1 INTRODUCTION

Wavefront sensors are devices widely used in different fields like optical-quality testing, adaptive optics, etc. Recently, the number of applications of adaptive optics has increased in fields such as lasers, confocal microscopy, LASIK (‘laser-assisted in situ keratomileusis’, also known as ‘refractive surgery’) surgery or human vision (Vdovin 1998; Neil, Booth & Wilson 2000; Fernandez, Iglesias & Artal 2001). Moreover, there are some specific applications of adaptive optics in astronomy like high-order compensation in large telescopes, coronagraphy or exoplanet detection (Canales & Cagigal 2000) that require a precise description of the incoming wavefront. This requirement drives the improvement on the standard wavefront sensing devices (Ribak 2001; Rha, Voelz & Giles 2004) and the development of new high-resolution wavefront sensors (Ragazzoni 1996; Vérinaud 2004). With this purpose the optical differentiation wavefront sensor (OD) was already presented in a previous work (Oti, Canales & Cagigal 2003). The optical differentiation technique has been sporadically used to retrieve phase information. Sirohi & Ram Mohan (1977) were the first that used a correct formulation of the optical differentiation method, but the wavefront phase retrieval was not raised in that early work. It is worth mentioning the paper of Bortz & Thompson (1983) where a phase retrieval technique related to that described here was presented. Ellerbroek (1987) and Horwitz (1994) both developed wavefront sensor devices with an arrangement similar to the one described here. Nevertheless, both concepts were based on geometrical optics and used a linearly increasing transmittance mirror. Hence, these two papers do not present an appropriate formulation for the wavefront estimate from an optical differentiation set-up.

The OD sensor takes advantage of the differentiation property of the Fourier transform (FT; Iizuka 1987). It consists of a telescopic system with a variable transmittance mask at the intermediate plane. The incoming field is Fourier transformed by the first lens, then, it is multiplied by the mask and, finally, is Fourier transformed again. When the mask amplitude increases linearly along a certain direction, the light detected at the telescopic system image plane is related with the wavefront phase derivative of the incoming field along that direction. To obtain the derivative in two orthogonal directions, required to correctly estimate the incoming wavefront, a rotating filter was proposed.

In this paper, we present the actual implementation of this novel concept along with an analysis of the required focal plane masks. We analyse the expressions for the wavefront phase derivative estimate and the signal-to-noise ratio (S/N) of the technique. The multi-object wavefront sensor configuration is also described but is not

ABSTRACT

We describe a novel concept for high-resolution wavefront sensing based on the optical differentiation wavefront sensor (OD). It keeps the advantages of high resolution, adjustable dynamic range, ability to work with polychromatic sources and, in addition, it achieves good performance in wavefront reconstruction when the field is perturbed by scintillation. Moreover, this new concept can be used as multi-object wavefront sensor in multiconjugate adaptive optics systems. It is able to provide high resolution and high sampling operation, which is of great interest for the projected extreme adaptive optics systems for large telescopes.

Key words: instrumentation: adaptive optics – instrumentation: miscellaneous.
thoroughly analysed. Next, we investigate the effect of scintillation in the wavefront estimation error and we study the performance of the sensor for different spatial resolution and sampling. Finally, we compare its performances with that of the Hartmann–Shack (H–S) sensor and the pyramid wavefront sensor (PWFS).

The main drawback of the optical differentiation technique is the energy loss due to absorption at the transmittance mask. This weakness has been improved, with regard to the OD sensor, with the simultaneous use of reflectance and transmittance masks. In spite of the energy absorption, the ODII sensor presents a performance similar or even higher than that of the H–S sensor when a proper design of the masks is made, in particular, when it operates in high resolution as required in the projected extreme adaptive optics systems.

In Section 2, the theoretical description of the ODII sensor is developed providing the expressions for the masks and the expression used to estimate the wavefront phase derivative. In Section 3, an analysis of the ODII sensor is carried out. We examine the S/N of the technique in photon noise and its capability to operate with several simultaneous reference objects. Then we check its performance when the incoming field presents scintillation by means of a computer simulation procedure. Next, we study the behaviour of the ODII sensor when different resolution or sampling of the wavefront phase is used by means of a computer simulation procedure. Then, we compare the ODII sensor with other existing wavefront sensors, such as the H–S wavefront sensor, in Section 4, or the PWFS, in Section 5. Finally, in Section 6 several interesting conclusions are derived.

2 THE OPTICAL DIFFERENTIATION SENSOR ODII

The novel ODII sensor consists of three achromatic lenses forming two 4-f systems, and a variable reflectance mirror and mask both placed in the common focal plane of the lenses as described in Fig. 1. The first lens performs the FT of the incoming electric field on to the mirror and the mask. Then the product of the transformed field times the mask and the mirror is Fourier transformed again by the second lens. In the same way, the field reflected by the mirror is also Fourier transformed by the third lens. Finally, the light from these two telescopic systems is detected by means of a detecting system such as a CCD. To avoid problems involved in the calibration of two CCD cameras, a plane mirror could be used to redirect the light to form the two images of the pupil on to the same CCD. The variable reflectance mirror must be slightly rotated in order to minimize the error produced by not applying the amplitude masks at exactly the focal plane and, thus, not providing the correct multiplication of the Fourier transformed field times the masks. This error could be reduced increasing the focal length of the transforming lenses.

This configuration of the sensor allows making use of the energy efficiently because both, the reflected and the transmitted light, are used to obtain information about the field derivative.

2.1 Mask design

Let us define four masks that will be used to obtain the phase derivative estimate. The masks $M_{x+}$ and $M_{x-}$ that have a linearly increasing and decreasing amplitude, respectively, in the $x$ direction, and the masks $M_{y+}$ and $M_{y-}$ that present a linearly increasing and decreasing amplitude, respectively, in the $y$ direction. The masks used to obtain the derivative in the $x$ direction can be described as

$$M_{x+} = 2\pi b_r r_x + a = 2\pi b u_x + a$$

and, similarly, for the $y$ direction

$$M_{y+} = 2\pi b_r r_y + a = 2\pi b u_y + a$$

where $r_x$ and $r_y$ represent real distances in the mask plane. The masks can be also expressed in terms of the spatial frequencies of coordinates $x$ and $y$ of the pupil plane, $u_x$ and $u_y$, where $b = \lambda / f$.

Moreover, $a$ and $b$ (or $b_r$) are two constant parameters that determine the mask behaviour, $\lambda$ is the wavelength and $f$ is the focal length of the first lens. We assume that the centre of the masks lies on the optical axis. Furthermore, to minimize the energy absorption by the masks, the maximum value of the transmittance is fixed to 1. Considering these conditions, the width of the masks can be expressed in terms of the mask parameters as

$$W = \frac{2(1 - a)}{2\pi b_r} = \frac{2(1 - a)\lambda f}{2b}$$

(3)

With the amplitude masks described in equations (1) and (2), placed at the intermediate focal plane of the 4-f systems the derivative of the incoming field along the $x$ or $y$ direction can be determined. We will take the value of parameter $a$ equal to 0.5, which implies that only amplitude masks are considered. Then, the expression of the mask width could easily be expressed as $W = 1/(2\pi b_r) = \lambda f/(2\pi b_r)$.

In the ODII sensor, a variable reflectance mirror plays the role of $M_{x+}$ for the reflected field and $M_{x-}$ for the transmitted one. It is worth noting that a mirror with reflectance $M_{x+}$ has a transmittance $[1 - (M_{x+})^2]^{1/2}$ and, hence, it is necessary to place an additional amplitude filter $AM$ so that $M_{x+} = AM \cdot [1 - (M_{x+})^2]^{1/2}$. The shape of the variable transmittance mirror and mask are plotted in Fig. 2. To obtain the derivative estimate in the orthogonal direction, it is only necessary to rotate the filter and mask $\pi/2$. Hence, with this reflectance/transmittance configuration only two filter positions are needed to obtain the wavefront-phase derivative estimate.

2.2 Theoretical sensor description

Let us consider the input electric field in the most general case $E(x, y) = A(x, y)e^{i\phi(x, y)}$, where $A(x, y)$ is the amplitude of the field affected by scintillation and $\phi(x, y)$ is the wavefront phase.

If we consider fields not affected by scintillation, $A(x, y)$ becomes a constant magnitude. When this electric field is coming into the...
the wavefront is not linked to the spatial resolution of the sensor. Hence, the wavefront phase derivative is sampled at each pixel of the sensor. The expression for the wavefront phase derivative given by equation (5) because it provides the best results at low-light level and it can be neglected. The results are the following expressions:

\[ I_{x}(x, y) = \left| \mathcal{F}^{-1}\{ \mathcal{F}\{ E(x, y) \} \cdot M_{+)}\} \right|^2 = \left( -jb \frac{\partial E(x, y)}{\partial x} + aE(x, y) \right)^2, \]

\[ I_{y}(x, y) = \left| \mathcal{F}^{-1}\{ \mathcal{F}\{ E(x, y) \} \cdot M_{-}\} \right|^2 = \left( jb \frac{\partial E(x, y)}{\partial y} + aE(x, y) \right)^2. \]  

(4)

Thus, the wavefront phase derivative can be estimated introducing the field expression into equation (4) and taking into account that, due to the requirement of minimizing the energy loss, the corresponding value of \( b \) must be small and, therefore, terms of order \( b^2 \) can be neglected. The results are the following expressions:

\[ \alpha_x = \frac{\partial \phi(x, y)}{\partial x} = \frac{I_{x} + I_{-}}{I_{x} + I_{-}}, \quad a \frac{I_{x} - I_{-}}{2b} = \frac{a}{2b\lambda f}, \]

\[ \alpha_y = \frac{\partial \phi(x, y)}{\partial y} = \frac{I_{y} + I_{-}}{I_{y} + I_{-}}, \quad a \frac{I_{y} - I_{-}}{2b} = \frac{a}{2b\lambda f}. \]  

(5)

A similar expression was proposed by Ellerbroek (1987), who used intensity (not amplitude) linear filters, but based it entirely on geometrical optics. In the following study, we only consider the expression for the wavefront phase derivative estimate given by equation (5) because it provides the best results at low-light level and it is derived for the most general case: fields affected by scintillation. Hence, the wavefront phase derivative is sampled at each pixel of the CCD, which means that high sampling of the wavefront is possible and it is limited only by the size of the pixels. Moreover, in contrast with other wavefront sensors like the H-S sensor, the sampling of the wavefront is not linked to the spatial resolution of the sensor. As a consequence, the ODII sensor is able to change its wavefront phase sampling to adapt it to the current sensing conditions without changing its resolution. Besides, a proper election of the mask parameters allows high spatial resolution and high dynamic range as it was shown in a previous paper (Oti et al. 2003). Moreover, the sensor provides an average of the wavefront phase derivative in each sensing area. When using polychromatic sources, the sensor also provides an average over the whole source bandwidth.

3 ODII SENSOR ANALYSIS

3.1 Signal-to-noise ratio

The measured magnitude is the wavefront phase derivative along one direction as expressed in equation (5). Using the standard error propagation formula to compute its variance and assuming that the terms of order \( b^2 \) can be neglected,

\[ \sigma^2 = \frac{2I}{4a^2 |A|^4} \left( \frac{a}{2b} \right)^2 \sigma^2. \]  

(6)

Then, the S/N when detection is affected by Poison noise can be expressed:

\[ \frac{S}{N}_{ODII} = \frac{\langle \alpha \rangle}{\sigma_\alpha} = \frac{\langle \alpha \rangle}{\frac{\langle \alpha \rangle 4abn_{ODII}}{\sqrt{2\lambda n_{ODII}}}} = \frac{\langle \alpha \rangle 2b\sqrt{n_{ODII}}}{\sqrt{2\lambda n_{ODII}}}. \]  

(7)

where \( \langle \ldots \rangle \) means ensemble average, \( n_{ODII} \) is the number of photons arriving at the corresponding area in the entrance pupil of the sensor and \( a \) is set to 0.5. The split of the incoming light on two channels, one for each orthogonal direction, has been taken into account to develop this expression. Parameter \( b \) could be expressed in terms of the number of Airy rings covered by the mask, \( N_\Lambda \), which is the number of times that the first Airy ring, of size \( 2\times1.22f/D_{lens} \) in the focal plane, is enclosed by the mask, where \( D_{lens} \) is the diameter of the first transforming lens. Then, the expression for the S/N of the ODII sensor can be written as

\[ \frac{S}{N}_{ODII} = \frac{\langle \alpha \rangle}{\sqrt{4\pi n_{ODII}}} \frac{(1 - a)D_{lens}}{\pi f_{ODII}}. \]  

(8)

As the high-frequency signals are blocked by the differentiation mask, the spatial resolution of our sensor is determined by its size. The maximum spatial frequency that the sensor could estimate is \( f_{max ODII} = 1.22 \frac{N_\Lambda}{D_{lens}} \). Increasing the size of the differentiation mask, or equivalently, increasing the number of Airy rings \( N_\Lambda \), the spatial resolution is also increased. Following this frequency analysis, the effect of the finite size of the differentiation mask at the focal plane is to smooth the wavefront phase derivative estimated with the ODII sensor. To maximize the S/N, the actual filter should have a value of \( b \) as large as possible, which implies a decrease in the mask size. However, as the size of the masks decreases, the energy corresponding to the larger slopes is blocked and its high-resolution information is not taken into account and, consequently, they will not be correctly estimated. On the contrary, if the size of the mask increases,
the value of $b$ decreases and the S/N falls. As a result, a compromise between the loss of energy and resolution and S/N must be achieved implying that an appropriate size of the mask should be selected for each sensing condition to provide good results. Equation (8) shows that very high spatial resolution is possible without loss of S/N if the number of available photon, $n_{\text{ODII}}$, is large enough.

3.2 Multi-object design

The ODII sensor can be used with reference stars out of the optical axis by means of a slightly modified expression for the wavefront-phase derivative estimate that takes into account the position of these reference stars. This fact gives the opportunity to employ this sensor as a multi-object wavefront sensor for multiconjugate adaptive optics. To do this, it is necessary to use a system of relay lenses and mirrors to re-image the telescope pupil on to the ODII sensor entrance pupil. The wavefronts of the stars must be tilted to assure that the final detection images do not overlap between them. Then, as an example (Fig. 3), let us consider that two reference stars are used, one lying on the optical axis of the sensor (S1) and the other star (S2) slightly displaced out of the optical axis. Specific expressions for the wavefront phase derivative estimate can be easily derived for every reference star used because their images lay at slightly different positions on to the differentiation mask and, hence, parameter $a$ is different for each star but the value of parameter $b$ remains the same (Fig. 3b). The light from the central star forms a centred image over the differentiation mask and at the CCD plane. On the other hand, the image corresponding to the second star lies out of the optical axis. Consequently, from each of those two separated images it is possible to estimate the derivative of the wavefront phase for each reference star. The main drawback this method presents is that the wavefront phase estimate obtained with the stars lying out of the optical axis has a lower dynamic range and spatial resolution. Following this scheme, it is possible to use our sensor with as many reference stars as available. Because only one wavefront sensor is required to estimate the whole wavefront phase, the design of multiconjugate adaptive optics systems could be greatly simplified.

![Figure 3](https://example.com/figure3.png)

**Figure 3.** The ODII sensor used as a multi-object wavefront sensor. (a) The light coming from an on-axis reference star S1, on the optical axis, forms a centred image I1, while a reference star is slightly out of the optical axis, S2, and gives an image I2 displaced from the centre. (b) Detail of the differentiation mask M. The on-axis star forms a centred image over the mask, S1, and the displaced star, S2, forms a displaced image.

3.3 Scintillation performance of the wavefront sensor

To demonstrate the capabilities of the new design of the sensor, we study its performance when scintillation is present. To this purpose, we have simulated atmospherically distorted fields fulfilling the statistics of the atmosphere, with perturbations both in amplitude and phase, using the random wave vectors (RWV) algorithm (Kouznetsov, Voitsekhovich & Ortega Martinez 1997). The distorted wavefront phases were corrected for the first three Zernike modes (piston, tip and tilt; Noll 1979). The number of samples of the field are $(\pi/4) \times 361 \times 361$. The wavefront phase derivatives of the incoming fields were estimated using both the ODII and the OD sensors. Then, from these derivatives, the wavefront phase was reconstructed in terms of Zernike polynomials using a standard procedure (Cubalchini 1979). In this work, there is no intention of correcting or determining the amplitude perturbation generated by the atmosphere on the field. The variable that we evaluate is the reconstruction accuracy of the sensor. The error in the whole detection process is estimated using the residual variance of the reconstructed wavefront phase, defined as

$$\sigma_{\text{rec}}^2 = \int [\phi(r) - \phi_{\text{rec}}(r)]^2 dr,$$

where $\phi(r)$ is the wavefront phase of the original field and $\phi_{\text{rec}}(r)$ is the reconstructed wavefront phase. We compare the preceding OD sensor (Oti et al. 2003) with the novel ODII sensor in terms of the wavefront phase reconstruction error resulting from the computer simulation described above. The optimal sizes of the masks were selected for both sensors. The number of sensing areas used in the wavefront phase reconstruction was 112 corresponding to a $12 \times 12$ sampling of the wavefront. 95 Zernike polynomials were estimated. Fig. 4 shows the simulated wavefront phase reconstruction error as a function of the number of photons. Because the OD sensor is not designed to deal with scintillation, it presents an inferior performance. Whereas the ODII sensor, specifically designed to manage with scintillation, presents a better behaviour. Hence, the enhanced new design of the sensor provides a good way to solve the wavefront sensing difficulties when strong atmospheric distortion is present.

3.4 Resolution analysis

We have already seen (equation 9) that the size of the differentiation mask determines the wavefront spatial frequencies that the

![Figure 4](https://example.com/figure4.png)

**Figure 4.** Residual variance of the reconstructed wavefronts for the ODII (solid curve) and the OD sensor (dashed curve) as a function of the number of photons at the entrance pupil of the sensors. The incoming fields are affected by scintillation.
The OD\textsubscript{II} sensor is able to detect: that is, the mask size controls the sensor resolution. In this paragraph, we analyse, by computer simulation, the effect of the differentiation mask size in the estimation of the wavefront phase. Atmospherically distorted wavefronts with constant amplitude were simulated using the Roddier algorithm (Roddier 1990). The conditions of the atmosphere were determined by the ratio of telescope diameter and Fried parameter \((D/r_0)\). Moreover, the generated wavefront phases were corrected for the first three Zernike modes. The number of samples of the incoming wavefront was \((\pi/4) \times 361 \times 361\), then the wavefront phase derivatives were estimated by our sensor. The number of samples of the wavefront derivatives was \((\pi/4) \times N_S \times N_S\) that is the total number of sensing areas, where \(N_S\) is the number of sensing areas across the sensor entrance pupil diameter. Then, wavefronts were reconstructed in terms of Zernike polynomials using a standard procedure (Cubalchini 1979) and compared with the incoming wavefronts. Lastly, the residual variance of the reconstructed wavefront phase was computed by means of equation (10). The number of estimated Zernike coefficients in the reconstruction procedure is 95.

To study the effect of the resolution in the reconstruction accuracy, 100 atmospherically distorted wavefronts were generated with ratio \(D/r_0 = 25\). The OD\textsubscript{II} sensor was simulated with differentiation masks of different sizes and \(N_S = 20\). In all cases, we considered the detector was affected by photon noise. In Fig. 5, the residual variance of the reconstructed wavefront phase is plotted for masks of different sizes. The wavefront phase estimated with small differentiation masks contains significant errors because the OD\textsubscript{II} sensor reduced spatial resolution does not consider high-order wavefront fluctuations. In addition, part of the energy is blocked by the mask resulting in a decrease of the available photons for detection. As the size of the differentiation mask increases, the residual variance of the estimated wavefront decreases until it reaches a minimum when the mask size is around \(D/r_0\). With this mask size, almost all the available energy is used to get information of the wavefront phase slope. Even though the spatial resolution of the sensor improves when increasing its mask size, providing a sharper estimation of the wavefront phase derivative, beyond this mask size, the wavefront phase starts to slowly degrade. This increase in reconstruction error is due to the decrease on the slope of the differentiation mask and hence a decrease of parameter \(b\), which in turn means a decrease in the \(S/N\). Fig. 5 shows that this \(S/N\) reduction could be partially compensated for a large number of available photons and, as a consequence, the raise of wavefront phase residual variance is less steeped.

3.5 Sampling

Our sensor provides a wavefront phase sampling that it is not related or limited by its spatial resolution. The wavefront sampling is only limited by the size of the pixels of the CCD and could be easily adjusted during sensor operation by adding the signal from adjacent CCD pixels. The next step in our analysis is to study the effect of wavefront sampling in the wavefront phase estimation by means of a computer simulation as described in Section 3.4. 100 wavefronts perturbed by atmospheric distortions were simulated with a \(D/r_0\) ratio of 20. The OD\textsubscript{II} sensor is simulated with a mask size of 20 Airy rings. With this mask size, it is expected to provide its best performance. As shown in Fig. 6, the residual variance of the estimated wavefront sharply decreases for a sampling around \(N_S = 20\). It can be seen that to obtain the best sensor performance it is enough to take \(N_S \approx D/r_0\). This behaviour is reproduced for other \(D/r_0\) values, although this analysis has not been included here.

![Figure 5](https://example.com/figure5.png)

**Figure 5.** Residual variance of the reconstructed wavefronts with the OD\textsubscript{II} sensor as a function of the size of the differentiation mask. The size of the mask is expressed in terms of Airy rings, \(N_A\), that it encircles. The residual variance is computed for different numbers of photons at the entrance pupil of the OD\textsubscript{II} sensor, 30 000 photons (solid curve), 75 000 photons (dashed curve) and 130 000 photons (dotted curve). The dot-dashed curve represents the residual variance of the reconstructed wavefronts for a perfect wavefront sensor without errors and noise.

![Figure 6](https://example.com/figure6.png)

**Figure 6.** Residual variance of the reconstructed wavefronts for the OD\textsubscript{II} sensor as a function of the number of sensing areas. The residual variance is computed for 30 000 photons (solid curve), 75 000 photons (dashed curve) and 130 000 photons (dotted curve).

4 COMPARISON WITH THE H–S SENSOR

First, we perform a frequency analysis of both wavefront sensors. In equation (9), we show that the spatial frequency resolution of the OD\textsubscript{II} sensor is limited by the size of the differentiation mask. Then, for the H–S sensor this spatial frequency limit is equal to the Nyquist critical sampling \(f_c = 1/(2d)\), where \(d = D_{\text{ens}}/N_S\) is the sampling of the wavefront that is equal to the separation between the microlenses. Hence,

\[
 f_{\text{max H–S}} = \frac{N_S}{2D_{\text{ens}}}. \tag{11}
\]

Then, comparing the expressions, equations (9) and (11), developed for the maximum spatial frequency estimated by both sensors, it follows that \(N_S \approx 2.44N_A\). This implies that to obtain equal resolution...
the H–S sensor must have a number of microlenses around 2.5 times the number of Airy rings covered by the differentiation mask.

Next, we compare our sensor with the H–S sensor in terms of S/N performance. The expression for the S/N of the H–S sensor with squared microlenses affected by Poisson noise can be found elsewhere (Welsh & Gardner 1989; Primot, Rousset & Fontanella 1990):

$$\frac{S}{N}_{H-S} = (\alpha) \frac{d}{n_{H-S} \sqrt{0.74N}},$$

where $d$ is the size of the microlens and $n_{H-S}$ is the number of photons at the microlens that is equal to $r_{ODII}$. This expression is developed for a perfect H–S sensor. From equations (8) and (13) and assuming that $\alpha = 0.5$, the ratio of the S/N for both sensors can be derived:

$$\frac{S}{N}_{ODII} = \frac{D_{bas} 0.74}{d 2 \times 1.22 \times N_A} = \frac{0.303}{N_A} N_S, \tag{13}$$

where $N_S$ is the number of sensing areas/microlenses across the entrance pupil of the sensor. For both sensors with equal spatial resolution, the ratio of the S/N is $(S/N)_{ODII} / (S/N)_{H-S} = 0.75$. As a consequence, the H–S sensor has a better performance in terms of the S/N. Nevertheless, this expression is derived for a perfect H–S sensor and under the conditions where this sensor presents its best performance. However, our sensor is especially well suited for extreme adaptive optics, which involve a sampling of at least 100 × 100 sensing areas or microlenses. This high wavefront sampling represents a trouble for the H–S sensor because, as the number of microlenses increases, its size decreases leading to an enlarged size of the image on the CCD. As a result of this fact, its estimation accuracy of the wavefront decreases.

### 4.1 Computer simulation

We have performed a computer simulation to compare both sensors when the spatial resolution varies. We follow the previously described simulation and reconstruction procedure, but now we include the H–S sensor in our simulation routine. The number of samples of the wavefront was $(\pi/4) \times 361 \times 361$. The wavefront phase derivatives were estimated with our sensor and the H–S wavefront sensor. The number of samples is $(\pi/4) \times N_S \times N_S$ that is the number of sensing areas or microlenses. The size and position of the sensing areas on the ODII sensor is matched to that corresponding to the microlenses of the H–S. The ratio $D/r_0 = 40$ and the number of estimated Zernike polynomials is 95. The size of the masks is equal to 40 Airy rings at the focal plane of the 4-f systems.

The computer simulation of the H–S sensor presents some complexity because there are many ways to simulate the CCD used to estimate the centroid of the image. One can vary the number of pixels of the CCD, which in turns means a variation, sometimes very important, on the reconstruction accuracy. Our simulation has been constructed in such a way that intends to reproduce accurately a real H–S with an interchangeable microlens array. Another point to be taken into account is that our computer simulation does not concern the light from adjacent microlenses that may overlap between them or go into the reserved CCD area of the corresponding microlenses. As a consequence, the estimated wavefront reconstruction accuracy of the H–S could be a rather optimistic value. The reconstruction accuracy resulting from the simulations is plotted as a function of the number of microlenses across the entrance pupil, $N_S$, and the size of the differentiation mask expressed in Airy rings, $N_A$, for a fixed number of photons (Fig. 7). This figure corresponds to a variation of

$$\sigma^2_{\text{res, ODII}} = 2 \sigma^2_{\text{res, pyr}}, \tag{14}$$

where $\sigma^2_{\text{res, ODII}}$ and $\sigma^2_{\text{res, pyr}}$ are, correspondingly, the residual variance in the determination of the tilt for the ODII sensor and the PWFS, respectively. The factor 2 in equation (14) shows that the absorbing differentiation mask degrades the performance of the ODII sensor with respect to that of the PWFS. Nevertheless, our sensor offers some interesting advantages with respect to the PWFS. For example, our sensor is able to work in open loop adaptive optics thanks
to its linear relation between the wavefront phase derivative and the detected signal in contrast with the PWFS that presents a sinusoidal relationship (Ragazzoni 1996; Esposito & Riccardi 2001). Another significant difference is that our sensor is able to work with several reference objects, which represents a relevant advantage in multiconjugated adaptive optics. This is especially interesting in the extremely large telescope designs in which multiconjugate adaptive optics systems and atmospheric tomography are the only way to get their full potential capabilities.

6 CONCLUSIONS

We introduce a novel concept on the design of the OD. The main advantages of this sensor are that the dynamic range, resolution and sampling can be easily adjusted. Furthermore, this new design can be used with various reference stars allowing its use in multiconjugate adaptive optics systems. We have shown that its spatial resolution is not linked to the sampling of the wavefront phase. Therefore, its sampling and its resolution could be adjusted separately to adapt them to the current sensing conditions. Besides, it is able to work with fields affected by scintillation. Furthermore, the new scheme performs better in high-resolution operation providing a new device for the demanding requirements of high-resolution wavefront sensing, which is of interest for the projected extreme adaptive optics devices.

ACKNOWLEDGMENTS

This research was supported by Ministerio de Ciencia y Tecnología grant AYA2004-07773-C02-01.

REFERENCES

Bortz J. C., Thompson B. J., 1983, SPIE, 351, 71
Horwitz B. A., 1994, SPIE, 2201, 496
Iizuka K., 1987, Engineering Optics. Springer-Verlag, Berlin
Sirohi R. S., Ram Mohan V., 1977, Optica Acta, 24, 1105

This paper has been typeset from a TeX/LaTeX file prepared by the author.