

DISCUSSION

$$(l - m) \cos \theta + i(l + m) \sin \theta = 2c(\cos \alpha - i \sin \alpha)$$

which equation has two different roots. Therefore I is a node and the same holds for J . It is possible that one of the roots is real; in that case we have

$$(l - m) \cos \theta = 2c \cos \alpha, \quad (l + m) \sin \theta = -2c \sin \alpha$$

which leads to the relation

$$(l^2 - m^2)^2 - 4c^2(l^2 + m^2) - 8c^2ml \cos 2\alpha = 0$$

between the parameters l , m , c , and α , which states that there is a position of the mechanism for which B and D coincide, so that the two circles $B(a)$ and $D(b)$ are concentric.

Authors' Closure

The authors appreciate the thoughtful discussions and generous comments made by K. Hain, and O. Bottema and G. R. Veldkamp.

As shown by K. Hain, there are a number of gear-link mechanisms based on the five-bar loop which are hardly known and which seem to possess promising design possibilities. In addition, the variety of motions obtainable from the geared five-bar linkage still remains to be fully exploited. The authors will look forward with interest to the further publications on this subject envisaged by the discussor.

Professors O. Bottema and G. R. Veldkamp have given a geometrically simple and useful interpretation of the curve $W = 0$ in the minus-one case, as the locus of the point of intersection of cranks AB and ED . The further discussion on the curve C_6 and the points I , J are well taken and sheds additional light on the nature of the minus-one curve.

Subsequent to the presentation of this paper, one of the authors (Ferdinand Freudenstein) received a communication from Academician I. I. Artobolevskii of the Institute of Machine Design, Moscow, USSR, calling attention to the additional references.^{5,6,7}

We have been unable to obtain copies of the first two of these references, but were able to secure a copy of the third, in which the results of the other two appear to be summarized. Included in these investigations is the equation and degree of the general five-bar curve and an analysis of the double points. The authors are pleased to have this opportunity to acknowledge the priority of the above investigations and to note at the same time that to the extent that the results overlap—and this extent is not believed to be major—the results of Professor Dobrovolskii have been confirmed.

A second communication, received from Mr. Frederick W. Seybold of Westfield, N. J., advised us of an ingenious application of a geared five-bar motion of gear ratio 3:1 in the drive of the platen of a printing press. This application is covered in U. S. Patent #2,403,760, assigned to American Type Founders of Elizabeth, N. J. In order to secure the appropriate drive characteristics (i.e., a nearly uniform translational velocity of the platen over a portion of its motion), a slider-crank drive was modified by essentially eliminating the third harmonic in the slider motion via additional mechanism. We were glad to learn of this interesting application.

In conclusion, the authors wish to express their appreciation for the constructive discussions and comments they have received.

⁵ V. V. Dobrovolskii, "Trajectories of five-link mechanisms" (in Russian), Proc. Moscow Machine Tool Construction Institute, 1937, vol. I.

⁶ V. V. Dobrovolskii, "Theory of mechanisms with one and more degrees of freedom" (in Russian), *ibid.*, vol. IV, 1939.

⁷ S. S. Biushgens, "Method of complex variables in the kinematics of plane mechanisms" (in Russian), Akad. Nauk USSR, Moscow-Leningrad, 1939, 72 pp., in particular, pp. 54-63.

Resonant Oscillations of a Beam-Pendulum System¹

T. J. HIGGINS.² The analysis given in this paper is clear and straightforward. The whole comprises a well-detailed illustration of the use of the small-parameter expansion procedure; and what can be gained therefrom by conjunction with some reasonable assumptions as to probable physical functioning of an elastic system.

Authors' Closure

The authors wish to thank Professor Higgins for his kind remarks. The equations discussed in this paper are fairly representative of a large class of pendulous type systems. For example, the referee pointed out that "they are the exact equations for small oscillations of a simple pendulum suspended from a linear spring, the spring being constrained to move vertically." They are closely related to the equations of a simple pendulum of variable length which the authors have called an *extensible pendulum*.³

¹ By R. A. Struble and J. H. Heinbockel, published in the June, 1963, issue of the JOURNAL OF APPLIED MECHANICS, vol. 30, TRANS. ASME, vol. 85, Series E., pp. 181-188.

² Professor, Department of Electrical Engineering, University of Wisconsin, Madison, Wis.

³ John H. Heinbockel and Raimond A. Struble, "Resonant Oscillations of an Extensible Pendulum," *Journal of Applied Mathematics and Physics (ZAMP)*, vol. 14, no. 3, 1963, pp. 262-269.

Extension of Iteration Method for Determining Strain Distributions to the Uniformly Stressed Plate With a Hole¹

A. MENDELSON² and S. S. MANSON.² The author presents an interesting extension of the general iteration method proposed by the writers several years ago for the practical solution of certain classes of plastic flow problems (footnote 1³ of the paper). There appears, however, to be some misunderstanding as to how this method was applied in that reference, which the writers would like to clarify.

With regard to the method of the previous paper³ the author states: "The method, however, had simplifications that made it inapplicable to situations where the plastic strains were small or of the same order of magnitude as the elastic strains. The limiting simplification was that the authors used the same coefficient in their expression for total effective strain as they used in their expression for the plastic part of the effective strain. This could be correct only when Poisson's ratio was 1/2."

The foregoing statement, however, is based on a misunderstanding. The authors of the previous paper³ did not use an *effective total strain* in the manner implied by the author. They used an *equivalent strain*, defined by (equation 3 of footnote 1)

$$\epsilon_{et} = \frac{\sqrt{2}}{3} [(\epsilon_r - \epsilon_\theta)^2 + (\epsilon_r - \epsilon_z)^2 + (\epsilon_\theta - \epsilon_z)^2]^{1/2} \quad (1)$$

as a purely convenient mathematically defined quantity without direct physical significance. It does not represent the abscissa of

¹ By E. A. Davis, published in the June, 1963, issue of the JOURNAL OF APPLIED MECHANICS, vol. 30, TRANS. ASME, vol. 85, Series E, pp. 210-214.

² Technical Consultant and Chief, Materials and Structures Division, respectively, National Aeronautics and Space Administration, Lewis Research Center, Cleveland, Ohio.

³ A. Mendelson and S. S. Manson, "Practical Solution of Plastic Deformation Problems in the Elastic-Plastic Range," NACA TN 4088, September, 1957.