The effect of non-isothermality on the gravitational collapse of spherical clouds and the evolution of protostellar accretion

E. I. Vorobyov\textsuperscript{1,2} and Shantanu Basu\textsuperscript{1}

\textsuperscript{1}Department of Physics and Astronomy, University of Western Ontario, London, Ontario, N6A 3K7, Canada
\textsuperscript{2}Institute of Physics, Stachki 194, Rostov-on-Don, Russia

Accepted 2005 August 15. Received 2005 August 5; in original form 2005 May 29

\section*{ABSTRACT}
We investigate the role of non-isothermality in gravitational collapse and protostellar accretion by explicitly including the effects of molecular radiative cooling, gas–dust energy transfer and cosmic ray heating in models of spherical hydrodynamic collapse. Isothermal models have previously shown an initial decline in the mass accretion rate $\dot{M}$ during the accretion phase of protostellar evolution, as a result of the gradient of the infall speed that develops in the prestellar phase. Our results show that: (1) in the idealized limit of optically thin cooling, a positive temperature gradient is present in the prestellar phase which effectively cancels out the effect of the velocity gradient, producing a near-constant (weakly increasing with time) $\dot{M}$ in the early accretion phase; and (2) in the more realistic case including cooling saturation at higher densities, $\dot{M}$ may initially be either weakly increasing or weakly decreasing with time, for the low dust temperature ($T_d \sim 6$ K) and high dust temperature ($T_d \sim 10$ K) cases, respectively. Hence, our results show that the initial decline in $\dot{M}$ seen in isothermal models is definitely not enhanced by non-isothermal effects, and is often suppressed by them. In all our models, $\dot{M}$ does eventually decline rapidly due to the finite mass condition on our cores and a resulting inward-propagating rarefaction wave. Thus, any explanation for a rapid decline of $\dot{M}$ in the accretion phase probably needs to appeal to the global molecular cloud structure and possible envelope support, which results in a finite mass reservoir for cores.

\textbf{Key words:} hydrodynamics -- stars: formation -- ISM: clouds -- dust, extinction -- ISM: molecules.

\section*{1 INTRODUCTION}
There is good reason to believe that the central regions of gravitationally bound prestellar cores are somewhat cooler than their outer parts. This is implied by the recent far-infrared observations of starless cores (Ward-Thompson, Andr\`{e} & Kirk 2002). Furthermore, dust radiative transfer calculations by Zucconi, Walmsley & Galli (2001) predict that there should be a positive radial dust temperature gradient, with values in the range $T_d \sim 5$–7 K at the core centre to $T_d \sim 15$ K at the core edge. Since the temperature of the gas $T_g$ is coupled to that of the dust for gas densities $n > 10^4$ cm$^{-3}$ (see e.g. Galli, Walmsley & Gon\c{c}alves 2002), one may expect that the radial distribution of the gas temperature in dense prestellar cores is also characterized by a positive temperature gradient. This implies that the effective polytropic index $\gamma$ of the gas may be less than unity, certainly during the prestellar phase.

A positive temperature gradient in the prestellar phase, and continued non-isothermality of a protostellar envelope during the accretion phase, may have important consequences for protostellar evolution. Consider the distinct differences in self-similar solutions of spherical protostellar accretion. Although the mass accretion rate $\dot{M}$ on to a protostar is time-independent in isothermal similarity solutions (Shu 1977; Hunter 1977), the similarity solutions for gravitational collapse of polytropic spheres (Yahil 1983; Suto & Silk 1988) show that $\dot{M}$ should increase with time if $\gamma < 1$ and decrease with time if $\gamma > 1$. The numerical modelling of the collapse of polytropic spheres by Ogino, Tomisaka & Nakamura (1999) also shows this tendency. If the cooling in an envelope due to direct emission of photons or frequent gas–dust collisions is efficient enough to reduce the effective value of $\gamma$ below unity, then the mass accretion rate $\dot{M}$ on to the protostar is expected to increase with time. However, in this case, it is difficult to explain the observations of Bontemps et al. (1996), who have suggested that the mass accretion rate of Class I objects (i.e. protostars in the late accretion stage) falls off by an order of magnitude compared to that of Class 0 objects (i.e. protostars in the early accretion phase). One may then need to appeal to other effects that limit the mass infall, such as a finite mass reservoir (see Vorobyov & Basu 2005, hereafter Paper I). Conversely, the thermal balance of a protostellar envelope may be dominated by the

\section*{Acknowledgements}
E.I.V. is supported by a Postdoctoral Fellowship from NSERC. We are grateful to an anonymous referee for useful comments.

\begin{thebibliography}{9}
\end{thebibliography}
compressional heating of matter rather than by radiative cooling of
gas or dust–gas energy transfer. This could raise the effective value of
γ above unity and perhaps lead to a declining $M$. In this case, at
least some of the inferred decline in $M$ during the accretion phase
may be attributed to non-isothermal effects.

Our motivation for this paper is to model the details of radiative
cooling, cosmic ray heating and gas–dust energy transfer in a model
of hydrodynamic collapse, in order to settle the above issues. In
Paper I, we studied spherical isothermal collapse and found that,
although an initially declining $M$ is present, as a result of the gradient
of the infall speed in the prestellar phase, the observational tracks of
envelope mass $M_{\text{envelope}}$ versus bolometric luminosity $L_{\text{bol}}$ could only be
explained by the effect of an inwardly propagating rarefaction wave
due to a finite mass reservoir. In fact, the Class I phase of protostellar
evolution was identified with the period of rapidly declining $\dot{M}$
due to a finite mass reservoir. In fact, the Class I phase of protostellar
evolution was identified with the period of rapidly declining $\dot{M}$
occuring after the rarefaction wave reaches the protostar. Here, we
investigate whether non-isothermality can change this conclusion
in any way. Specifically, can non-isothermality explain any part
(or even all) of the inferred drop in $M$ during the accretion phase
(Bontemps et al. 1996)?

The paper is organized as follows. The numerical code used to
model the gravitational collapse, as well as the initial and boundary
conditions, are briefly discussed in Section 2. The temporal evolu-
tion of the mass accretion rate for an assumption of optically thin
gas is studied in Section 3.1. The influence of the radiative cooling
saturation and the gas–dust energy transfer (Goldsmith 2001) on the
mass accretion rate are investigated in Section 3.2. Our main results
are summarized in Section 4.

2 Model Assumptions

We consider the gravitational collapse of spherical non-isothermal
clouds composed of molecular hydrogen with a 10 per cent admix-
ture of atomic helium. The evolution is calculated by solving the
hydrodynamic equations in spherical coordinates:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \rho v_r \right) = 0,$$

$$\frac{\partial (\rho v_r)}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \rho v_r v_r \right) = -\frac{\partial p}{\partial r} - \frac{GM}{r^2},$$

$$\frac{\partial e}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 e v_r \right) = -\frac{\rho}{r^2} \frac{\partial}{\partial r} \left( r^2 v_r \right) + \Gamma_{\text{cr}} - \Lambda_{\text{rc}} - \Lambda_{\text{gd}},$$

where $\rho$ is the gas density, $v_r$ is the radial velocity, $M$ is the enclosed
mass, $e$ is the internal energy density and $p = e(\gamma_1 - 1)$ is the
gas pressure. We use the ratio of specific heats $\gamma_1 = 5/3$ to link pressure
and internal energy density and modify the energy equation for the
effects of cooling and heating. The details of the radiative cooling
rate $\Lambda_{\text{rc}}$, gas–dust energy transfer rate $\Lambda_{\text{gd}}$ and cosmic ray heating
$\Gamma_{\text{cr}}$ are given in Section 3. The gas dynamics of a collapsing cloud
is followed by solving the usual set of hydrodynamic equations in
spherical coordinates using the method of finite differences with a
time-explicit, operator split solution procedure similar to that of
the ZEUS-1D numerical hydrodynamics code described in detail in
Stone & Norman (1992). Because the time-scales of cooling and
heating are usually much shorter than the dynamical time, the en-
ergy equation update due to cooling and heating requires an implicit
scheme. Explicit schemes usually fail due to a strict limitation on the
numerical time-step set by the Courant–Friedrich–Levy condition.
Therefore, cooling and heating of gas are treated numerically using
Newton–Raphson iterations, supplemented by a bisection algorithm

for occasional zones where the Newton–Raphson method does not
converge. In order to monitor accuracy, the total change in the
internal energy density in one time-step is kept below 15 per cent.
If this condition is not met, the time-step is reduced and a solution
is again sought. When the gas number density in the collapsing core
exceeds $10^{11} \text{ cm}^{-3}$, we gradually reduce the cooling and heating so as to establish
adiabatic evolution with $\gamma_1 = 5/3$ for $n > 10^{12} \text{ cm}^{-3}$.
This simplified treatment of the transition to an opaque protostar
misses the details of the physics on small scales. Specifically, a
proper treatment of the accretion shock and radiative transfer ef-
fects is required to predict accurately the properties of the stellar
core [see Winkler & Newman (1980) for a detailed treatment and
review of work in this area]. However, our method should be ade-
quate to study the protostellar accretion rate, and has been used
successfully by, for example, Foster & Chevalier (1993) and Ogino
et al. (1999) for this purpose. The numerical grid has 600 points,
which are initially uniformly spaced, but then move with the gas un-
til the central core is formed. This provides an adequate resolution
throughout the simulations.

We impose boundary conditions such that the gravitationally
bound cloud core has a constant mass and constant volume. The
assumption of a constant mass appears to be observationally justi-
fied. For instance, the observations by Bacmann et al. (2000) have
shown that in at least some cases, such as L1544 in Taurus, individ-
ual prestellar clouds are characterized by sharp edges defining typ-
ical outer radii $\approx 0.05$–0.5 pc; implying that these prestellar clouds
represent finite reservoirs of mass for subsequent star formation.
Physically, this assumption may be justified if the core decouples
from the rest of a comparatively static, diffuse cloud due to a shorter
dynamical time-scale in the gravitationally contracting central con-
densation than in the external region. A specific example of this,
due to enhanced magnetic support in the outer envelope, is found in
the models of ambipolar diffusion induced core formation (see e.g.
Basu & Mouschovias 1995). The constant-volume condition of a
collapsing cloud core is mainly an assumption of a constant radius
of gravitational influence of a subcloud within a larger parent diffuse
cloud.

The radial gas density distribution of a self-gravitating isother-
mal cloud that is in hydrostatic equilibrium can be conveniently
approximated by a modified isothermal sphere, with gas density

$$\rho = \frac{\rho_c}{1 + (r/r_c)^2},$$

(Binney & Tremaine 1987), where $\rho_c$ is the central density and
$r_c$ is the radial scalelength. We choose a value $r_c = c_s/\sqrt{\pi G \rho_c}$,
so that the inner profile is close to that of a Bonnor–Ebert sphere
(Bonnor 1956; Ebert 1955), $r_c$ is comparable to the Jeans length,
and the asymptotic density profile is twice the equilibrium singular
isothermal sphere value $\rho_{\text{SIS}} = c_s^2/(2\pi G r_c^2)$. The latter is justified on the
grounds that core formation should occur in a somewhat non-
equilibrium manner [an extreme case is the Larson–Penston flow
(Larson 1969; Penston 1969), in which case the asymptotic density
profile is as high as 4.4 $\rho_{\text{SIS}}$, and also by observations of protostellar
envelope density profiles that are often overdense compared to $\rho_{\text{SIS}}$
(Andr{é}, Motte & Belloche 2001). We also add a (moderate) positive
density perturbation of a factor $\alpha = 1.7$ (i.e. the initial gas density
distribution is increased by a factor of 1.7) to drive the cloud (es-
specially the inner region, which is otherwise near-equilibrium) into
gravitational collapse. For the purpose of comparison with isother-
mal simulations, we determine the size of the central flat region
assuming a constant temperature $T = 10$ K, which yields $r_c = 0.03$ pc.
3 Results of cloud collapse

In this section we study how the non-isothermality of gas can affect the temporal evolution of the mass accretion rate. Our results should be interpreted in the context of models of one-dimensional radial infall. Hence, our calculated mass accretion rates really represent the infall on to the inner protostellar disc that would be formed due to rotation. Since disc masses are not observed to be greater than protostellar masses, it is likely that the protostellar accretion is at least proportional to the mass infalling on to the disc. The mass accretion rate is computed at a radial distance 600 au = 0.003 pc.1 We neglect heating due to the central source at this distance. First, we consider a simplified assumption of optically thin gas heated only by the cosmic rays. We further take into account the saturation of the radiative cooling of gas at \( n > 10^7 \text{ cm}^{-3} \) and the gas–dust energy transfer (Goldsmith 2001).

3.1 Optically thin limit

Our optically thin model serves as a example limiting case that builds our intuition about the effect of temperature gradients in a cloud core and serves as a comparison to the more realistic cooling models presented in Section 3.2. In this idealized limit, the radiative cooling of gas can be expressed as

\[
\Lambda_{\text{rad}} = L(T) n^2 \text{ erg cm}^{-3} \text{ s}^{-1},
\]

where \( L(T) \) is the radiative cooling rate in \( \text{ erg cm}^{-3} \text{ s}^{-1} \) and \( n \) is the number density. For the temperature dependence of the radiative cooling rate \( L(T) \) we have implemented the cooling function of Wada & Norman (2001, their fig. 1) for solar metallicity and gas temperature \( T_g < 10^4 \text{ K} \). The cooling function simplifies the implementation of cooling by collecting the effects of various coolants.

The cooling processes taken into account are: (1) vibrational and rotational excitation of H\(_2\); (2) atomic and molecular cooling due to fine-structure emission of C and O; and (3) rotational line emission of CO. We do not consider cooling due to C\(_3^+\), since the complete conversion of C\(_3^+\) to CO in cloud cores occurs when the number density rises beyond \( 10^7 \text{ cm}^{-3} \) (Nelson & Langer 1997). Moreover, this chemical change appears to have little effect on the dynamics of self-gravitating cloud cores. This is because the cooling rate of C\(_3^+\) is similar to that of CO, particularly in the density–temperature region typical for the cloud cores (Nelson & Langer 1997).

Since we are interested in well-shielded regions, we can ignore the direct photoelectric heating of gas by the incident ultraviolet radiation. The cosmic ray heating is thus the only process that heats the gas directly. We adopt the cosmic ray heating per unit volume (Goldsmith 2001)

\[
\Gamma_{\text{cr}} = 10^{-27} \left( \frac{n}{\text{cm}^{-3}} \right) \text{ erg cm}^{-3} \text{ s}^{-1}.
\]

The initial gas temperature distribution is obtained by solving the equation of thermal balance \( \Lambda_{\text{rad}} = \Gamma_{\text{cr}} \) for the adopted radial gas density profile of model N11. The thin solid line in Fig. 1 shows the resulting gas temperature \( T_g \). It varies from 5.6 K in the cloud’s centre to 7 K at the outer edge for the cloud with \( r_{\text{out}} = 0.13 \text{ pc} \). For a larger cloud of radius \( r_{\text{out}} = 0.4 \text{ pc} \) (model N12), the temperature grows from 5.6 K in the centre to 8.5 K at the outer edge, as shown by the dashed line in Fig. 1.

After the collapse is initiated, some aspects of the temporal evolution are shown in Fig. 2. The solid lines in Figs 2(a) and (b) show the evolution of the mass accretion rate \( \dot{M} \) at a radial distance of 600 au from the centre for models N11 and N12, respectively. The accretion rates of the corresponding isothermal clouds \( (T_g = 10 \text{ K}) \) are plotted by the dashed lines for comparison. An obvious difference is seen in the behaviour of \( \dot{M} \) of the non-isothermal clouds as compared to that

\[1\text{ We note that the accretion rate is not expected to vary significantly in the range 0.1–1000 au, according to radiation hydrodynamic simulations of spherical collapse by Masunaga & Inutsuka (2000).} \]
of the isothermal ones: no peak in the mass accretion rate is seen at the time of the hydrostatic core formation ($t \approx 0.18$ Myr). This is because the effect of progressively warmer layers falling on to the protostar cancels the effect of inner mass shells having greater initial infall speeds; we discuss this effect again in Section 3.2.2. Instead, $\dot{M}$ of the non-isothermal clouds grows monotonically and appears to stabilize at $\sim 2.1 \times 10^{-3} M_\odot$ yr$^{-1}$ if the size of the cloud is sufficiently large (model NI2). Note that the positive temperature gradient developing in the non-isothermal pre-collapse clouds shortens the duration of the pre-core-formation phase by $\sim 0.07$ Myr as compared to that of the isothermal clouds. A sharp drop of the mass accretion rate at $\approx 0.31$ Myr in model NI1 (at $\approx 0.83$ Myr in model NI2) is due to a rarefaction wave propagating inward from the cloud’s outer edge (see Paper I for details). In the case of a larger cloud of $r_{\text{out}} = 0.4$ pc (model NI2), it takes a longer time of $\sim 0.8$ Myr for the rarefaction wave to reach the innermost regions. As a consequence, model NI2 exhibits a longer period of nearly time-independent mass accretion rate than model NI1. At the time when the rarefaction wave reaches the radius of $r \sim 600$ au, roughly half of the envelope mass has accreted on to the central hydrostatic core.

### 3.2 Cooling saturation at higher densities

Calculations of thermal balance in dark, well-shielded molecular cores by Goldsmith & Langer (1978) and Goldsmith (2001) have shown that the cooling from main coolants such as CO, C$^{13}$O, H$_2$O, C and others is proportional to $n^2$ only at low densities of molecular hydrogen, i.e. $n \leq 10^2$–$10^4$ cm$^{-3}$. At $n \geq 10^5$ cm$^{-3}$, the cooling from these species saturates (see e.g. Goldsmith 2001). This is because the transitions that can contribute to the cooling are thermalized due to the combination of a higher collision rate and radiative trapping. Noticeable exceptions are the cooling from C$^{18}$O and CS, which do not saturate even at densities $n \sim 10^6$ cm$^{-3}$ due to a sufficiently large spontaneous decay rate. As a result, the total cooling has a complicated dependence on the temperature and density of molecular hydrogen, expressed by Goldsmith (2001) as

$$\Lambda_{\text{cc}} = \alpha \left( \frac{T_g}{10 \text{K}} \right)^\beta \text{erg cm}^{-3} \text{s}^{-1},$$

where $\alpha$ and $\beta$ are given in Table 2 of Goldsmith (2001) for the undepleted (standard) molecular abundances.

As in Section 3.1, we assume that the cosmic ray heating is the only mechanism that heats the gas directly. The dust grains heated by the diffuse visible–infrared radiation field may be an additional source of indirect gas heating/cooling depending on the difference between the gas and dust temperatures. For the gas–dust energy transfer, we adopt the expression given in Goldsmith (2001),

$$\Lambda_{\text{gd}} = 2 \times 10^{-33} \left( \frac{n}{\text{cm}^{-3}} \right)^2 \left( \frac{T_g - T_d}{\text{K}} \right) \left( \frac{T_g}{10 \text{K}} \right) \text{erg cm}^{-3} \text{s}^{-1},$$

where $T_g$ and $T_d$ are the temperatures of gas and dust, respectively. When the dust temperature is greater than that of the gas, the dust heats the gas, and vice versa.

#### 3.2.1 Initial gas temperature profile

The radial distribution of the gas temperature depends on the adopted temperature of the dust. As demonstrated by Zucconi et al. (2001), the dust temperature in the prestellar cores is sensitive to the centre-to-edge visual extinction $A_v$ of a parent cloud. We consider a simple model in which the dust temperature is constant throughout the prestellar core and equal to 6 K or 10 K. These two limits correspond to cores that are heavily shielded ($A_v \sim 100$.0 mag) or only moderately shielded ($A_v \sim 5.0$ mag) from the external radiation.

The initial temperature distribution of gas is obtained by solving the equation of thermal balance $\Lambda_{\text{cc}} + \Lambda_{\text{gd}} = \Gamma_{\text{cr}}$. In model NI3, the gas temperature $T_g$ decreases from 12.6 K near the cloud’s centre to 10.3 K at the outer edge ($r_{\text{out}} = 0.13$ pc) as shown in Fig. 3 by the thin solid line. The development of the negative temperature gradient in the 0.03–0.13 pc range is due to radiative cooling saturation at higher densities $n > 10^4$ cm$^{-3}$, where the cosmic ray heating dominates the radiative cooling. At the same time, the gas–dust energy

![Figure 2](https://example.com/figure2.png)

**Figure 2.** The temporal evolution of the mass accretion rates (solid lines) obtained in (a) model NI1 and (b) model NI2. The dashed lines give the mass accretion rates for the corresponding isothermal ($T_g = 10$ K) clouds.

![Figure 3](https://example.com/figure3.png)

**Figure 3.** The initial radial gas temperature distributions in four models listed in Table 1: model NI3 (thin solid line), model NI4 (thin dashed line), NI5 (thin dot-dashed line) and model NI6 (thin dotted line). Note that the radial distributions of the gas temperature for models NI3 and NI4 and models NI5 and NI6 merge at $r < 0.13$ pc. The thick solid line plots the initial radial distribution of the gas density.

transfer becomes an efficient coolant at \( n > 10^4 \) cm\(^{-3} \) and the gas temperature stabilizes at \( T_g \approx 12.6 \) K in the innermost region \( r < 0.02 \) pc. In the case of a larger cloud with \( r_{\text{out}} = 0.4 \) pc (model NI4), the gas temperature exhibits a more complicated behaviour as shown in Fig. 3 by the dashed line (note that it merges with the thin solid line at \( r < 0.13 \) pc). It decreases from 12.6 K near the cloud’s centre and reaches a minimum value of 9.6 K at \( r \approx 0.25 \) pc. Further out, the gas temperature grows again to 10.0 K at \( r_{\text{out}} = 0.4 \) pc. The latter indicates that the gas becomes effectively optically thin at \( n \leq 10^3 \) cm\(^{-3} \) and the radial temperature distribution of gas develops a positive temperature gradient, as was indeed found in Section 3.1 for the optically thin radiative cooling.

For a lower value of the dust temperature \( T_d = 6 \) K, the initial radial distribution of gas temperature is shown in Fig. 3 by the dot-dashed and dotted lines for the clouds of \( r_{\text{out}} = 0.13 \) pc (model NI5) and \( r_{\text{out}} = 0.4 \) pc (model NI6), respectively. The dust–gas energy transfer reduces the gas temperature in the cloud’s innermost regions by \( \approx 2 \) K as compared to the previously considered case of \( T_d = 10 \) K. At the lower densities, the gas temperature is mainly determined by the balance of radiative cooling of gas and cosmic ray heating. As a consequence, the gas temperature profiles for both adopted values of \( T_d \) become very similar for \( n < 10^3 \) cm\(^{-3} \).

3.2.2 Mass accretion rate

The solid lines in Figs 4(a) and (b) show the temporal evolution of the mass accretion rate \( M \) for models NI5 and NI6, respectively. The corresponding accretion rates of isothermal \(( T_g = 10 \) K) clouds are plotted by the dashed lines for comparison. At the lower dust temperature \( T_d = 6 \) K, the accretion rate resembles that of the optically thin cloud: \( M \) has no well-developed peak at the time of the hydrostatic core formation at \( t = 0.26 \) Myr. Thus, the gas–dust energy exchange acts as an effective thermostat; the gas collisionally transfers its energy to the dust instead of directly radiating it by photon emission. We note here that such low dust temperatures of \( T_d \approx 6 \) K may indeed be present in dense prestellar cores as suggested by the self-consistent modelling of Zucconi et al. (2001) and Galli et al. (2002).

The self-similar solutions for the accretion phase of isothermal cloud collapse (Shu 1977; Hunter 1977) predict that the mass accretion rate is time-independent and that

\[
M = \frac{k c^3}{G},
\]

where \( c_s \) is the isothermal sound speed. The coefficient \( k \) is equal to 0.975 in the Shu solution, whereas in the Hunter (1977) extension (to the accretion phase) of the Larson–Penston solution, \( k = 46.9 \). This difference is due to the different velocity \( v(r) \) and density \( \rho(r) \) radial profiles of gas at the time when the central hydrostatic core forms: they are \( v(r) = 0 \) and \( \rho(r) = c_s^2/2\pi Gr^2 \) in the Shu solution and \( v(r) = -3.3 c_s \) and \( \rho(r) = 4.4 c_s^2/2\pi Gr^2 \) in the Larson–Penston–Hunter solution. However, numerical simulations of both isothermal and non-isothermal collapse show that the gas velocity at the time of stellar core formation is not constant with radius; the absolute value of gas velocity only approaches \( 3.3 c_s \) near the hydrostatic core, while decreasing at larger radii and converging to zero at the outer boundary. As a result, the peak in \( M \) appears right after the formation of a central hydrostatic stellar core.

The absence of the peak in models NI5 and NI6 can be understood if one considers the local polytropic index \( \gamma = d \ln P/d \ln \rho \) obtained from the model’s known radial distributions of pressure and density. Fig. 5(a) plots the radial distribution of \( \gamma \) obtained for model NI6 at four different evolutionary times. Shortly after the formation of the hydrostatic core at \( t \approx 0.26 \) Myr, the gas flow in the inner 0.04 pc is characterized by \( 0.8 < \gamma < 1.0 \). Since the polytropic law implies that \( T_g c_s^{\gamma-1} \rho^{\gamma} = \) constant during the collapse, and the gas density in the infalling envelope decreases with radius as \( r^{-1.5} \), the gas temperature \( T_g \) and the associated sound speed \( c_s \) will increase with radius as long as \( \gamma < 1.0 \). Because the similarity solutions (see equation 9) predict that \( M \) is directly proportional to the sound speed, the mass accretion rate will increase with time, as progressively warmer layers of gas (characterized by increasing \( c_s \)) fall on to the central hydrostatic core. As a result, the increase of \( M \) due to the strong positive temperature gradient appears to compensate the decrease in \( M \) due to the non-constant radial velocity profile and the mass accretion rate in this early phase becomes a monotonically growing function of time as shown by the solid line in Fig. 4(b). At \( t \approx 0.32 \) Myr the polytropic index in the infalling envelope becomes greater than unity throughout the infalling envelope. At roughly the same time, the accretion rate attains the maximum value of \( 2.35 \times 10^{-3} M_\odot \) yr\(^{-1} \). Further evolution is characterized by a slowly declining \( M \). In this intermediate phase, the mass accretion rate appears to converge to that of the isothermal cloud. The late phase of accretion \(( t > 0.8 \) Myr \)) is characterized by a rapid and terminal decline when the rarefaction wave caused by a finite mass reservoir arrives at the centre. This effect was discussed in detail in Paper I. The transition from early-phase to late-phase accretion occurs smoothly for the smaller clouds (i.e. model NI5), but the intermediate phase of slowly declining mass accretion is still clearly visible for larger clouds in which the effect of the rarefaction wave is delayed due to a larger radius.

![Figure 4.](https://academic.oup.com/mnras/article-abstract/363/4/1361/1048028)
The solid lines in Figs 6(a) and (b) show the temporal evolution of the mass accretion rate $\dot{M}$ for models NI3 and NI4, respectively. The corresponding accretion rates for isothermal ($T_d = 10$ K) clouds are plotted by the dashed lines. At a higher dust temperature $T_d = 10$ K, the accretion rate resembles that of the isothermal cloud: a well-developed peak is visible at the time of hydrostatic core formation $t \approx 0.33$ Myr. Again, as in the case of lower dust temperature, we consider the radial profiles of local polytropic index obtained from the model’s known pressure and density profiles. Fig. 5(b) shows the radial distribution of $\gamma$ in model NI4 at three times. Shortly after the formation of hydrostatic core at $t \approx 0.33$ Myr, the gas flow in the inner envelope $r < 0.03$ pc has local polytropic index that is only slightly below unity ($0.9 \lesssim \gamma \lesssim 1.0$). As a consequence, the very weak positive temperature gradient is not capable of compensating the decrease in the mass accretion rate due to the non-constant radial velocity profile and the behaviour of the accretion rate is qualitatively similar to that of the isothermal cloud. At $t \approx 0.36$ Myr the polytropic index grows above unity in the infalling envelope, as is seen from the dashed line in Fig. 5(b). In this phase, the accretion rate declines due to both the combined action of cooling and heating and the non-constant radial velocity profile developed in the prestellar phase. However, as in the case of lower dust temperature, a sharp and terminal decline of accretion at the late protostellar stages $t > 0.8$ Myr is due to the rarefaction wave rather than to any other effect.

### 3.2.3 Evolution of gas temperature

In the post-core-formation epoch, the gas is virtually in a free-fall motion near the central hydrostatic core and the characteristic dynamical time becomes comparable to or even shorter than that of the cooling. As a result, the flow becomes nearly adiabatic with $\gamma \approx 1.5$ and the gas temperature in the central region grows with time. The latter tendency is clearly seen in Fig. 7, where we plot the radial gas temperature distribution in model NI5 obtained at four different evolutionary times. The initial radial gas temperature
distribution shown by the dot-dashed line in Fig. 3 is characterized by a mild increase towards the outer edge of a cloud, so that it reaches a maximum of $T_g \approx 11.3$ K at $r \approx 0.05$ pc and decreases to roughly the central value of $T_g = 10.2$ K at the outer edge $r = 0.13$ pc. This behaviour of the gas temperature remains qualitatively similar until and shortly after the formation of the central hydrostatic core at $t \approx 0.26$ Myr. However, in the post-core-formation stage, the radial distribution of the gas temperature develops a negative gradient, the slope of which gradually grows with time as shown in Fig. 7 by the thin dashed, dot-dashed and dotted lines for $t = 0.32$, 0.4 and 0.5 Myr, respectively. The development of the negative temperature gradient is due to compressional heating $P(\nabla \cdot \mathbf{v})$ that overtakes other heating/cooling processes at $r < 0.04$ pc. We note that, as a result of some residual cooling in the interior of the infalling envelope, the polytropic index $\gamma$ tends to a value of 3/2 instead of 5/3. The thick dashed line in Fig. 7 shows the temporal evolution of the gas temperature at the radial distance of 600 au from the cloud’s centre. The initial decrease of the gas temperature prior to the formation of the central hydrostatic core is followed by an increase and then a saturation at $T_g \approx 60$ K in the post-core-formation stage.

4 CONCLUSIONS
We have numerically followed the gravitational collapse of spherical prestellar cores of finite mass and volume down to the protostellar stage. The influence of the cooling and heating on the temporal evolution of the mass accretion rate $\dot{M}$ has been investigated. We summarize our results as follows.

(i) Optically thin gas heated only by cosmic rays. In a simplified approach where the radiative cooling of gas is proportional to the square of the gas density, the mass accretion rate monotonically increases with time and appears to attain a constant value after the formation of the hydrostatic core, if the size of a cloud is sufficiently large. This temporal evolution of $\dot{M}$ is qualitatively different from that found in isothermal simulations, which show the development of a peak and subsequent decline in $\dot{M}$ shortly after the formation of the hydrostatic core (see e.g. Hunter 1977; Foster & Chevalier 1993; Ogino et al. 1999; Paper I). The later evolution of the mass accretion rate, after roughly half of the envelope has accreted on to the central hydrostatic core, shows a sharp drop due to the gas rarefaction wave propagating inward from the cloud’s outer edge. This drop in $\dot{M}$ is a direct consequence of the assumed finite mass and volume of the gravitationally bounded non-isothermal cloud.

(ii) Effects of radiative cooling saturation and gas–dust energy transfer. In a more realistic approach where the radiative cooling of the gas saturates at $n \gtrsim 10^4$ cm$^{-3}$ (Goldsmith 2001) and the gas–dust energy transfer is taken into account, the temporal evolution of the mass accretion rate depends on the dust temperature. If the dust temperature is sufficiently low ($T_d \approx 6$ K), which roughly corresponds to the gravitationally bounded subcloud being deeply embedded within a parent diffuse (i.e. non-gravitating) molecular cloud and heavily shielded from the interstellar radiation field ($A_v \approx 100.0$ mag), the gas behaves as being effectively optically thin due to efficient cooling by dust. Specifically, no well-developed peak in $\dot{M}$ is observed shortly after the formation of the hydrostatic core, and the temporal evolution of the mass accretion rate is qualitatively similar to that considered in the optically thin case. The absence of the peak is due to the positive temperature gradient developing in the cloud’s innermost regions before and shortly after the formation of the hydrostatic core. This positive temperature gradient effectively increases the mass accretion rate with time and appears to compensate its decrease due to the non-uniform radial gas velocity profile. In the opposite case of a subcloud being only moderately shielded from the interstellar radiation field with $A_v \approx 5.0$ mag and $T_d \approx 10$ K, the temporal evolution of the mass accretion rate is qualitatively similar to that of the isothermal $(T_d = 10$ K) cloud.

Irrespective of the effects of radiative cooling and dust temperature, the later evolution of the mass accretion rate, after roughly half of the envelope has accreted on to the hydrostatic core, is characterized by a fast decline due to a rarefaction wave associated with the finite mass reservoir. In Paper I, we associated this phase (which has a temporally declining bolometric luminosity $L_{bol}$) with the empirically defined Class I phase of protostellar evolution. The inclusion of non-isothermal effects in this paper does not change this conclusion. We have shown that the initial decline in $\dot{M}$ is weakened or may even be absent in a model with a realistic treatment of cooling. Therefore, the ultimate decline in $\dot{M}$ due to the finite mass reservoir is even more necessary to explain the Class I phase.

Our simulations also demonstrate that the evolution of prestellar cores down to the late protostellar stage cannot be described by a polytropic law with a fixed index $\gamma = d \ln P/d \ln \rho$. Instead, before and shortly after the formation of the hydrostatic core, the local polytropic index $\gamma$ in the inner envelope $r < 0.03$ pc is below unity and $\sim 0.8-1.0$, mainly due to effective cooling by the dust. However, in the post-core-formation epoch, the gas is virtually in a free-fall near the central hydrostatic core and the compressional heating overtakes other energetic processes. As a consequence, the flow becomes nearly adiabatic with $\gamma \sim 1.5$ and the gas in the accreting envelope attains a negative temperature gradient.

ACKNOWLEDGMENTS
EIV gratefully acknowledges present support from a CITA National Fellowship and past support by the NATO Science Fellowship Program administered by the Natural Sciences and Engineering Research Council (NSERC) of Canada. SB was supported by a research grant from NSERC.

REFERENCES
Ebert R., 1955, Z. Astrophys., 37, 217
Irrespective of the effects of radiative cooling and dust temperature, the later evolution of the mass accretion rate, after roughly half of the envelope has accreted on to the hydrostatic core, is characterized by a fast decline due to a rarefaction wave associated with the finite mass reservoir. In Paper I, we associated this phase (which has a temporally declining bolometric luminosity $L_{bol}$) with the empirically defined Class I phase of protostellar evolution. The inclusion of non-isothermal effects in this paper does not change this conclusion. We have shown that the initial decline in $\dot{M}$ is weakened or may even be absent in a model with a realistic treatment of cooling. Therefore, the ultimate decline in $\dot{M}$ due to the finite mass reservoir is even more necessary to explain the Class I phase.

Our simulations also demonstrate that the evolution of prestellar cores down to the late protostellar stage cannot be described by a polytropic law with a fixed index $\gamma = d \ln P/d \ln \rho$. Instead, before and shortly after the formation of the hydrostatic core, the local polytropic index $\gamma$ in the inner envelope $r < 0.03$ pc is below unity and $\sim 0.8-1.0$, mainly due to effective cooling by the dust. However, in the post-core-formation epoch, the gas is virtually in a free-fall near the central hydrostatic core and the compressional heating overtakes other energetic processes. As a consequence, the flow becomes nearly adiabatic with $\gamma \sim 1.5$ and the gas in the accreting envelope attains a negative temperature gradient.

ACKNOWLEDGMENTS
EIV gratefully acknowledges present support from a CITA National Fellowship and past support by the NATO Science Fellowship Program administered by the Natural Sciences and Engineering Research Council (NSERC) of Canada. SB was supported by a research grant from NSERC.

REFERENCES
Ebert R., 1955, Z. Astrophys., 37, 217

Gravitational collapse of spherical clouds 1367


This paper has been typeset from a TeX/LaTeX file prepared by the author.