involving $\psi_m$ ($m > 0$) on the right-hand side of equation (2) can be ignored, one then has

$$\psi_n(\beta, \tau) \simeq -\frac{1}{\beta_n^2} c_n \left[ (1 - e^{-\beta_n \lambda}) \frac{\partial \psi_0}{\partial \gamma} + \frac{1}{\beta_n^2 \rho_0^2} \left[ 1 - e^{-\beta_n \lambda} (1 + \beta_n^2 \tau) \right] \frac{\partial \psi_0^3}{\partial \rho_1} + \cdots \right],$$

$$n = 1, 2, 3, \cdots \quad (3)$$

Substitution of $\partial \psi_n/\partial \tau = \partial \psi_n/\partial \gamma$, which can be obtained from equation (3), into equation (19) of the paper yields, by noting that $c_0 = \beta_0 = 0$, for the mean concentration

$$\frac{\partial \psi_0}{\partial \tau} = \frac{1}{\rho_0^2} \frac{\partial \psi_0}{\partial \rho_1} + \sum_{n=1}^{\infty} \beta_n^2 c_n c_{n0} \left[ (1 - e^{-\beta_n \lambda}) \frac{\partial \psi_0}{\partial \rho_1} + \frac{1}{\beta_n^2 \rho_0^2} \left[ 1 - e^{-\beta_n \lambda} (1 + \beta_n^2 \tau) \right] \frac{\partial \psi_0^3}{\partial \rho_1} + \cdots \right].$$

By using equations (15) and (16) in the paper for the product $c_n c_{n0}$, comparison between equations (1) and (4) shows that

$$k_{2n+1}(\tau) = 0, \quad n = 0, 1, 2, \cdots \cdots, \quad (5a)$$

$$k_2(\tau) = \frac{1}{\rho_0^2} + \frac{1}{\beta_1^2} \sum_{n=1}^{\infty} \beta_n^2 \sum_{n=1}^{\infty} \beta_n^2 \sum_{n=1}^{\infty} \beta_n^2 e^{-\beta_n \lambda}, \quad \sum_{n=1}^{\infty} \beta_n^2$$

The expression for $k_2(\tau)$ in equation (5b) becomes identical to that in equation (19) of reference [17] through the use of the relations

$$(6a) \quad \sum_{n=1}^{\infty} \beta_n^{-6} = 1/3072,$$

and

$$(6b) \quad \beta_n = \frac{4 \rho_0}{\beta_0} \left[ \frac{\partial \psi_0}{\partial \gamma} \right]_{\beta_0 = \beta_n}$$

The present results show, however, that all the $k$'s with an odd index have a value identical to zero.

The Taylor's series expansion for $\psi_0$ as given by equation (20) in the paper is exact. It is apparent from the present comparison that Dr. Gill's dispersion model expressed in terms of the mean concentration and its longitudinal derivatives is a valid one whenever terms other than $\partial^2 \psi_0/\partial \gamma^2$ on the right-hand side of equation (20) can be neglected. The contribution to the variation of $\psi_0$ due to the existence of such terms is difficult to assess from first principles. It is hopeful that results of numerical calculations will be made available for quantitative clarification.

References


Authors' Closure

The authors regret that the title of the paper may have caused some misunderstanding for the discusser. We should like to point out that the references cited in the discussion do not concern torsion but rather problems of torsionless axial symmetry.

2 Mechanical Engineer, Potomac Electric Power Company, Washington, D.C. Mem. ASME.
3 Numbers in brackets designate References at end of Discussion.

DISCUSSION

W. A. Vachon.\footnote{1} 1 Most cables have predisposition to hockling at lower values of torque or higher values of tension because they have a preset curvature arising from the spool or drum on which they've wound them.

2 Flexible cables, capable of going over small diameter sheaves, have lower $EI$'s and will hockle more easily. There may be an optimum $EI$ for given applications.

3 I recall that torsional elastic instability would set in as described at modest tensions but hockling would not occur unless end restraints could travel (axially) enough to afford the wire for hockling. It takes only very low tensions (after elastic instability sets in) to keep the cable from hockling. What this is I don't know.

Author's Closure

The author would like to thank Mr. Vachon for his interesting Discussion. In a subsequent telephone conversation, particularly about his third point, we seemed to agree on the following observations concerning the behavior of tensioned and twisted cables: When a weighted cable is twisted to a torque approaching the Greenhill value (on the $y = 0$ curve in Fig. 1 of the paper, or when $M = 2\nu TEI$ for long cables), then additional twisting deformation tends to occur at a nearly constant twisting torque while the cable center line develops a nonhocking small deflection helical pattern. The cable

2 Project Engineer, Charles Stark Draper Lab, Mail Station 38, Cambridge, Mass.
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will then hockle if the tension is reduced slightly, but not necessarily if the twisting deflection is further increased at constant tension and approximately constant torque. In either case, however, the Grenhill torque turns out to be the maximum sustainable torque at the particular tension. In the first case (additional twisting deformation), the load point in Fig. 1 travels along a horizontal line until it approaches the $\gamma = 0$ curve, and then comes to a stop even though the twisting deflection is further increased. In the second case (lowered tension), the load point is allowed to come down toward a lower value of tension, and this results in a violent instability and hockling when the load point reaches the $\gamma = 0$ curve.

These observations are consistent with the calculations performed to develop Fig. 1 of the paper. The only question which seems to remain is whether the small deflection helical pattern which develops as $\gamma = 0$ is approached in a horizontal direction is predicted by the theory as developed here. We have not performed sufficient computations near that curve to answer this question definitively.

Transverse Vibration of a Uniform Simply Supported Timoshenko Beam Without Transverse Deflection

X. Markenscoff. This paper contends that pure thickness-shear modes exist in pure form (i.e., not coupled with flexure) for bounded beams. This is wrong. Pure thickness-shear modes may only exist in beams or plates of infinite length [1-4]. The resonant frequencies of thickness-shear modes in an infinite plate were found by Koga in 1932 [2] and the coupling of thickness-shear and flexural vibration was later exhaustively investigated by Mindlin in 1950's and subsequently by several other authors (e.g., [5]). As for the author's contention that the frequency is independent of the length of the beam we may quote Mindlin [1]: "the resonances in a bounded plate commonly designated as thickness-shear are simply local regions in the spectrum of flexural resonances over which the frequency does not change, as rapidly as elsewhere, with change of plate dimensions."

References


Author's Closure

The author would like to thank X. Markenscoff for his interest and for providing comparisons with the findings of Mindlin plate theory.

Whether or not what is correct for an analysis based on finite plates can be directly applied to beams seems a matter for conjecture. Some work has already been done by the author to extend the application of the "dynamic discretization" technique to plate vibration. Although this is based as yet on simple plate theory it shows that there are marked differences in behavior as between beams and plates.

If the technique ultimately proves capable of working with Mindlin plate theory then perhaps the author may, at some future date, be able to confirm the Mindlin statement quoted by Mr. Markenscoff.

The quotation from Mindlin places no figure on the degree of variation of thickness-shear mode frequency with plate geometry, so that it is not possible to deduce whether the resolution of the experiments conducted by the author was sufficient to detect frequency variation with beam length. The inference from the experiments is that conventional Timoshenko beam theory describes the vibrational behavior of engineering beams quite well and this theory results in a thickness-shear mode frequency for a simply supported uniform beam which is independent of length.

Perhaps the most comprehensive paper dealing with Timoshenko beam theory is that of Traill-Nash and Collar* who discuss at length the simply supported case and say "Thus the motion corresponding to $r = 0$ is a pure shearing oscillation of the beam, with the rotational inertia forces at any section in equilibrium with the shearing motions and with zero lateral displacement. It may be noted that the results just established are independent of $L$, and accordingly may relate either to a simply supported beam or to an infinite beam."

Surface Integral Method Applied to the Problems of an Elastic Strip With Periodically Spaced Holes

A. J. Durelli and A. Assa. It is assumed that the author uses the standard convention that $r$ is the maximum of all shear stresses at a point, and $\sigma_1$, $\sigma_2$ are the maximum and minimum of all normal stresses at the point. It would seem that the author does not use the convention that stresses are graded by their algebraic values. The following questions are raised about Figs. 3(a-d).

1. It is understood that the boundary of the hole is free of tractions. How can it be explained that at several regions of this boundary neither of the principal stresses is zero? This is particularly true at the region surrounding the two bottom corners, and the region of the straight top boundary of the hole.

2. The situation is not as clear on the top boundary which is also understood free of tractions with the exception of the applied concentrated load. If values of $\sigma_{min}$ are interpolated, it also seems that both principal stresses are finite at several points on that boundary.

The following question is raised about Fig. 4. It seems that the problem is geometrically and mechanically symmetric with respect to the vertical axis of coordinates. How can it be explained that there are shears on the axis of symmetry?

The discussers conducted a photoelastic test for the case of the hole free of stresses. The load at the top boundary was applied through a disk reproducing approximately the author's loading condition. The

References

2 Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.
3 Numbers in brackets designate References at end of Discussion.

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