The authors have very elegantly exploited the interference method of determining the film shape in point contact by introducing the concept of oil starvation. They have thus started to study the practical ball bearing where oil starvation probably often does occur.

Fig. 8 shows very clearly the importance of having an extensive and complete oil meniscus in the inlet. The presence of an oil starvation must seriously delay the pressure build-up and hence reduce film thickness.

In the practical bearing, the cage must influence the availability of the oil supply to each ball. A proper cage design can ensure that each ball is locally supplied with oil to make up for the starvation present in its inlet region, these being formed from the wake of the previous ball.

R. Gohar

D I S C U S S I O N

Y. P. Chiu

The authors are to be congratulated for having performed an extremely interesting experimental and analytical investigation on a very important phenomenon in rolling contact.

First, the discussor wishes to show two slides of contact maps of steel ball-glass contacts in a slow speed 12rpm lightly loaded with a polybutene H1900 (viscosity 500,000 cP at operating temperature). These slides show contact shapes similar to that observed by Cameron in tests at higher speeds. Also, starvation has been observed when there is short oil supply. Similar to what Di Wedewen, et al., have observed, the starvation results in thinner EHD film thickness. Since starvation corresponds to a lower film thickness and thus to an increase in the percent of metallic contact area, there is no doubt that severe starvation (or small values of \( s \) in the authors' paper) can induce lower fatigue life. In rolling bearing application, it is an important area for research to optimize the amount of starvation in contact.

The authors have made remarks that starvation provides a higher friction traction in contact and that therefore it is desirable in rolling contact. The discussor believes this is important in high speed lightly loaded rolling element bearings which show a tendency toward skidding. Partial starvation may reduce the possibility of gross roadway skidding.

Lastly, the author has found very little different in contact shape when using glass and sapphire flats and operating at the same conditions. We have run a series of tests using three transparent materials (glass, sapphire, and Plexiglas) at constant speed with polybutene oil and have found that even at the same load the contact shape is somewhat different. Therefore, there is some dependence between contact shape and material elastic modulus. K. L. Johnson has shown in two dimensional contact that the contact shape depends on the parameter \( \text{p} = \frac{p_{\text{ave}}}{p_{\text{ave,quad}}} \).

The rest of the point contact has been established by a linear correlation relationship between the shape ofconstiction and \( \text{p} = \frac{p_{\text{ave}}}{p_{\text{ave,quad}}} \). The latter is equal to 8th power of the parameter \( W^{1/4}/U^{1/4} \) where:

\[
W = \frac{Q}{ER} \quad U = \frac{Q}{ER}
\]

Therefore, the contact shape has a weak dependence on \( E \).

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Authors’ Closure

The authors wish to thank the discussers for their comments on this paper.

Both the comments of R. Gohar about the influence of the cage design on the local oil supply and the comments of Y. P. Chu about the optimum amount of starvation to eliminate gross raceway skidding but yet maintain satisfactory fatigue life emphasize the practical importance of controlling the lubricant supply in the inlet region. An additional problem, particularly for gas turbine engines, is that the lubricant must also function as a coolant. Design techniques such as under-race cooling and inner-race feed slots described by Brown [26] have shown that more efficient lubrication is achieved through proper distribution of lubricant within the bearing. More work in this direction may lend to control and optimization of starvation in practice.

Considering further the application of the results to practice, it should be emphasized that although the paper deals primarily with the point contact case, the general features of the results apply equally to line and elliptical contacts which may be encountered more frequently in practice. This is supported by considering the recent work of Wolveridge, Baglin and Archard [27] who have computed the EHD starvation problem for line contact using Grubin assumptions as was done here for point contacts. They use a parameter $\psi$ to describe their results. In terms of our variables

$$\psi = \frac{S_0^{1/3}}{(2R(h_0))}\sqrt{s}$$

where $b$ is one half the Hertzian width. This is almost identical to the starvation parameter used in Fig. 15

$$\frac{S}{S_f} = \frac{S_0^{1/3}}{3.52R(h_0)\sqrt{s}}$$

It can be shown that $\psi = 2.21S/S_f$. To show the generality of the results, the authors would like to take this opportunity to replot the data of Fig. 15 and include also the theory for point contact and line contact. This is shown in Fig. 24. There is very good agreement between point contact experiment and theory; and, there are only small differences between point and line contact theories. These differences are probably the result of slightly different inlet geometries since at $h_0/h_0 = 9$, $S/S_f = 1.185$ for line contact and $S/S_f = 1$ for point contact. Note also that when $h_0/h_0 = 9$, both line and point contact theories predict a film thickness which is approximately 95-percent of its flooded value.

Also, for completeness we would like to include the full range of theoretical results for the flooded point contact case. Table 3 gives the first order approximations of $q_{max}$ for log$_B$ $B$ ranging from $-2$ and $3$.

<table>
<thead>
<tr>
<th>$\log_B B$</th>
<th>$q_{max}$</th>
<th>$\log_B B$</th>
<th>$q_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3.0$</td>
<td>not accurate</td>
<td>$0.5$</td>
<td>0.0447</td>
</tr>
<tr>
<td>$-2.0$</td>
<td>0.461</td>
<td>1.0</td>
<td>0.0216</td>
</tr>
<tr>
<td>$-1.5$</td>
<td>0.305</td>
<td>1.5</td>
<td>0.0102</td>
</tr>
<tr>
<td>$-1.0$</td>
<td>0.272</td>
<td>2.0</td>
<td>0.0048</td>
</tr>
<tr>
<td>$-0.5$</td>
<td>0.163</td>
<td>2.5</td>
<td>0.0022</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0884</td>
<td>3.0</td>
<td>0.0009</td>
</tr>
</tbody>
</table>

A good straight line of log$_B q_{max}$ against log$_B B$ exists over the range between $-0.2$ and $1.0$. Table 4 gives the second order approximations of $q_{max}$ for log$_B B$ ranging from $-0.2$ to $1.0$.

<table>
<thead>
<tr>
<th>$\log_B B$</th>
<th>$q_{max}$</th>
<th>$\log_B B$</th>
<th>$q_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.2$</td>
<td>0.118</td>
<td>0.4</td>
<td>0.0547</td>
</tr>
<tr>
<td>$0.0$</td>
<td>0.0926</td>
<td>0.6</td>
<td>0.0416</td>
</tr>
<tr>
<td>$0.2$</td>
<td>0.0714</td>
<td>1.0</td>
<td>0.0221</td>
</tr>
</tbody>
</table>

Numbers 20-29 in brackets designate Additional References at end of Closure.

The straight line relationship over this range is given by equation (9) which leads to the film thickness formula of equation (11). Finally, we would like to consider the results of Fig. 7 in light of the comments of Y. P. Chu about the contact shape. Since the present point contact theory predicts the central film thickness $h_0$ rather than the minimum film thickness $h_0$, it is the intent of Fig. 7 to show that $h_0$ can be substantially less than $h_0$. At the lower film thicknesses ($h_0 = 0$ micron) the minimum film thickness, which occurs at the lateral constrictions, is almost 70-percent less than $h_0$. If the data of Forood, et al. [10] for large film thicknesses ($h_0 = 225$ micron) are included in Fig. 7 the minimum film thickness, which now occurs at the rear constrictions, is only about 30-percent lower than $h_0$.

Johnson [28] suggests that the EHD pressure distribution may be approximately a function of a single parameter $g_0$. For line contact

$$g_0 = \left(\frac{\mu^3}{\mu_s^3E'R'^2R}\right)^{1/3}$$

He furthermore shows that for previously published theoretical results there is a reasonable correlation between $g_0$ and the position of the pressure peak.

The location of the pressure peak is located just upstream of the rear constriction it is expected that the location of the upstream edge of the rear construction will also correlate with $g_0$. This has been shown by Chu, et al. [29] for point contacts where

$$g_0 = \left(\frac{\mu^3}{\mu_s^3E'R'^2R}\right)^{1/3}$$

In addition, the location of the pressure peak should give an indication of the overall pressure shape and therefore the film shape. Preliminary analysis of experimental point contact results, however, shows that a correlation between $h_0/h_0$ and $g_0$ is not very good. In general, $h_0/h_0$ decreases with $g_0$, but decreases more rapidly for the higher values of the material modulus $E'$. It appears, therefore, that while the film shape along the center line may be a function of $g_0$ only, the film shape at the edge of the contact (line or point), where the minimum film thickness most often occurs, is sensitive to additional variables. Since local film thickness is determined by the inlet conditions immediately upstream, these variables must include the effects of side leakage and the geometry at the edges of the inlet region. Thus, the lateral constrictions appear to be somewhat different “animal” than the rear constriction; and, a rigorous correlation between $h_0/h_0$ and the operating variables may not be a simple one.

Additional References