown of $SU(3)$ symmetry for hadrons and the origin of the large mass of the intermediate weak boson. This theory is interesting and has no conspicuous disagreement with the experimental

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* When the bare mass of $\nu_\mu$ vanishes exactly and the $\chi$ field is a vector field, the Lagrangian is invariant under the $\gamma_5$ transformation. Then the physical mass of $\nu_\mu$ vanishes. However, it is difficult to ascribe the electron mass to the electro-magnetic interaction which is also left invariant by the $\gamma_5$ transformation.
In this article, we shall analyze, following theory (II), the cosmic ray data so far obtained: penetrating showers with large transferred energies\(^7\),\(^8\) muon pairs\(^9\),\(^10\) and muon bundles\(^11\) produced by high energy muons.

The coupling constant \(f\) of the \(z-\mu\) interaction, assuming the boson to be a vector field, is restricted by\(^12\)

\[
f^2/m_x^2 < 4 \times 10^{-1}/m_N^2
\]

from a comparison of the observed anomalous magnetic moment of the muon with that calculated by quantum electro-dynamics. The inequality (1) may be strengthened by more precise measurements of the muon magnetic moment. In this formalism, the \(z\) boson makes virtual or real transitions to a \(\mu\) pair or to \(n\) pions through a virtual \(A\) pair. The ratio of \(\mu\)-pair production cross section via a virtual \(z\) to that via a virtual photon (\(\sigma_{\text{em.}}\)) for a high energy muon (Fig. 1) is expressed in the form\(^12\)

\[
\frac{\sigma_{r.x}}{\sigma_{\text{em.}}} \propto \left(\frac{f^2}{e^2}\right)^2 \left(\frac{q^2}{m_x^2}\right)^2 \lesssim 0.3 \left(\frac{q^2}{m_N^2}\right)^2
\]

provided \(m_x^2 \gg q^2 \gg m_\mu^2\), where \(q\) stands for the four-momentum of the \(\mu\) pair produced, that is to say, of the virtual \(z\) or \(\gamma\). We have used the condition (1) at the last step in (2). In higher energy phenomena, real \(z\)'s would be created which would decay rapidly into a \(\mu\) pair or \(n\) pions. The cross section for this process \((\sigma_{r.x})\) can be compared with that for ordinary bremsstrahlung \((\sigma_\gamma)\) with the same transferred momentum:\(^12\)

\[
\frac{\sigma_{r.x}}{\sigma_\gamma} \sim \frac{f^2}{e^2} \lesssim 0.5 \frac{m_x^2}{m_N^2}.
\]

As is easily seen from (2) and (3), in phenomena with small transferred momenta \((q \leq 1 \text{ GeV}/c)\), the effect of the \(z\) boson may be negligible, while in the case of very large transferred momenta, so that real \(z\)'s are produced, the contribution from the \(z\) boson turns out to be dominant.

In § 2 we shall estimate the mass of the boson, using (3), from the data on penetrating showers with large transferred energies, and obtain \(m_x \approx 10m_N\). Assuming this mass value, we shall, in § 3, interpret the large production cross section for \(\mu\) pairs with high energies as the effect of \(z\). Furthermore the data on muon bundles will be shown not to contradict the existence of \(z\).

Section 4 will be devoted to theory (III). If we take \(g^2 \leq 1\), following
Kitazoe, where \( g \) is the coupling constant of the muon current with the intermediate boson pair (see Eq. (8)), the expected value of the cross section for \( \mu \)-pair production or for muon bundle production is not inconsistent with the experimental data.

In the last section we shall discuss the results obtained and some related problems.

§ 2. Penetrating showers with large energy transfer produced by high energy muons

By the use of two cloud chambers set at a depth of 40 meters water equivalent (m.w.e.) underground, the muon-study group of Osaka City University (OCU) has observed one event of penetrating showers with large energy transfer (\( \geq 300 \) GeV), produced by a high energy muon. Using the value \( 8.0 \times 10^{-7} \) cm\(^{-2}\) sec\(^{-1}\) st\(^{-1}\) as the flux of muons with energies \( \geq 300 \) GeV at this depth, the resulting cross section for the event is found to be \( 8.25 \times 10^{-30} \) cm\(^2\)/nucleon. The cross section value comes out to be one or two orders of magnitude larger than the production cross section (\( \sim 10^{-31} \) cm\(^2\)/nucleon) for phenomena with small energy transfers due to the electro-magnetic interaction of the muons. Furthermore the Institute for Nuclear Study (INS) group has very recently observed one event of an almost horizontal air shower with an energy transfer of \( 3 \times 10^{14} \) eV which suggests a nuclear interaction of a muon with an energy of more than \( 3 \times 10^{14} \) eV. The production cross section is assumed to be nearly \( 1 \times 10^{-30} \) cm\(^2\)/nucleon and, again, cannot possibly be regarded as the ordinary electromagnetic interaction of the muon in the Weizsäcker-Williams treatment.

In addition, according to a private communication from the emulsion group at Konan University, they have found one muon-interaction event with a transferred energy of \( \geq 1 \) TeV, the corresponding cross section being estimated to be \( 1.8 \times 10^{-30} \) cm\(^2\)/nucleon. Also, the K.G.F. neutrino experiments show one event (No. 13) of very big electron showers at 7,500 m.w.e. underground. These experimental results give us the impression that high energy muons may have some anomalous interactions other than the ordinary electromagnetic one.

In order to interpret such extraordinary interactions in terms of production of the real \( z \) boson, the mass of the \( z \) boson should be determined by comparison of the observed cross section with the ordinary bremsstrahlung cross section as given in expression (3). That is,

A) The OCU case

\[
\frac{\text{the obs. cross section}}{\text{the ord. brems. cross section}} = \frac{(8.25) \times 10^{-30} \text{A in rock cm}^2/\text{nucleus}}{1.65 \times 10^{-30} \text{cm}^2/\text{nucleus}} \leq 0.5 \left( \frac{m_x}{m_y} \right)^2,
\]

where \( A \) is the atomic number of the target material.
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\[ m_X \geq 6.4 m_N. \]

B) The INS case

the obs. cross section

the ord. brems. cross section

\[ \frac{1 \times 10^{-28} A}{8.13 \times 10^{-30} \text{ cm}^2/\text{nucleus}} < 0.5 \left( \frac{m_X}{m_N} \right)^2 \]

\[ \therefore m_X > 12 m_N. \]

As these are order of magnitude estimates, since both observations are only of one event each, we shall postulate that \( m_X \sim 10m_N \).*).

\[ \text{§ 3. Discussion of the experimental results on muon pairs} \]

If the existence of the \( \chi \) boson with mass \( m_X \sim 10m_N \) is real, not too infrequent production of muon pairs via the virtual or real boson should be expected. This expectation is now discussed in connection with the experimental data on muon pairs so far obtained.

A) The data of the OCU group

The OCU group has observed, by means of two (upper and lower) cloud chambers, a forked type shower resembling muon pairs with two or three secondary penetrating particles without any heavily ionizing particles. All events originated from lead plates 1 cm thick contained in the upper chamber and their secondaries penetrated 15 iron plates 1 cm thick in the lower chamber, a lead absorber 6 cm thick between the two chambers and a lead absorber 6 cm thick below the lower chamber. Out of 4 events, in the Table, it is impossible to determine in the case of two events whether or not the secondaries interact with the lead absorber between the chambers. The energies of the secondaries are estimated.

\[ \begin{array}{cccc}
\text{Number of event} & \sigma_{\text{exp}} & \sigma_{\text{theor}} & \text{Threshold energy} \\
2 \ or \ 4 & (4.4 \sim 8.7) \times 10^{-30} \text{ cm}^2/\text{nucleus} & 1.1 \times 10^{-29} \text{ cm}^2/\text{nucleus} & 1.0 \text{ GeV} \\
\end{array} \]

* On the other hand, from (1) and the expression for the muon self-energy due to \( \chi \)

\[ \delta m_\mu = \frac{3f^2}{4\pi} \ln \frac{A^2}{m_X^2}, \]

where \( A \) is a cutoff parameter, we get

\[ m_X \geq 30m_N/\sqrt{\ln(A/m_X^2)}. \]

Thus the assumption \( m_X \sim 10m_N \) means \( A/m_X^2 \geq 90. \)
timated to be $\gtrsim 1.0$ GeV from a thickness traversed through the absorbers. Moreover, the energies of incident muons are presumed to be larger than 5 GeV, from the experimental triggering condition for the cloud chambers. Since the total four-momentum $q$ of the muon pairs is the sum of the energies of the secondary pairs in their c.m.s., the four-momentum values of the majority of events are assumed to be $q^2 = (2E^*)^2 \gtrsim (0.7 \text{ GeV})^2$. Inserting this $q$ value into the expression (2),

$$\frac{\sigma_{v,\chi}}{\sigma_{\text{e.m.}} \text{ (pairs)}} = \left( \frac{f^2}{(10m_N)^2} \right) \times 137 q^2 \sim 0.1. \quad (4)$$

On the other hand, the ratio of the cross section observed by the OCU group to the cross section calculated by Murota et al.\textsuperscript{10} for electromagnetic production of muon pairs by a muon shows

$$\frac{\sigma_{\text{observed}} \text{ (pairs)}}{\sigma_{\text{e.m.}} \text{ (pairs)}} = \frac{(4.4 \sim 8.7) \times 10^{-30}}{1.1 \times 10^{-29}} = 0.4 \sim 0.8. \quad (5)$$

From this comparison, it can be asserted that the experimental cross section of the OCU group for production of muon pairs, being not very large, is inconsistent with the existence of the $\chi$ boson with mass $10m_N$.

B) The data of Barrett et al.

At a depth of 1,600 m.w.e. below ground, Barrett et al. found\textsuperscript{10} 43 events in which two or three parallel penetrating particles entered simultaneously two detectors separated by a distance of a few meters, while, at the same time of observation, 21,160 single muons were recorded. The two-particle case applied to 39 events and the three-particle case to 4 events. The angular resolution for each of the parallel particles in these events was $\pm 16$ degrees because the counters in detectors were of 2” diameter with a vertical separation of 7”. Provided that these events may be explained by muon-pair productions by muons, we can consider the positions of their production origins to be a distance of at least several meters above the detectors, from the crossing point of two convergent lines joining the discharged counters. Therefore, the energies of the secondary muons traversing a 75 cm thickness of rock and lead absorbers surrounding the detectors would exceed 5 GeV. If we postulate that the primary energy of the muon is 10 GeV, the number of events estimated from the electromagnetic interaction theory of Murota et al. is at most 0.5 event. This is not enough to explaining even the number observed in the three-particle case. That is, the ordinary electromagnetic productions mechanism for muon pairs are not valid to explain such events.

If we take into account that the primary energy of muon is $\gtrsim 10$ GeV and the secondary energies $\gtrsim 5$ GeV, the square of the four-momentum transfer, $q^2$, in the present events is inferred to be $\gtrsim (2.8 \text{ GeV})^2$. If the events may be explained by virtual production of a $\chi$ with mass $10m_N$, the expression (2) gives
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\[
\frac{\sigma_x}{\sigma_{\text{e.m.}}} = \left( \frac{f^2}{(10m_N)^2} \times 137q^3 \right)^2 = 24.
\]  

(6)

Therefore, the expected number of events produced via \(Z\) bosons is 12 (= 0.5 events \(\times 24\)).

This value is not inconsistent\(^a\) with the observed number 4\(\sim\)43.

C) The data of Vernov et al.

Vernov et al. have observed\(^1\) 59 muon bundle events by means of 16 groups of muon detectors of total sensitive area 6.3 m\(^2\), each group consisting of G-M counter trays of 3 layers, at a depth of 40 m.w.e. underground. “Muon bundle” means that three neighbouring counters at two or three layers are discharged, in association with extensive air showers of \(4 \times 10^4\) events in the range \(5 \times 10^4 \leq N_e \leq 2 \times 10^7\) (\(N_e\) = number of electrons).

The events may be understood in terms of muon pairs. Taking account of the vertical distance of 45 cm between the upper and lower G-M counter layers and their diameter of 2", we can say that the production points of the pairs should be a distance of at least 2 meters above the detectors. Also, as the detectors are surrounded by absorbers of 10 cm lead and 16 cm iron, the energies of the secondary muons are assumed to be above 1.3 GeV. On the assumption that the primary energies in the events are over 5 GeV, their \(q^2\) values may be estimated from the energies of the secondaries in the c.m.s of the latter as \(q^2 \approx (2E_p^*)^2 \geq (0.9 \text{ GeV})^2\). Therefore,

\[
\frac{\sigma_x}{\sigma_{\text{e.m.}}} = \left( \frac{f^2}{(10m_N)^2} \times 137q^3 \right)^2 = 0.3.
\]  

(7)

From the expression of Murota et al. for the ordinary electromagnetic cross section for similar values of the energy, we find

\[
\sigma_{\text{e.m.}} \approx 1.7 \times 10^{-31} \text{ cm}^2/\text{nucleus in rocks}
\]

at a depth of 40 m.w.e. Accordingly,

\[
\sigma_x \approx 5.1 \times 10^{-32} \text{ cm}^2/\text{nucleus in rocks}.
\]

Data on extensive air showers tells us that air showers of \(4 \times 10^4\) events with \(5 \times 10^4 \leq N_e \leq 2 \times 10^7\) contain \(8.8 \times 10^9\) muons with energies \(\geq 5\) GeV. The number, \(n_{\text{e.m.}}\), of muon pairs produced via the ordinary electromagnetic interaction may thus be estimated to be

\[
n_{\text{e.m.}} = \sigma_x \cdot \frac{N}{A} \cdot \tau \cdot n_p \approx 4.3 \times 10^4,
\]

where \(\tau\) is the thickness corresponding to secondary energies of above 1 GeV.

As the events were observed by muon detectors of 6.3 m\(^2\), 86 events = 4.3 \(\times 10^4\)

\(^a\) Wolfendale et al. have asserted\(^1\) that the observed frequencies can be explained in terms of the pions generated locally by the ordinary nuclear interactions of energetic muons, together with muons forming the remnants of extensive air showers.
$x(6.3/\pi R^2)$ can fall on to the detectors, where $R$ is a diameter of air showers; it is known that $R<30$ meters from the experimental results of Vernov et al.

The expected number of muon pair produced via the $X$ boson is about three times less than this number. Although this number is only a rough estimate, it seems that the $X$ boson with the mass of $10m_N$ gives no large contribution to muon-bundle phenomena.

§ 4. On the model of quadratic coupling of the intermediate bosons

Kitazoe's proposal is attractive in that it goes some way towards unifying the weak interaction and the $\mu$-$e$ problem. According to his scheme, the quadratic coupling of the intermediate weak boson $W$ with the muon is taken as a scalar coupling, as follows:

$$\mathcal{L}_i = \frac{g}{m_w} \bar{\mu} \mu W^+ W,$$

(in reference 6), the Lagrangian is defined by using $m_N$ instead of $m_w$, where the dimensionless coupling constant is

$$g^2 = m_w^2/m_N^2.$$

Now, when a closed loop of $W$, instead of the $X$ propagator in Fig. 1 (b), is inserted as illustrated in Fig. 2 (a), we get for the $\mu$-pair production

$$\frac{\sigma_W}{\sigma_{c.m.}} \approx \left(\frac{g^2}{\epsilon^2}\right)^2 \frac{L^3}{3m_\mu} \left(\frac{q^2}{m_w^2}\right)^2$$

under the condition that $m_w^2 \gg q^2 \gg m_\mu^2$ and the two four-momenta of $\mu^+$ and $\mu^-$ are nearly equal. $E_0$ stands for the incident muon energy and $L$, which comes from the $W$-closed loop, reduces, in the present approximation, to a constant independent of $q$:

$$L \approx 1/(4\pi)^2 \text{ for } A^2 \approx m_w^2 \approx q^2,$$

$$\approx \frac{1}{(8\pi)^2} \left(\frac{A^2}{m_w^2}\right)^2 \text{ for } A^2 \gg m_w^2 \gg q^2.$$

For the data of the OCU group in § 3. A) ($q^2 \gtrsim (0.7 \text{ GeV})^2, E_0 \gtrsim 5 \text{ GeV}$) from Eqs. (10) and (11),

$$\sigma_W/\sigma_{c.m.} \approx 8g^4 \left(\frac{m_N}{m_w}\right)^4 \text{ for } A^2 \approx m_w^2,$$

$$\approx 30g^4 \left(\frac{m_N}{m_w}\right)^4 \left(\frac{A^2}{m_w^2}\right)^4 \text{ for } A^2 \gg m_w^2 \gg q^2.$$
Substituting Eq. (9) into (12a), we get \( \sigma_w/\sigma_{e.m.} \approx 8 \), while 0.4\(<0.8\) in Eq. (5) is the observed value for this ratio. Hence the magnitude of the coupling constant given by (9) seems to be too large by about a factor 4. However, no serious difficulty would arise from taking \( g^2 \) smaller than that in Eq. (9).

On the other hand, in the case \( A^2 > m_w^2 \), the coupling constant should be taken as \( g = 34 (m_N/m_w)^2 (m_w^2/A^2)^2 \) from the muon self-energy. Thus Eq. (12b) gives

\[
\frac{\sigma_w}{\sigma_{e.m.}} \approx 30 \times 34^4 \left( \frac{m_N}{m_w} \right)^2 \left( \frac{m_w^2}{A^2} \right)^4 \cdot
\]

Up to now, experiment shows \( m_w > 2.2\) GeV. Assuming \( m_w \approx 3 m_N \), we have \( A^2 \approx (3\sim4) m_w^2 \).

The penetrating showers with large energy transfer produced by a high energy muon can also be interpreted in terms of real \( W \)-pair production process if the \( W \)-bosons decay, through the intermediate weak interaction, into \( 2\pi \) or \( 3\pi \) states with a probability nearly equal to lepton decay probabilities. Although multi-\( \nu \)-pairs are produced in the process shown in Fig. 2(b), this does not give any extraordinary cross section.

§ 5. Concluding remarks

From the results of the preceding sections, the muon should have an anomalous interaction other than the electro-magnetic interaction, if the cosmic ray data quoted there are accepted. Thus we are led to introduce, for example, a neutral \( z \) boson coupled with the muon in order to understand these phenomena and the \( \mu-e \) mass difference.

Unfortunately, so far we have not been able to determine from the cross section data whether the \( z \) boson participates in the anomalous interaction or the \( W \) boson does. In order to choose the correct theory from the two possibilities, we shall have to measure the invariant mass of a \( \mu \) pair with high energy, which is constant for a \( \mu \) pair produced via a real \( z \) boson, but not constant for a \( \mu \) pair produced via a virtual or a real \( W \) boson. As the OCU and INS groups are planning a very accurate experiment on penetrating showers with large momentum transfer, we may in the near future reach a definite conclusion on this problem.

If no deviation from quantum electrodynamics is found from a more accurate measurement of the muon anomalous magnetic moment, the restriction on the coupling constant and the mass of the \( z \) boson would be strengthened over the condition (1). (The coupling constant of the quadratic coupling of the \( W \) bosons would also be affected.) We would intend tentatively to introduce an interaction form factor which vanishes at zero momentum transfer, and is effective at large transferred momenta.

For example, if the form factor is assumed to be of the form
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\[ f(q^2) = \frac{q^2}{(q^2 + \kappa^2)} \]  

(13)

for convenience of calculation, where \( \kappa^2 \) is a parameter associated with the spread of the interaction, the muon anomalous magnetic moment due to the \( Z \) boson reduces to

\[ g_X = f^2 \frac{1}{2\pi} \left( \frac{m_r}{m_x} \right)^4, \]  

(14)

provided \( \kappa^2 = m_x^2 \). Thus the upper limit of \( f^2 \) is relaxed to

\[ \frac{f^2}{m_x^4} \leq 0.2/m_N^4. \]  

(15)

Hence, by a method similar to that in § 2 A), using (15) we have

\[ m_x \geq 1.3m_N. \]

In contrast to the magnetic moment, the muon self-energy is not affected by this interaction form factor (13) because the main contribution to the self-energy comes from large \( q^2 \) where \( f(q^2) \approx 1 \).

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References