Disc–wind field matching in accretion discs with magnetically influenced winds

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ABSTRACT

A solution is presented for the nearly force-free magnetic field in the inner region of an accretion disc wind, well inside the Alfvén surface. This is matched to a dynamo-generated disc field and the consequences for the disc and wind structures are investigated. The radial and vertical structures of the disc are found, together with the essential properties of the wind flow. Solutions result for which the wind angular momentum loss is at least comparable to the viscous radial advection of angular momentum.

Key words: accretion, accretion discs – magnetic fields.

1 INTRODUCTION

Accretion discs have strong differential rotation and turbulent motions and hence are likely to generate magnetic fields by a dynamo process (see Campbell 1997, and references therein). Winds may flow from the disc surfaces and be influenced by the large-scale magnetic field. As in the stellar case, a magnetically channelled wind of desirable geometry can result in a large flux of angular momentum per unit poloidal magnetic flux. In the disc case, this loss of angular momentum drives an inflow which leads to accretion on to the central object. It is only necessary to lose a small amount of mass in the wind to give the desired angular momentum loss, so most mass reaches the central region of the disc surrounding the accretor. The original model of Blandford & Payne (1982) only considered the wind structure, treating the disc as a thin boundary layer. Subsequent work has also considered the structure of the disc (see Campbell 2003 for references).

In Campbell (2003) a detailed model was presented for a disc with a magnetic wind. The radial and vertical structures of the disc were calculated, together with the essential properties of the wind. The magnetic field was generated by a simple \( \alpha \omega \)-dynamo in the disc, producing a steady field of the required dipolar symmetry. The disc inflow bends the poloidal magnetic field away from the central vertical axis of the disc. For sufficient field bending material has enough thermal energy to surmount the centrifugal–gravitational potential barrier and flow through the sonic surface. Beyond this surface centrifugal force exceeds gravity along the poloidal magnetic field and the wind flow reaches speeds comparable to the rotational velocity when it passes through the Alfvén surface. The continuous removal of disc angular momentum plays a major part in driving the inflow. Turbulent viscous stresses can also contribute to this by advecting angular momentum in the outward radial direction through the disc.

The model of Campbell (2003) used a simple parabolic field structure in the wind region well inside the Alfvén surface, in order to find the coordinates of the sonic point and then the wind mass flux. However, this over-estimates the curvature of the field in this nearly force-free region. The present analysis presents an improved model by self-consistently calculating the inner wind field and matching it to the disc field solution. In Section 2 the disc–wind model is summarized and then in Section 3 the inner wind region field is calculated and matched to the disc field. In Section 4 the resulting sonic point coordinates and the wind mass flux are found. Section 5 uses the results to calculate the radial and vertical structures of the disc, and the essential properties of the wind. Section 6 presents the conclusions.

2 THE DISC–WIND MODEL

The detailed model is presented in Campbell (2003), so only the main features are given here. Cylindrical coordinates \((\sigma, \phi, z)\) are used, centred on the accretor of mass \(M\). It is assumed that the accretor has no significant magnetic field that could affect the structure of the disc or wind. Through most of the disc, the stellar gravity is the dominant force in the \(\sigma\)-direction and hence the angular velocity in a thin disc has the Keplerian form

\[
\Omega = \Omega_K = \left( \frac{GM}{\sigma^3} \right)^{1/2}. \tag{1}
\]

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The thin nature of the disc is characterized by the small aspect ratio \( \frac{h}{\varpi} \), where the disc surfaces are \( z = \pm h(\varpi) \) and \( dh/d\varpi \ll 1 \). The large difference between the radial and vertical length-scales enables separation of variables to a good approximation.

The large-scale magnetic field is generated in the disc by a simple \( \alpha \omega \)-dynamo, satisfying the equations

\[
B_{\varpi} = -\frac{\eta}{\varpi^2} \frac{\partial^2 B_{\varphi}}{\partial \varpi^2},
\]

\[
\frac{\partial^3 B_{\varphi}}{\partial \varpi^3} + \frac{\varpi \Omega_{\varpi}^2 \alpha}{\eta^3} B_{\varphi} = \frac{\varpi \Omega_{\varpi}^2}{\eta^3} v_{\varpi} B_{\varphi},
\]

where \( \eta \) is the magnetic diffusivity and the primes, here and below, denote differentiation with respect to the relevant independent variable.

The \( \alpha \)-effect function is taken as

\[
\alpha(\varpi, z) = \begin{cases} 
-\epsilon c_s, & 0 < z < h, \\
0, & z = 0, \\
\epsilon c_s, & -h < z < 0,
\end{cases}
\]

where \( \epsilon \) is a dimensionless parameter related to the turbulence, and \( \eta \) is parametrized by

\[
\eta = \frac{\epsilon}{N_{\alpha}} c_s h.
\]

and

\[
c_s = \left( \frac{R_\mu T_c}{h} \right)^{1/2}
\]

is the central plane sound speed and \( N_{\alpha} \) is a turbulent magnetic Reynolds number. The field components \( B_{\varpi} \) and \( B_{\varphi} \), and the inflow speed \( v_{\varpi} \), can be expressed in the separable forms

\[
B_{\varpi} = B_{\varpi s}(\varpi_0) f''_{\varphi}(\zeta)
\]

\[
B_{\varphi} = B_{\varphi s}(\varpi_0) f_{\varphi}(\zeta),
\]

\[
v_{\varpi} = v_{\varpi c}(\varpi_0) f_{v}(\zeta),
\]

where subscripts s and c denote surface and central values, \( \varpi_0 \) refers to the base of a field line at \( z = 0 \) and

\[
\zeta = \frac{z}{h}.
\]

To leading order

\[
B_{\varphi} = B_{\varphi s}(\varpi_0).
\]

The magnetic field has dipole symmetry, as required for wind launching, with \( B_{\varpi} \) and \( B_{\varphi} \) anti-symmetric about \( z = 0 \). The function \( f_{\varphi}(\zeta) \) obeys the equation

\[
f''_{\varphi} + K^3 f_{\varphi} = -K^3 F f_{v},
\]

where

\[
K^3 = \frac{3 \Omega_{\varpi}^2 c_s^3}{2 \eta^2}
\]

and

\[
F = \frac{v_{\varpi c} B_{\varphi s}}{\alpha B_{\varpi s}}.
\]

The boundary conditions are

\[
f_{\varphi}(0) = 0, \quad f'_{\varphi}(0) = 0, \quad f_{\varphi}(1) = 0.
\]

The first two conditions arise from the anti-symmetry of \( B_{\varphi} \) and \( B_{\varpi} \) about \( z = 0 \), while the third condition is a result of the nearly force-free nature of the magnetic field well inside the Alfvén surface (see Campbell 1999). The quantities \( K \) and \( F \) are constant, with \( K^3 \) being the dynamo number. The induction equation yields

\[
\frac{B_{\varpi s}}{B_{\varphi s}} = \frac{K^3}{N_{\alpha} f'_{\varphi}(1) \tan i_s},
\]

with

\[
\tan i_s = \frac{B_{\varphi s}}{B_{\varpi s}}
\]

where \( i_s \) is the angle that the poloidal magnetic field makes with the horizontal at \( z = h \). The central plane inflow speed is

\[
v_{\varpi c} = -\frac{\epsilon K^3 F}{N_{\alpha} |f'_{\varphi}(1)| \tan i_s} c_s.
\]
The radial variations of the magnetic field components can be found from the induction equation, mass conservation, vertical equilibrium and (17). These give

\[ B_{\phi}(R_0) = \left( \frac{I}{I_i} \right) \frac{1}{2} \frac{1}{M^{1/4} M^{1/2} \frac{\tan i_0}{\sigma_0^{3/4}}} \]

(19)

\[ B_{\phi}(R_0) = -\left( \frac{I}{I_i} \right) \frac{1}{2} \frac{1}{M^{1/4} M^{1/2} \frac{\tan i_0}{\sigma_0^{3/4}}} \]

(20)

\[ B_{\phi}(R_0) = \left( \frac{I}{I_i} \right) \frac{1}{2} \frac{1}{M^{1/4} M^{1/2} \frac{\tan i_0}{\sigma_0^{3/4}}} \]

(21)

where

\[ I = \frac{|f_\rho(1)|}{K^3 F} \]

(22)

\[ I_1 = \int_0^1 f_\rho f_\phi \, d\zeta \]

(23)

\[ k_1 \text{ is a separation constant and } M \text{ is the accretion rate. The density and temperature have the separable forms} \]

\[ \rho = \rho_0(T_0) \rho_i(\zeta) \]

(24)

\[ T = T_i(T_0) f_T(\zeta) \]

(25)

The quantities \( h(R_0), T_i(T_0), \rho_0(T_0) \) and \( v_\phi(R_0) \) can be solved for algebraically (see Campbell 2003).

The thermal problem and vertical equilibrium lead to an integro-differential equation for \( f_\rho(\zeta) \) given by

\[ f_\rho = - \left( \frac{1}{k_1 A} \frac{1}{f_T} \right) \left\{ 2k_1 \left[ f_\rho - \frac{N_u^2}{K^2} (F + f_\phi) f_\phi \right] + \zeta f_\phi \right\} + \frac{3}{20} \frac{1}{k_2} \frac{f_T^2}{f_T^{15/2}} \int_0^\zeta f_\rho \, d\zeta + \frac{8}{9} k_1 \frac{1}{N_0} \int_0^\zeta \frac{f_T}{f_T^{15/2}} \int_0^\zeta f_\rho \, d\zeta \]

(26)

where

\[ f_T = \left( \frac{1}{k_1 A} \frac{1}{f_\phi} \right) \left[ k_1 A - k_1 \left( \frac{N_u^2}{K^2} f_\phi^2 + f_\phi^2 \right) - \int_0^\zeta \zeta f_\phi \, d\zeta \right] \]

(27)

\[ I_m(\zeta) = \frac{1}{K^2 T} f_\phi^2 + 2f_\phi f_\theta + F(\phi - f_\phi(\zeta)) \]

(28)

\[ k_1 A = \frac{9}{4} \frac{N_u^4}{e^2 K^2} \]

(29)

and \( k_2 \) is a thermal separation constant. The vertical dependent part of the angular momentum equation leads to

\[ f_\phi(\zeta) = f_\phi(1) + \left( \frac{1 - f_\phi(1)}{f_\phi(0)} \right) \frac{f_\phi(0)}{f_\phi(\zeta)} \]

(30)

where

\[ f_\phi(1) = \frac{I_1 f_\phi(0) - 1}{I_2 f_\phi(0) - 1} \]

(31)

with \( I_1 \) given by (23) and

\[ I_2 = \int_0^\zeta f_\rho \, d\zeta \]

(32)

Equations (26)–(29) can be solved numerically to obtain \( f_\rho(\zeta) \) and \( f_T(\zeta) \). Equation (30) then gives \( f_\phi(\zeta) \) for the vertical variation of the inflow.

3 THE WIND REGION MAGNETIC FIELD

The wind flow region must be considered in order to calculate the angular momentum loss which plays a part in driving the disc inflow. This is related to the mass flux, \( m \), through the sonic point, and to the Alfvén point coordinate \( \sigma_\lambda \). The quantity \( M \sigma_\lambda^2 \) is relevant to the wind angular momentum transport. Ideally, a full solution for the wind magnetic field and flow should be found, but this poses a presently intractable problem for magnetic winds (see Mestel 1999). However, it was shown by Campbell (2003) that the essentials of the disc can be found from conservation laws and consideration of the field structure between the disc and sonic surfaces. The quantities \( M \) and \( \sigma_\lambda \) can then be found and related to the disc structure.

If the wind angular momentum transport is to be at least comparable to the viscous transport, the sonic surface must not lie too far above the disc surface. Suitable sonic points have coordinates \((\sigma_m, z_m)\) which satisfy \(z_m/\sigma_m \ll 1\). The poloidal magnetic field structure must be found for the region \(h \leq z \leq z_m\), in order to calculate the sonic point coordinates and \(m\). A simple parabolic field structure was used by Campbell (2003), but this tends to over-estimate the field line curvature. The present analysis improves on this by solving the relevant equation to calculate the local wind field.

For the magnetic torque to be significant, the Alfvén surface must lie well beyond the disc surface. Magnetic stresses dominate material stresses at points for which \((\sigma_m/\sigma_m)^2 \gg 1\), and then the magnetic field has a nearly force-free structure. The region between the disc and sonic surfaces satisfies this condition. The poloidal magnetic field can be expressed in terms of a flux function \(\psi_m\) by

\[
B_p = \nabla \times \left( \frac{\psi_m}{\sigma} \hat{\phi} \right),
\]

which gives components

\[
B_m = -\frac{1}{\sigma} \frac{\partial \psi_m}{\partial z}, \quad B_z = \frac{1}{\sigma} \frac{\partial \psi_m}{\partial \sigma}.
\]

The use of these components in the force-free condition \(J \times B = 0\) leads to the Grad–Shafranov equation

\[
\sigma \frac{\partial}{\partial \sigma} \left( \frac{1}{\sigma} \frac{\partial \psi_m}{\partial \sigma} \right) + \frac{\partial^2 \psi_m}{\partial z^2} = -\frac{1}{2} \frac{d}{d\psi_m}(W^2),
\]

where

\[
W(\psi_m) = \sigma B_p.
\]

Equation (33) yields

\[
B_p = \frac{1}{\sigma} (\nabla \psi_m) \times \hat{\phi}
\]

and hence

\[
B_p \cdot \nabla \psi_m = 0.
\]

Equation (36) then gives

\[
B_p \cdot \nabla W = \frac{dW}{d\psi_m} B_p \cdot \nabla \psi_m = 0
\]

and so

\[
B_p \cdot \nabla (\sigma B_p) = 0
\]

for an axisymmetric force-free region.

The wind field must match the dynamo-generated disc field at \(z = h\) for all \(\sigma\). The matching of the radial dependences is achieved by taking a flux function of the form

\[
\psi_m = C \sigma^{-3/4} f(\xi),
\]

where \(C\) is a constant and \(\xi = z/h\). Because \(h\) depends on \(\sigma\), the derivative \((\partial \psi_m/\partial \sigma)_z\) obtained from (39) should account for the \(\sigma\) variation of \(h\) in \(f(\xi)\) as well as for the explicit variation \(\sigma^{-3/4}\). However, it is shown below that the implicit variation is ignorable to leading order in \(h/\sigma\). The solution for \(h(\sigma)\) is very nearly linear (see Campbell 2003) and so

\[
h(\sigma) = \bar{K} \sigma,
\]

where \(\bar{K}\) is a constant. It follows from (39) that

\[
\left( \frac{\partial \psi_m}{\partial \sigma} \right)_z = \frac{3}{4} C \sigma^{-1/4} f + C \sigma^{-3/4} f' \left( \frac{\partial \xi}{\partial \sigma} \right)_z = C \sigma^{-1/4} \left( \frac{3}{4} f - \xi f' \right),
\]

and

\[
\left( \frac{\partial \psi_m}{\partial \xi} \right)_\sigma = \frac{\sigma}{h} C \sigma^{-1/4} f',
\]

where \(f' = df/d\xi\). Equations (34a,b) then yield

\[
B_m = -\frac{\sigma}{h} C \sigma^{-3/4} f', \quad B_z = C \sigma^{-5/4} \left( \frac{3}{4} f - \xi f' \right)
\]

and hence

\[
\frac{B_z}{B_m} = -\frac{3}{4} \frac{h f}{\sigma f'} + \frac{h}{\sigma} \xi.
\]

However, \(B_z/B_m \sim 1\) is required for suitable wind flows and \((h/\sigma)\xi \ll 1\) for the region of interest. It then follows that only the first term in (41) can satisfy this, with \(f' \sim -(h/\sigma)f\), and the small second term \((h/\sigma)\xi\) can be dropped to leading order. The variation of \(\xi\) with \(\sigma\) at constant \(z\) can therefore be ignored to a good approximation, giving

\[
B_m = -\frac{\sigma}{h} C \sigma^{-3/4} f', \quad B_z = \frac{3}{4} C \sigma^{-3/4} f
\]

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\[
\frac{B_z}{B_\psi} = -\frac{3}{4} \frac{h f}{\sigma f'}. \tag{43}
\]

Equations (40) and (43) show that \(B_z/B_\psi\) is independent of \(\sigma\).

Substitution of (39) into (35) leads to

\[
f'' - \frac{15}{16} \frac{h^2}{\sigma^2} f = -\frac{1}{2} \frac{h \sigma^{-3/4}}{C} \frac{d}{d \psi_m} (W^2). \tag{44}
\]

Then using (39) to eliminate \(\sigma^{-3/4}\) gives the separated equations

\[
f'' - \frac{15}{16} \frac{h^2}{\sigma^2} f = Df^{-5/3}, \tag{44}
\]

and

\[
\frac{d}{d \psi_m} (W^2) = -2 \frac{C^{8/3} D}{\bar{K}^2} \psi_m^{-5/3}, \tag{45}
\]

where \(D\) is a constant. Integrating (45), and using the definition (36) for \(W\), yields

\[
\sigma B_\phi = -\left(\frac{3C^{8/3} D}{\bar{K}^2}\right)^{1/2} \psi_m^{1/3}, \tag{46}
\]

where the negative root is taken to facilitate surface matching to the disc field. Using (39) for \(\psi_m\) in (46) yields

\[
B_\phi = -\left(\frac{3D}{\bar{K}^2}\right)^{1/2} \frac{C}{\sigma^{5/4} f^{1/3}}. \tag{47}
\]

Equations (42a,b) and (47) give the components of the wind magnetic field. Matching these to the components of the disc field at \(\zeta = 1\) determines the constants \(C\) and \(D\).

Matching \(B_z\) at the disc surface, using the wind field component (42b) with the scaling

\[
f(1) = 1, \tag{48}
\]

yields

\[
C = \frac{4}{3} \sigma^{5/4} B_{zs}, \tag{49}
\]

with \(B_{zs}\) given by the disc field component (21). It follows from (16) and (17) that

\[
B_{zs} = \frac{N}{\bar{K}} f_\phi''(1) B_\psi s. \tag{50}
\]

Equating this to the surface value of (42a) gives

\[
f''(1) = -\frac{N}{\bar{K}^2} \frac{f_\phi''(1)}{\sigma \sigma^{5/4} B_\psi s}. \tag{51}
\]

Substituting for the wind value of \(B_\psi s\) from (47), noting that \(\bar{K} = h/\sigma\), then leads to

\[
f''(1) = -\frac{N}{\bar{K}^2} |f_\phi''(1)| (3D)^{1/2}. \tag{51}
\]

Another expression for \(f''(1)\) follows from equating (17) to the surface value of (43) as

\[
f''(1) = -\frac{3}{4} \frac{h}{\tan i_s} \frac{1}{\sigma}. \tag{52}
\]

Equating (51) and (52) yields

\[
D = \frac{3}{16} \frac{K^6}{N^2} \frac{1}{|f_\phi''(1)|^3 \tan^4 i_s \sigma^2}. \tag{53}
\]

and the use of (16) enables this to be expressed as

\[
D = \frac{3}{16} \left(\frac{B_{zs}}{B_{\psi s}}\right)^2 \frac{h^2}{\sigma^2}. \tag{53}
\]

The use of (49) and (53) for \(C\) and \(D\), together with (40) and (43) for \(\bar{K}\) and \(B_z/B_\psi\), in (42a,b) and (47) gives the wind field components in the force-free inner region as

\[
B_\sigma = B_{zs} \left(\frac{\sigma_s}{\sigma}\right)^{5/4} \frac{f''(\zeta)}{|f''(1)|}, \tag{54}
\]

\[
B_\phi = B_{\psi s} \left(\frac{\sigma_s}{\sigma}\right)^{5/4} \frac{1}{f(\zeta)^{1/3}}, \tag{55}
\]

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\[ B_c = B_\infty \left( \frac{\sigma_c}{\sigma} \right)^{5/4} f(\zeta). \]  

(56)

Equation (44) must be solved to obtain \( f(\zeta) \) subject to the surface conditions (48) and (52). Solutions of interest here will have \( |B_{\phi s}/B_c| < 1 \) and \( f \sim 1 \), so the ratio of the the third to the second term in (44) will be \( \sim (\sigma/h)^2 \). Using (53) for \( D \) then gives this ratio as \( \sim 0.2(B_{\phi s}/B_c)^2 \), so it is a reasonable approximation to drop the third term in (44). The equation then reduces to

\[ f'' - k^2 f = 0, \]

(57)

where

\[ k = \frac{\sqrt{15}}{4} \frac{h}{\sigma}. \]

(58)

This is equivalent to ignoring \( J_o \) relative to the poloidal components of the current density. The poloidal magnetic field is then approximately current-free.

The general solution of (57) is

\[ f(\zeta) = c_1 e^{k\zeta} + c_2 e^{-k\zeta}, \]

where \( c_1 \) and \( c_2 \) are constants. Applying the boundary conditions (48) and (52) then determines \( c_1 \) and \( c_2 \) and leads to

\[ f(\zeta) = \cosh[k(\zeta - 1)] - \frac{3}{\sqrt{15} \tan t} \sinh[k(\zeta - 1)]. \]

(59)

The poloidal field line equations can be found by integrating

\[ \frac{dz}{d\sigma} = B_c/B_\infty. \]

(60)

It follows from (10) for \( \zeta \) that on field lines

\[ \frac{dz}{d\sigma} = \frac{h}{\sigma} \frac{dc}{d\sigma} + \frac{h}{\sigma} \zeta. \]

(61)

Since \( dz/d\sigma \sim 1 \) is required for suitable wind flows, it follows that for \( \zeta \sim 1 \) the last term in (61) can be ignored. Equations (43), (60) and (61) then yield

\[ \frac{f'}{f} \frac{d\zeta}{d\sigma} = -\frac{3}{4} \frac{d\sigma}{\sigma}, \]

(62)

which integrates to give the field line equations

\[ f(\zeta) = \left( \frac{\sigma_c}{\sigma} \right)^{3/4}. \]

(63)

using \( f(1) = 1 \). It is noted that (62) remains valid if the higher order terms are retained in \( B_c/B_\infty \) and \( dz/d\sigma \) in (41) and (61), since they cancel.

It follows from (58) that \( k \ll 1 \), so use of the quadratic Taylor expansion of (59) in (63) leads to the field line equation

\[ \frac{\sigma}{\sigma_c} = 1 + \frac{h}{\tan t} \frac{1}{\sigma} \left( \zeta - 1 \right) - \frac{5}{8} \left( 1 - \frac{7}{5} \tan^2 t \frac{h^2}{\sigma^2} \right) \frac{1}{\sigma^2} \left( \zeta - 1 \right)^2, \]

(64)

valid for \( \zeta \ll \sigma/h \). It follows that, to first order in \( h/\sigma \), the wind poloidal field is straight in this region. The curvature is a second-order effect, and the variation of \( h/\sigma \) with \( \sigma \) in \( \zeta = z/h \) must be accounted for at this order. It can be shown that the field line curvature changes from negative in the disc to positive in the wind. Hence the wind poloidal field slowly starts to collimate parallel to the vertical axis. The field lines must inflect as they pass from the disc to the wind region. The vertical length-scale of the horizontal field changes from \( \sim h \) in the disc to \( \sim \sigma \) in the wind. A thin transition region will exist near \( z = h \) through which the vertical length-scale rapidly changes as the field lines inflect. The details of this region would involve a higher order problem and need not be considered for the purposes of the present work.

Noting that \( h/\sigma \) is a constant which can be expressed as \( h(\sigma_\infty)/\sigma_\infty \), while \( \zeta = z/h(\sigma) \), the linear form of (64) is

\[ \frac{\sigma}{\sigma_c} = 1 - \frac{h(\sigma_\infty)}{\tan t} \frac{1}{\sigma_c} \frac{1}{\sigma_c} + \frac{1}{\tan t} \frac{1}{\sigma_c} \frac{z}{\sigma_c}. \]

(65)

4 THE SONIC POINT AND WIND MASS FLUX

It was shown by Campbell (2002) that provided \( \sigma_A \lesssim 10 \sigma_c \), and \( |B_{\phi s}/B_c| < 1 \), the slow magnetosonic and sonic points are essentially identical. The sonic point can then be found from the condition of balance of the gravitational and centrifugal forces along the poloidal

magnetic field. The effective potential (gravitational plus centrifugal) for matter corotating with a field line at angular velocity \( \Omega_k(\sigma_s) \) is

\[
\psi = \frac{GM}{\sigma_s} \left[ \left( \frac{\sigma^2}{\sigma_s^2} + \frac{\sigma^2}{\sigma_s^2} \right)^{-1/2} + \frac{1}{\sigma_s^2} \right].
\]

The condition \( B_p \cdot \nabla \psi = 0 \) yields

\[
\sigma \cos i_s + z \sin i_s = \frac{\sigma}{\sigma_s} \cos i_s,
\]

which can be expressed as

\[
\frac{\sigma}{\sigma_s} + \frac{z}{\sigma_s} \tan i_s = \frac{\sigma}{\sigma_s} \left[ \left( \frac{\sigma}{\sigma_s} \right)^2 + \left( \frac{z}{\sigma_s} \right)^2 \right]^{3/2}.
\]

Using (65) for \( \sigma / \sigma_s \), and expanding (68) to second order in \( h / \sigma_s \) and \( z / \sigma_s \), leads to a quadratic equation for \( z / \sigma_s \). The roots of this equation give sonic point solutions as

\[
z_{sa} = \frac{2 \tan i_s \left( \tan^2 i_s - 3 \right)}{3 \left( \tan^2 i_s + 4 \right)} \sigma_s \quad \text{and} \quad z_{sa} = \frac{3}{3 - \tan^2 i_s} h.
\]

Solutions must satisfy \( z_{sa} / \sigma_s < 1 \) for self-consistency, but (69a) only does this for a narrow range of angles just above \( i_s = 60^\circ \). Hence (69b) is taken as the relevant solution which, together with (65), yields the sonic point coordinates as

\[
\sigma_{sa} = \sigma_s + \frac{\tan i_s}{3 - \tan^2 i_s} h, \quad z_{sa} = \frac{3}{3 - \tan^2 i_s} h.
\]

The mass flux through the sonic point is given by

\[
m = \rho_0 a^2,
\]

where \( a \) is the sound speed in the isothermal wind. The subsonic flow region can be treated as approximately hydrostatic and this yields

\[
\rho_a = \rho_0 \exp \left\{ - \left[ \frac{(\psi_{sa} - \psi)}{a^2} + \frac{1}{2} \right] \right\},
\]

where \( \rho_0 \) is the density at the disc surface and \( \psi \) is the effective potential given by (66). Evaluating \( \psi \) at the sonic point \( (\sigma_{sa}, z_{sa}) \) and at the disc surface \( (\sigma_s, h) \), using (66) and (69a), and expanding \( (\psi_{sa} - \psi) \) to second order leads to

\[
rho_a = \rho_0 \exp \left\{ - \left[ \Omega_{k_s}^2 \frac{h^2}{2a^2} \frac{\tan^2 i_s}{3 - \tan^2 i_s} + \frac{1}{2} \right] \right\},
\]

where \( \Omega_{k_s} = \Omega_k(\sigma_s) \).

The Alfvén coordinate \( \sigma_A \) can be found from the angular momentum equation for the wind, which gives

\[
\sigma_A^2 = \frac{k_1 I_f}{3 \pi I_1} \left( f_\sigma(1) \right) \frac{M}{\dot{m}} \left( 1 + \tan^2 i_s \right)^{1/2} \tan i_s,
\]

with \( \dot{m} \) given by (71) and (73) as

\[
m = a \rho_0 \exp \left\{ - \left[ \Omega_{k_s}^2 \frac{h^2}{2a^2} \frac{\tan^2 i_s}{3 - \tan^2 i_s} + \frac{1}{2} \right] \right\}.
\]

5 DISC–WIND SOLUTIONS AND VERTICAL STRUCTURE

Equations (26) and (27) are integrated numerically to find \( f_\sigma(\xi) \) and \( f_T(\xi) \). An improved Euler method is used together with Taylor expansions about \( z = 0 \) to start the solutions. Expressions for the constants \( k_1 \) and \( k_2 \) are given by Campbell (2003). The function \( f_\xi(\xi) \) follows from (30). Figs 1–3 show the vertical variations of \( f_\sigma, f_T \) and \( f_s \), while Table 1 shows the corresponding disc and wind properties. Fig. 3 illustrates that the wind flow speed decreases sharply as the disc surface \( \xi = 1 \) is approached. This would be consistent with \( f_\sigma(\xi) \) passing through zero in a thin boundary layer just beyond \( \xi = 1 \). As the wind region is entered \( v_m \) becomes positive in the outflow. This would be essentially the same boundary layer discussed above through which the poloidal field lines inflect as the sign of their curvature changes. The dynamo quantity \( K = 1 \) is uses here to give field bending sufficient for wind launching, with \( i_s \) below the critical \( 60^\circ \) angle. This is still well below the maximum value of \( K = \sqrt{3} / 3 \) discussed by Campbell (1999).

The magnetic wind torque \( T_w \) is comparable to the viscous torque \( T_v \), so the wind plays a major part in driving the inflow. The value of the ratio \( 4 \pi \sigma_s^2 m / M \) shows that only a small amount of mass is lost in the wind. The requirement of \( |B_\phi / B_{sa}| < 1 \) is met, while the smallness of the ratio \( B_{sa}^2 / 2 \mu_0 P_c \) illustrates that the magnetic vertical compression of the disc is small. The value of the optical depth \( \tau_D \) through the disc is consistent with the use of the radiative diffusion equation for the heat flux.
Figure 1. The vertical variation of the density.

Figure 2. The vertical variation of the temperature.

Figure 3. The vertical variation of the inflow speed.
Table 1. Disc and wind quantities.

| $f_{\rho}(1)$ | $f_{T}(1)$ | $f_{v}(1)$ | $i_s$ | $T_{in}/T_v$ | $\sigma_\Lambda/\sigma_0$ | $4\pi\sigma_\Lambda^2 m/M$ | $|B_{\phi}/B_{z}|$ |
|--------------|-------------|-------------|-------|---------------|----------------|---------------------------|------------------|
| 0.43         | 0.70        | 0.45        | 58.0° | 1.04          | 3.09           | $3.14 \times 10^{-2}$    | 0.91             |
| $B_{\phi}/2\mu_0 P_c$ | $\tau_D$ | $I_1$ | $I_2$ | $I_3$ | $k_1$ | $k_2$ | $A k_1$ |
| $3.43 \times 10^{-2}$ | 10.39       | 0.73       | 0.79  | 0.38          | $1.81 \times 10^{-2}$ | 0.27           | 0.53             |

6 CONCLUSIONS

The disc–wind model presented here improves on that of Campbell (2003) in that it solves for the magnetic field in the inner wind region, well inside the Alfvén surface, in a way which is self-consistent with a force-free structure. The nearly straight geometry of the wind field means that greater bending of the poloidal field is necessary to ensure $i_s < 60°$. However, this requirement is easily accommodated with the dynamo solution used. The required wind mass flux results without the need to invoke a hot disc corona, photospheric temperatures being sufficient to allow material to surmount the potential barrier associated with reaching the sonic surface.

REFERENCES


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