Jets, structured outflows and energy injection in gamma-ray burst afterglows: numerical modelling

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ABSTRACT

We investigate numerically the ability of three models (jet, structured outflow and energy injection) to accommodate the optical light-curve breaks observed in 10 gamma-ray burst (GRB) afterglows (980519, 990123, 990510, 991216, 000301c, 000926, 010222, 011211, 020813 and 030226), as well as the relative intensities of the radio, optical and X-ray emissions of these afterglows. We find that the jet and structured outflow models fare much better than energy injection model in accommodating the multiwavelength data of the above 10 afterglows. For the first two models, a uniform circumburst medium provides a better fit to the optical light-curve break than a wind-like medium with a \( r^{-2} \) stratification. However, in the only two cases where the energy injection model may be at work, a wind medium is favoured (an energy injection is also possible in a third case, the afterglow 970508, whose optical emission exhibited a sharp rise, but not a steepening decay). The best-fitting parameters obtained with the jet model indicate an outflow energy of \( 2 \times 10^{50} \) to \( 6 \times 10^{50} \) ergs and a jet opening of \( 2^\circ - 3^\circ \). Structured outflows with a quasi-uniform core have a core angular size of \( 0^\circ.7 \)–\( 1^\circ \) and an energy per solid angle of \( 0.5 \times 10^{53} \) to \( 3 \times 10^{53} \) erg sr\(^{-1} \), surrounded by an envelope where this energy falls off roughly as \( \theta^{-2} \) with angle from the outflow axis, requiring thus the same energy budget as jets. Circumburst densities are found to be typically in the range \( 0.1 \)–\( 1 \) cm\(^{-3} \), for either model. We also find that the reverse shock emission resulting from the injection of ejecta into the decelerating blast wave at about 1 d after the burst can explain the slowly decaying radio light curves observed for the afterglows 990123, 991216 and 010222.

Key words: radiation mechanisms: non-thermal – shock waves – ISM: jets and outflows – gamma-rays: bursts.

1 INTRODUCTION

Since the prediction of radio (Paczynski & Rhoads 1993) and optical transients (Mészáros & Rees 1997) associated with gamma-ray bursts (GRBs), more than 100 afterglows have been observed (including X-ray transients). Good monitoring in all three frequency domains has been achieved for about 20 afterglows; the radio, optical and X-ray flux were observed to decay as a power law, \( F_\nu \propto t^{-\alpha} \) (\( \alpha > 0 \)), confirming the expectations. For 10 of these well-observed afterglows – 980519, 990123, 990510, 991216, 000301c, 000926, 010222, 012111, 020813 and 030226 – the optical light-curve decay exhibits a steepening at about 1 d after the burst, with an increase \( \Delta \alpha \) in the temporal index as low as 0.4 (afterglow 991216) and as high as 1.8 (afterglow 000301c). Optical light-curve breaks have or may have been observed in other afterglows; they are not included in the sample of afterglows modelled in this work because those afterglows have been adequately monitored at only one optical frequency.

With the exception of the afterglow 010222, for which a late (10 d) measurement suggests that a break occurred also in the X-ray light curve, currently available X-ray observations do not extend sufficiently before and after the optical break time to prove that the light-curve break is achromatic. Within the measurement uncertainties, a single power-law fits well the decay of all adequately monitored X-ray afterglows: 990123, 990510, 991216, 000926 (and 010222 until the last measurement).

Breaks, in the form of peaks, have been observed in the radio emission of all 10 afterglows above, usually at \( \sim 10 \) d after the burst, i.e. often occurring after the optical light-curve break. Given that the radio and optical breaks are not simultaneous, they cannot both arise from the dynamics of the afterglow, the structure of the GRB ejecta or some property of the circumburst medium (CBM), as these mechanisms should yield achromatic breaks. Furthermore, given that, before their respective breaks, the radio emission rises while the optical falls off, these temporal features cannot be due to the passage of the same afterglow continuum break through the observing band.

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With the possible exception of the afterglow 990123, there is no evidence for an evolution (softening) of the optical continuum across the light-curve break for the above 10 afterglows (e.g. Panaitescu 2005). Furthermore, the only spectral feature whose passage could yield the large steepening $\Delta \alpha$ observed in some cases – the peak of the forward shock (FS) continuum$^1$ – should yield a rising or flat optical light curve before the break, contrary to what is observed. Therefore, it is the radio peak that should be attributed to a spectral break crossing the observing domain. That spectral break should be the injection frequency $v_i$, which, for reasonable shock microphysical parameters, should reach the radio at around 10 d. Indeed, for the afterglow 991208, there is observational evidence (Galama et al. 2000) that the peak of the afterglow continuum crosses the 10–100 GHz domain at that time. Further evidence for the passage of a spectral break through radio is provided by that the peak time of the radio flux of the afterglow 030329 increases with decreasing observing frequency (Frail et al. 2005).

Today, the generally accepted reason for the optical light-curve break is the narrow collimation of the GRB ejecta. As predicted by Rhoads (1999), if the GRB ejecta are collimated, the afterglow light curve should exhibit a steepening when the jet begins to expand sideways. More than half of the steepening, $\Delta \alpha$, is due to the finite angular opening of the jet: as the GRB remnant is decelerated (by sweeping-up of the CBM) and the relativistic Doppler beaming of the afterglow emission decreases, an ever-increasing fraction of the emitting surface becomes visible to the observer; when the jet edge is seen, that fraction cannot increase any longer and the afterglow emission exhibits a faster decay (Panaitescu, Mészáros & Rees 1998). Because it arises from the blast wave dynamics, a light-curve break should also be present at radio wavelengths at the same time as the optical break. However, radio observations before 1 d are very scarce and strongly affected by Galactic interstellar scintillation (Goodman 1997) until after 10 d, thus they cannot disentangle the jet break from that arising from the passage of the FS peak frequency.

Another mechanism for the optical light-curve breaks has been proposed by Rossi, Lazzati & Rees (2002) and Zhang & Mészáros (2002): if GRB outflows are endowed with an angular structure (i.e. non-uniform distribution of the ejecta kinetic energy with direction), as first proposed by Mészáros (1998), then a steepening of the afterglow decay would arise when the brighter outflow symmetry axis becomes visible to the observer. In this model, the stronger the angular structure is, the larger the break magnitude, $\Delta \alpha$, should be.

A third mechanism for breaks rests on the proposal of Rees & Mészáros (1998) and Paczyński (1998) that the FS energizing the CBM could be refreshed by the injection of a substantial energy through some delayed ejecta that were released at the same time with the GRB-producing ejecta, but had a smaller Lorentz factor, or were ejected sometime later, and which catch up with the decelerating FS during the afterglow phase. Fox et al. (2003) have proposed that the early (0.003–0.1 d) slow decay of the optical emission of the afterglow 021004 is caused by such an injection process. In this scenario, when the energy injection (EI) episode ends, the FS deceleration becomes faster and the afterglow emission should exhibit a steepening.

Note the various origins of the afterglow light-curve break in each model. In the jet model, the break is caused by the changing outflow dynamics when the jet starts to spread and by the outflow’s geometry. In the structured outflow (SO) model, the origin is, evidently, the outflow’s anisotropic surface brightness. For both these models, special relativity effects play an important part. In the EI model, the break originates in the altered outflow dynamics at the time when the EI subsides.

The purpose of this work is to compare the ability of these three models in accommodating

(i) the shape of the light-curve breaks observed in the optical emission of the afterglows 980519, 990123, 990510, 991216, 000301c, 000926, 010222, 012111, 020813, 030226, and

(ii) the relative intensity of the radio, optical and X-ray emissions of these afterglows, for either a uniform (i.e. homogeneous) CBM or one with a $r^{-2}$ density radial stratification, as expected if GRB progenitors are massive stars (Woosley 1993; Paczyński 1998).

For the first task above, we performed an analytical test of the models (see table 2 of Panaitescu 2005), based on comparing

(i) the pre- and post-break optical light-curve decay indices, $\alpha_1$ and $\alpha_2$,

(ii) those in the X-rays,

(iii) the slopes $\beta_x$ and $\beta_o$ of the power-law optical and X-ray continua ($F_o \propto v^{-\beta_o}$ and $F_x \propto v^{-\beta_x}$), and

(iv) the optical to X-ray spectral energy distribution (SED) slope, $\beta_{ox} = \ln(F_x/F_o)/\ln(v_x/v_o)$,

with the relations among them expected for each model. We note that, in the framework of the relativistic fireballs (Mészáros & Rees 1997), the optical and X-ray decay indices and SED slopes are tightly connected as only one continuum feature, the cooling frequency $v_c$ (Sari, Narayan & Piran 1998) can be between these domains at the times when observations were usually made (0.1–100 d) and that the SED slope increases by a fixed amount, $\delta \beta = 1/2$, across this spectral break.

Including the radio afterglow emission in the analytical test of the three break models is less feasible and often unconstraining. First, that the afterglow radio flux is modulated by diffractive and refractive interstellar scintillation makes it difficult to determine accurately the radio SED slope [see figs 4 and 5 of Frail et al. (2000a) for the best-monitored radio afterglow – 970508]. Secondly, after the injection frequency has fallen below the radio (i.e. during the decay phase of the radio light curve), we do not expect, in general, any spectral break to be between radio and optical, hence the radio and optical light curve indices should be the same. This is indeed the case for most afterglows. Nevertheless, there are a few troubling exceptions: over 1–2 decades in time, the radio emission of the afterglows 991208, 991216, 000926 and 010222 exhibits a much shallower decay than that observed at optical wavelengths at the same time or prior to the radio decay. An analytical investigation of the various possible ways to decouple the radio and optical light curves (Panaitescu & Kumar 2004) has led to the conclusion that, the anomalous radio decay is due to a contribution from the reverse shock to the radio emission.

However, radio observations provide an indirect constraint on them because the flux and epoch of the radio peak determine the FS synchrotron peak flux, injection frequency and self-absorption frequency. These three spectral properties constrain the afterglow parameters pertaining to the outflow dynamics (energy per solid angle, jet initial opening and medium density) and emission

\footnote{This is the smallest of the synchrotron frequency $v_i$ corresponding to the typical post-shock electron energy (which we call 'injection frequency') and to the electrons that cool radiatively on a dynamical time-scale (the cooling frequency $v_c$).}
GRB afterglows: numerical modelling

2 MODELS DESCRIPTION

The basic equations employed in our numerical calculations of the outflow dynamics are presented in Panaitescu & Kumar (2000) for spherical outflows, Panaitescu & Kumar (2001) for the jet model, Panaitescu & Kumar (2003) for the SO model and Panaitescu et al. (1998) for the EI model. The basic equations for the calculation of the afterglow spectral features (synchrotron and inverse Compton peak fluxes, absorption, injection and cooling frequencies), and of the afterglow emission at any wavelength are given in Panaitescu & Kumar (2000, 2001). Equations for the spectral characteristics can also be found in Sari et al. (1998), for the dynamics and emission from spherical blast waves interacting with a uniform medium in Waxman, Kulkarni & Frail (1998), Granot, Piran & Sari (1999), Wijers & Galama (1999) and for a wind medium in Chevalier et al. (2000). The dynamics of blast waves with EI is also treated in Sari & Mészáros (2000). Rhoads (1999) provided a detailed treatment of the dynamics and emission from jets interacting with uniform media (see also Sari, Piran & Halpern 1999).

2.1 Jets

The jet dynamics is determined by the initial jet opening $\theta_{\text{jet}}$, the ejecta initial kinetic energy per solid angle $E_0$ (or, equivalently, by the jet energy $E_{\text{jet}} = \pi\theta_{\text{jet}}^2 E_0$), and the CBM particle density $n$ or, in the case of a wind, the mass-loss rate $\dot{M}$ (cf. Equations (4) and (5) but with $n$ replaced by $\dot{M}$). The deceleration of the jet, as it sweeps up the CBM, is calculated assuming that the post-shock gas has the same internal energy per mass (i.e. temperature) as that immediately behind the FS, which is equal to the bulk Lorentz factor of the shocked CBM, $\Gamma$. The lateral size of the jet is assumed to increase at the comoving sound speed, and the kinetic energy per solid angle is approximated as uniform during the spreading. Radiative (synchrotron and inverse Compton) losses are calculated from the electron distribution and magnetic field strength.

In the jet model, the light-curve break is seen when the jet edge becomes visible to the observer. For an observer located close to the jet axis, this time can be approximated by $\Gamma(t_{\text{jet}}) = \theta_{\text{jet}}$, using the initial jet opening and ignoring the lateral spreading that occurred until the jet-break time. If the FS dynamics were adiabatic, then energy conservation would lead to

$$\Gamma(t) = 8.6 \left(\frac{E_{0,53}}{n_0}\right)^{1/8} [t_d/(1 + z)]^{-3/8}$$

for a uniform medium and

$$\Gamma(t) = 15 \left(\frac{E_{0,53}}{A_{\gamma}}\right)^{1/4} [t_d/(1 + z)]^{-1/4}$$

for a wind, where $t_d$ is the observer time in days. Then the jet-break time is given by

$$t_{\text{jet}} \simeq 0.7(z + 1) \left(\frac{E_{0,53} n_0^{-1/8} \theta_{\text{jet}}^8}{\theta_{\text{jet}}^{53}}\right)^{1/3} d$$

for a uniform (‘unif’) CBM and

$$t_{\text{jet}} \simeq 5(z + 1) E_0 A_{\gamma}^{-1} t_d^{1/4} d$$

for a wind. Given that observer locations off the jet axis (but within $\theta_{\text{jet}}$) have a small effect on the resulting afterglow light curves (Granot et al. 2002), we consider that, in the jet model, the observer is always on the jet axis. Furthermore, we ignore the possible existence of a counter-jet whose emission would become visible to the observer when the semirelativistic dynamics sets, i.e. it would affect the radio afterglow emission beyond 100 d after the burst.

At the jet-break time, the afterglow decay index at observing frequency, $v$, steepens from that corresponding to a spherical outflow (Sari et al. 1998)

$$\alpha_1 = \begin{cases} 3p - 3, & v < v_c \text{ and unif CBM} \\ 3p - 2, & v_c < v \text{ and any CBM} \\ 3p - 1, & v < v_c \text{ and wind CBM} \end{cases}$$

to $\alpha_2 = p$ (Rhoads 1999). In equation (6), $p > 0$ is the power-law index of the post-shock electron distribution with energy (equation 16).

2.2 Structured outflows

The dynamics of SOs is calculated similarly to that of jets. To track the lateral fluid flow and the change of the kinetic energy per solid angle, the outflow surface is divided into infinitesimal rings and we consider that each ring spreads at a rate proportional to the local sound speed and to the ring width. This prescription for lateral flow intensity can be obtained if the observer is located within the core, but only numerically for outer locations. In the former case, which yields breaks with a smaller magnitude $\Delta z$. For this reason, in our previous analytical assessment of the three models (Panaitescu 2005), we have restricted our attention only to the $\theta_{\text{obs}} < \theta_{\text{core}}$ case, which yields breaks with a smaller magnitude $\Delta z$. 

for a homogeneous medium and
\[
\alpha_2 = \begin{cases} 
\frac{4}{q-1} & v_c < v \leq \frac{1}{2} q (p-1) \\
\frac{2(4-p)}{q} & v_c < v < \frac{1}{2} q (p-2) 
\end{cases}
\]
(9)
for a wind medium (Panaitescu & Kumar 2003), provided that the structural parameter \( q < \tilde{q} \), where, for a uniform medium, \( \tilde{q} = 8/(p + 4) \) if \( v < v_c \) and \( \tilde{q} = 8/(p + 3) \) if \( v > v_c \). For a wind, the two values of \( \tilde{q} \) are swapped. For \( q > \tilde{q} \), the core dominates the afterglow emission and the post-break decay index is
\[
\alpha_2 = \alpha_1 + \begin{cases} 
3/4 & \text{unif CBM} \\
1/2 & \text{wind CBM} 
\end{cases}
\]
(10)
i.e. \( \Delta \alpha \) is just the steepening produced by the finite angular extent of the emitting outflow. In this case, the structure outflow model reduces to that of a jet with sharp edges and no sideways expansion.

For observers located outside the core, the break magnitude \( \Delta \alpha \) can be larger (see figs 2–4 in Panaitescu & Kumar 2003).

2.3 Energy injection

The injection of energy in the FS has two effects: it mitigates the FS deceleration and generates a reverse shock (RS), which energizes the incoming ejecta and contributes to the radio afterglow emission.

For ease of interpretation, we consider an EI that is a power law of the form
\[
e^{-\nu_d t}
\]
(11)
where \( dE_i/dt \) is the rate of the influx of energy per solid angle. Evidently, this injection has an effect on the afterglow dynamics only if the total added energy, \( E_i \), is comparable or larger with that existing in the afterglow after the GRB phase, \( E_0 \). In this case, the light-curve decay index during the injection process is (Panaitescu 2005)
\[
\alpha_1 = \begin{cases} 
\frac{1}{4} (3 - e) p - \frac{1}{4} (1 + e) & v < v_c \text{ and unif CBM} \\
\frac{1}{2} & v_c < v \text{ and any CBM} \\
\frac{1}{4} & v < v_c \text{ and wind CBM} 
\end{cases}
\]
(12)
After the EI subsides, the post-break index \( \alpha_2 \) has the value given in equation (6) (for an adiabatic, spherical blast wave).

Numerically, the EI is modelled by first calculating the dissipated energy and that added as kinetic energy during the collision between an infinitesimal shell of delayed ejecta and the FS. The partition of the incoming ejecta energy is determined from energy and momentum conservation, and depends only on the ratio \( \Gamma_i/\Gamma \), with \( \Gamma_i \) being the Lorentz factor of the incoming ejecta. Numerically, this factor is obtained from the kinematics of the catching up between the freely flowing delayed ejecta and the decelerating FS. It can also be calculated analytically, if it is assumed that all the ejecta were released on a time-scale much shorter than the observer time when the catching up occurs. With this assumption, it can be shown that the ejecta–FS contrast Lorentz factor is constant,
\[
\Gamma_i/\Gamma = (1 + e)^{-1/2} \begin{cases} 
2 & \text{unif CBM} \\
\sqrt{2} & \text{wind CBM} 
\end{cases}
\]
(13)
Once the dissipated fraction is known, we track the adiabatic conversion of the internal energy into kinetic, as described in Panaitescu et al. (1998). Although the concept of adiabatic losses implies that there will be a dispersion in the Lorentz factor of the swept-up CBM, we ignore it and, just as for the other two models, assume that all the fluid behind the FS moves at the same Lorentz factor \( \Gamma \).

Even if the injected energy is negligible, the RS crossing the delayed ejecta can be of relevance for the lower frequency afterglow emission at days after the burst. To calculate this emission, we set the typical electron energy behind the RS by assuming that all the dissipated energy is in the shocked ejecta, which would be strictly correct only if the density of the shocked CBM were much larger than that of the delayed ejecta. Otherwise, this assumption leads to an overestimation of the post-RS electron energy. After the EI ceases, we track the evolution of the RS electron distribution, subject to adiabatic and radiative cooling. The tracked electron distribution is used for the calculation of the RS continuum break frequencies. In contrast, for all three models, the FS electron distribution is set using the current Lorentz factor \( \Gamma \) of the FS (see below). This last approximation is more appropriate for a uniform medium than for a wind medium, as in the former case \( dM/dr \propto r^2 \) while for the latter \( dM/dr = \text{constant} \).

2.4 Model parameters

As presented above, the jet model has two dynamical parameters: the initial kinetic energy per solid angle \( E_i \) and the initial jet opening angle \( \theta_{\text{jet}} \). The SO model has three such parameters: the energy per solid angle \( E_i \) on the axis (and in the uniform core), the core angular size \( \theta_{\text{core}} \), and the structural parameter \( q \) for the outflow envelope. The EI model has three dynamical parameters: the total injected energy \( E_i \), the temporal index \( e \) of the injection law and the time \( t_{\text{off}} \) when the injection episode ends, the initial \( E_0 \) being less relevant if \( E_0 \ll E_i \).

In addition, all models have another parameter that determines the outflow dynamics: the external medium density \( n \) (or the wind parameter \( A \)). The observer location \( \theta_{\text{obs}} \), relative to the outflow symmetry axis is relevant only for the SO model, but not for the jet model, as long as \( \theta_{\text{obs}} < \theta_{\text{jet}} \), or for the EI model, where the outflow is spherically symmetric.

The calculation of the synchrotron and inverse Compton emissions (and of the radiative losses) requires three more parameters: one for the magnetic field strength, parametrized by the fraction \( \varepsilon_B \) of the post-shock energy in it:
\[
B^2/(8\pi) = n[(4\Gamma_i^2 + 3)(\Gamma_i - 1)m_p c^2 ] \varepsilon_B,
\]
(14)
the comoving frame particle density in the unshocked fluid, \( m_p \) the proton mass, one for the minimal electron energy behind the shock, parametrized by the fraction \( \varepsilon_i \) of the post-shock energy in electrons if all had the same energy \( \gamma_i m_e c^2 \):
\[
\gamma_i m_e = (\Gamma_i - 1)m_p \varepsilon_i,
\]
(15)
and the index \( p \) of the electron power-law distribution with energy:
\[
dN/d\varepsilon \propto \varepsilon^{-p}.
\]
(16)
Equations (14) and (15) apply also to the RS if \( \Gamma \) is replaced by the Lorentz factor of the shocked ejecta as measured in the frame of the incoming (unshocked) ejecta. The parameters \( \varepsilon_i \) and \( \varepsilon_B \) determine the synchrotron characteristic frequency, \( \nu_i \), corresponding to the typical electron energy, as well as the self-absorption and cooling frequencies.

We assume that the three microphysical parameters \( \varepsilon_B \), \( \varepsilon_i \) and \( p \), have the same value both behind the forward and reverse shock.

Summarizing, the jet model has six free parameters, the EI model has seven and the SO model has eight. The V-band extinction due to dust in the host galaxy, $A_V$, which affects the observed slope of the optical SED and the overall optical flux, is an extra parameter for all models. To determine the host frame $A_V$ extinction, we assume an Small Magellanic Cloud (SMC)-like reddening curve\(^3\) for the host galaxy and use the parameters inferred by Pei (1992) for SMC.

All these parameters are constrained by the multwavelength afterglow data in the following way. The pre-break decay index $\alpha_1$ of the optical light curve determines the electron distribution index $p$ for the jet and SO models (equations 6, 8 and 9), and the EI index $e$ for the EI model (equation 12). The post-break decay index $\alpha_2$ overconstrains the electron index $p$ for the jet model, determines the structural parameter $q$ for the SO model and the electron index $p$ for the EI model.\(^4\) The jet opening $\theta_{\text{jet}}$ for the jet model and the observer location $\theta_{\text{obs}}$ for the SO model (if the observer is outside the core) or the core angular size $\theta_{\text{core}}$ (if the observer is within the core) are set by the epoch of the optical light-curve break (equations 4 and 5). The same observable determines the time $t_{\text{off}}$ when the EI ceases in the EI model. For the SO model, whenever the observer is outside the outflow core, the $\theta_{\text{out}}$ is not well constrained because it has a weak effect on the resulting afterglow emission. The outflow energy, $E_0$ (or the injected one $E_\ast$), the medium density, $n$ (or the wind parameter $A_\ast$) and the two microphysical parameters $e_\ast$ and $e_B$ are constrained by the afterglow spectral parameters – flux at the peak of the spectrum, self-absorption, injection and cooling frequencies—which in turn are constrained by the relative intensities of the radio, optical and X-ray emissions (e.g. Granot et al. 1999; Wijers & Galama 1999). That the self-absorption frequency is not always constrained by the radio observations accounts for part of the resulting parameter uncertainties, but note that the parameters $E_0$, $n$, $e_\ast$ and $e_B$ are also constrained by matching the radio emission at late times, when the blast wave is only mildly relativistic, i.e. after the self-similar relativistic dynamics phase employed in analytical calculations of afterglow parameters. For the EI model, the initial outflow energy $E_0$ is irrelevant, as the blast wave kinetic energy is that injected starting at a time before the first observation. Finally, the host extinction, $A_V$, is constrained by the ratio of the observed optical flux to the intrinsic optical flux that is consistent with the X-ray emission (for the other afterglow parameters).

3 RESULTS OF NUMERICAL MODELLING

Table 1 lists the reduced chi-square, $\chi^2/\nu$, for the best fits obtained with the jet, SO and EI models, for the two types of circumburst media: homogeneous and wind like. For a uniform CBM, the best fits with the EI model are very poor and, for almost all afterglows, they are substantially worse than those for a wind. This is so because, for a uniform CBM, the observed post-break optical decay index $\alpha_2$ [given by equation (6)] requires a higher electron index, $p$, than for a wind, leading to an optical SED slope $(\beta_\nu)$ and an optical to X-ray spectral index $(\beta_{\nu,0})$ larger than those observed. Also, the EI model has difficulties in accommodating both the radio and X-ray fluxes, with the FS emission either overestimating the observed radio flux or underestimating the X-ray flux.

Tables 2–5 list the parameters of the best fits with $\chi^2 < 4$ for the jet, SO, EI and jet+EI models, respectively, some of which are shown in Figs 1–17. From the parameter uncertainties determined by us (Panaitescu & Kumar 2002) for the jet model from the $\chi^2$-variation around its minimum for eight GRB afterglows, we estimate the following uncertainties for the best-fitting parameters given in Tables 2–5: $\sigma(\log E_0) = \sigma(\log E_\ast) = 0.3, \sigma(\log n) = 0.7, \sigma(\log A_\ast) = 0.3, \sigma(\theta_{\text{jet}}) = 0.3 \sigma(\theta_{\text{obs}}), \sigma(\theta_{\text{out}}) = 0.2, \sigma(\log e_\ast) = 1, \sigma(\theta_{\text{core}}) = 0.3 \sigma(e), \sigma(p) = 0.1, \sigma(q) = 0.3, \sigma(e) = 0.2, \sigma(A_V) = 0.2 A_V$.

Figs 1–17 display the best fits obtained with the jet model for the afterglows 010222, 011211, 020813 and 030226, which we did not present previously, and comparable or better fits obtained with the SO and EI models. With the exception of the jet model and a uniform CBM for the afterglow 990510, all best fits presented are not acceptable in a statistical sense, as they have $\chi^2 > 1$. In general, fits with $\chi^2 < 4$ appear adequate upon visual inspection, a significant fraction of the $\chi^2$ arising often from optical measurements (which have the smallest uncertainties), indicating either a small, short-lived afterglow fluctuation or, perhaps, an occasional outlier resulting from underestimated observational errors. In some cases, there are also systematic discrepancies between the model fluxes and observations, most often in the radio. As a general rule, if the fits are poorer than $\chi^2 > 4$ for both types of circumburst media then the fits are not shown, with the exception of those obtained with the jet and SO models for the afterglows 011211 and 030226, for which we want to show that the latter model is much better than the former in accommodating the 1-d, sharp optical light-curve break. Also as a general rule, if the fits obtained with the jet and SO models are comparable, only one is shown in the figures.

For clarity, the figures contain only the radio optical frequencies where observations span the longest time, but the fitted data set included few or several other bands in these two domains (see figure captions). With the exception of the afterglow 000926, the available X-ray measurements are in only one band, which we use to infer the X-ray flux at the mid energy of that band. The amplitude of interstellar scintillation, whose calculation is based on the treatment and maps given by Walker (1998), is indicated with vertical bars at the time of the radio observations. In all figures, the best fit for a uniform CBM is shown with continuous lines, while that for a wind medium with dotted lines.

3.1 980519

The best fit obtained with the jet model is discussed in Panaitescu & Kumar (2002). The addition of more radio data and some optical outliers (which were previously excluded) lead now to a worse fit ($\chi^2$-wise) but to the same afterglow parameters, within their uncertainty. The best fit for the SO model, shown in Fig. 1, has a $\chi^2$ comparable to that for the jet model (Table 1), but is qualitatively poorer, as it overestimates the observed early radio. Although the measured radio fluxes are within the amplitude of the fluctuations caused by the interstellar Galactic gas, such a constant offset cannot be caused by scintillation (unfortunately, $\chi^2$-statistics does not reflect the smaller probability of systematic discrepancies).

3.2 990123

The best fit obtained with the jet model and a uniform medium is presented in Panaitescu & Kumar (2001). There we assumed that the
radio flare seen at 1 d (Kulkarni et al. 1999) arises in the GRB ejecta energized by the RS, and included in the modelling only the radio measurements at 3–30 d, when the 8-GHz flux of this afterglow is less than 40 μJy. This sets an upper limit on the FS peak flux, $F_{p}$, which, together with that $F_{p} \propto n^{1/2}$ before the jet break and $F_{p} \propto n^{1/4}$ after that, requires a small CBM density: $n < 10^{-2} \text{ cm}^{-3}$.

Including the radio measurements before a few days, as well as the millimetre data, the $K$-band fluxes after 10 d (with which the

### Table 1.

Reduced chi-square, $\chi^2_r$, of the best fits obtained for 10 GRB afterglows with three models (SO = structured outflow, EI = energy injection) for light-curve breaks and for two type of circumburst media: uniform ($n = \text{const}$) and wind like ($n \propto r^{-2}$).

<table>
<thead>
<tr>
<th>GRB</th>
<th>$\Delta \alpha^a$</th>
<th>$N^b$</th>
<th>Jet $n = \text{const}$</th>
<th>Jet $n \propto r^{-2}$</th>
<th>SO $n = \text{const}$</th>
<th>SO $n \propto r^{-2}$</th>
<th>EI $n \propto r^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>980519</td>
<td>0.56 ± 0.37</td>
<td>73</td>
<td>2.6</td>
<td>1.8</td>
<td>2.4</td>
<td>1.4</td>
<td>3.0</td>
</tr>
<tr>
<td>990123</td>
<td>0.48 ± 0.21</td>
<td>112</td>
<td>2.0</td>
<td>2.3</td>
<td>1.8</td>
<td>2.2</td>
<td>1.5</td>
</tr>
<tr>
<td>990510</td>
<td>1.29 ± 0.10</td>
<td>101</td>
<td>0.78</td>
<td>3.1</td>
<td>2.1</td>
<td>4.6</td>
<td>3.0</td>
</tr>
<tr>
<td>991216</td>
<td>0.40 ± 0.20</td>
<td>84</td>
<td>2.0</td>
<td>1.8</td>
<td>1.2</td>
<td>1.2</td>
<td>3.4</td>
</tr>
<tr>
<td>000301c</td>
<td>1.83 ± 0.18</td>
<td>111</td>
<td>4.4</td>
<td>8.3</td>
<td>3.3</td>
<td>7.1</td>
<td>10</td>
</tr>
<tr>
<td>000926</td>
<td>0.64 ± 0.13</td>
<td>145</td>
<td>2.2</td>
<td>3.5</td>
<td>2.2</td>
<td>2.8</td>
<td>3.3</td>
</tr>
<tr>
<td>010222</td>
<td>0.88 ± 0.08</td>
<td>175</td>
<td>2.2</td>
<td>3.9</td>
<td>1.7</td>
<td>4.0</td>
<td>4.7</td>
</tr>
<tr>
<td>011211</td>
<td>1.70 ± 0.25</td>
<td>88</td>
<td>4.7</td>
<td>8.2</td>
<td>2.3</td>
<td>4.7</td>
<td>7.6</td>
</tr>
<tr>
<td>020813</td>
<td>0.54 ± 0.06</td>
<td>105</td>
<td>1.6</td>
<td>2.6</td>
<td>1.1</td>
<td>2.6</td>
<td>1.6</td>
</tr>
<tr>
<td>030226</td>
<td>1.48 ± 0.07</td>
<td>112</td>
<td>8.5</td>
<td>17</td>
<td>4.0</td>
<td>11</td>
<td>16</td>
</tr>
</tbody>
</table>

$^a$Magnitude of optical light-curve break, defined as the increase of the exponent of the power-law flux decay. $^b$Number of radio, optical and X-ray measurements.

### Table 2.

Jet model parameters for the best fits of Table 1 with $\chi^2_r < 4$.

<table>
<thead>
<tr>
<th>GRB</th>
<th>$n$ (cm$^{-3}$)</th>
<th>$A_s$</th>
<th>$E_0$ (10$^{53}$ erg s$^{-1}$)</th>
<th>$\theta_{\text{jet}}$ ($^\circ$)</th>
<th>$\lg \epsilon_B$</th>
<th>$\lg \epsilon_i$</th>
<th>$\rho$</th>
<th>$A_V$</th>
<th>Fig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>980519</td>
<td>0.1</td>
<td>0.8</td>
<td>2.3</td>
<td>−3.9</td>
<td>−1.2</td>
<td>2.8</td>
<td>0</td>
<td>0</td>
<td>fig. 2 in PK02</td>
</tr>
<tr>
<td>990123</td>
<td>0.8</td>
<td>0.9</td>
<td>2.3</td>
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<td>−2.7</td>
<td>1.5</td>
<td>0.18</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>990510</td>
<td>0.3</td>
<td>0.2</td>
<td>3.1</td>
<td>−2.3</td>
<td>−1.6</td>
<td>1.8</td>
<td>0.04</td>
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<td>fig. 3 in PK01</td>
</tr>
<tr>
<td>991216</td>
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<td>2.1</td>
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<td>−1.6</td>
<td>1.8</td>
<td>0.05</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>000926</td>
<td>0.1</td>
<td>0.2</td>
<td>2.6</td>
<td>−2.3</td>
<td>−1.9</td>
<td>1.7</td>
<td>0.05</td>
<td>14</td>
<td>fig. 5 in PK02</td>
</tr>
<tr>
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<td>0.09</td>
<td>0.8</td>
<td>2.5</td>
<td>−3.1</td>
<td>−2.0</td>
<td>1.8</td>
<td>0.28</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>020813</td>
<td>0.07</td>
<td>0.7</td>
<td>2.3</td>
<td>−2.9</td>
<td>−2.1</td>
<td>1.9</td>
<td>0.25</td>
<td>14</td>
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<tr>
<td>030226</td>
<td>0.2</td>
<td>0.4</td>
<td>4.8</td>
<td>−3.5</td>
<td>−2.3</td>
<td>1.5</td>
<td>0.29</td>
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</tr>
</tbody>
</table>

### Table 3.

Structured outflow parameters for the best fits of Table 1 with $\chi^2_r < 4$.

<table>
<thead>
<tr>
<th>GRB</th>
<th>$n$ (cm$^{-3}$)</th>
<th>$A_s$</th>
<th>$E_0$ (10$^{53}$ erg s$^{-1}$)</th>
<th>$\theta_{\text{core}}$ ($^\circ$)</th>
<th>$q$</th>
<th>$\theta_{\text{obs}}/\theta_{\text{core}}$</th>
<th>$\lg \epsilon_B$</th>
<th>$\lg \epsilon_i$</th>
<th>$\rho$</th>
<th>$A_V$</th>
<th>Fig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>980519</td>
<td>0.6</td>
<td>0.6</td>
<td>0.9</td>
<td>1.4</td>
<td>0.9</td>
<td>−3.4</td>
<td>−1.5</td>
<td>2.7</td>
<td>0.07</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>990123</td>
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<td>0.5</td>
<td>0.7</td>
<td>1.7</td>
<td>0.7</td>
<td>−3.2</td>
<td>−2.2</td>
<td>1.6</td>
<td>0.15</td>
<td>0</td>
<td></td>
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<tr>
<td>990510</td>
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<td>0.2</td>
<td>1.1</td>
<td>1.5</td>
<td>1.3</td>
<td>−3.0</td>
<td>−2.6</td>
<td>1.5</td>
<td>0.14</td>
<td>0</td>
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<tr>
<td>991216</td>
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<td>0.7</td>
<td>0.7</td>
<td>1.8</td>
<td>3.0</td>
<td>−2.7</td>
<td>−1.4</td>
<td>1.8</td>
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<td>0</td>
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<td>0.5</td>
<td>1.2</td>
<td>0.5</td>
<td>−3.8</td>
<td>−1.7</td>
<td>1.7</td>
<td>0.05</td>
<td>0</td>
<td></td>
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<tr>
<td>000926</td>
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<td>2.5</td>
<td>0.8</td>
<td>2.3</td>
<td>3.3</td>
<td>−2.6</td>
<td>−1.6</td>
<td>2.4</td>
<td>0</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>010222</td>
<td>0.09</td>
<td>0.6</td>
<td>1.7</td>
<td>2.1</td>
<td>0.7</td>
<td>−3.7</td>
<td>−1.9</td>
<td>1.8</td>
<td>0.27</td>
<td>9</td>
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<tr>
<td>011211</td>
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<td>0.9</td>
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<td>−3.9</td>
<td>−1.5</td>
<td>1.9</td>
<td>0.11</td>
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<tr>
<td>020813</td>
<td>0.06</td>
<td>1.3</td>
<td>0.8</td>
<td>1.9</td>
<td>1.4</td>
<td>−3.4</td>
<td>−2.0</td>
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<tr>
<td>030226</td>
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<td>1.0</td>
<td>2.0</td>
<td>−2.6</td>
<td>−1.6</td>
<td>2.0</td>
<td>0</td>
<td>17</td>
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</tr>
</tbody>
</table>

Table 4. Best-fitting parameters obtained with the energy injection model.

<table>
<thead>
<tr>
<th>GRB</th>
<th>n (cm(^{-3}))</th>
<th>A(_0) (10(^53) erg sr(^{-1}))</th>
<th>e</th>
<th>t(_{\text{off}}) (day)</th>
<th>lg (\varepsilon_B)</th>
<th>lg (\varepsilon_i)</th>
<th>p</th>
<th>(A_V)</th>
<th>Fig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>990123</td>
<td>0.003</td>
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<td>0.7</td>
<td>0.5</td>
<td>-2.1</td>
<td>-1.0</td>
<td>2.3</td>
<td>0.04</td>
<td>5</td>
</tr>
<tr>
<td>020813</td>
<td>0.13</td>
<td>0.7</td>
<td>0.3</td>
<td>0.9</td>
<td>-3.2</td>
<td>-2.5</td>
<td>1.8</td>
<td>0.22</td>
<td>15</td>
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</tbody>
</table>

Table 5. Best-fitting parameters obtained with the jet+EI model.

<table>
<thead>
<tr>
<th>GRB</th>
<th>(\chi^2)</th>
<th>n (cm(^{-3}))</th>
<th>A(_0) (10(^53) erg sr(^{-1}))</th>
<th>(\theta_{\text{jet}}) (°)</th>
<th>(E_i/E_0)</th>
<th>t(_{\text{off}}) (day)</th>
<th>lg (\varepsilon_B)</th>
<th>lg (\varepsilon_i)</th>
<th>p</th>
<th>(A_V)</th>
<th>Fig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>990123</td>
<td>1.9</td>
<td>10(^{-3})</td>
<td>1.9</td>
<td>0.7</td>
<td>0.1</td>
<td>0.7</td>
<td>-3.0</td>
<td>-1.2</td>
<td>2.3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>0.04</td>
<td>0.7</td>
<td>6.0</td>
<td>0.2</td>
<td>0.7</td>
<td>-3.5</td>
<td>-1.3</td>
<td>1.8</td>
<td>0.06</td>
<td>7</td>
</tr>
<tr>
<td>991216</td>
<td>2.2</td>
<td>10(^{-3})</td>
<td>1.4</td>
<td>2.1</td>
<td>0.1</td>
<td>1.5</td>
<td>-2.4</td>
<td>-1.1</td>
<td>2.4</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>0.04</td>
<td>0.8</td>
<td>3.6</td>
<td>0.2</td>
<td>1.8</td>
<td>-2.0</td>
<td>-1.5</td>
<td>2.0</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>010222</td>
<td>1.6</td>
<td>10(^{-4})</td>
<td>0.7</td>
<td>2.0</td>
<td>0.06</td>
<td>2.5</td>
<td>-1.8</td>
<td>-1.3</td>
<td>2.3</td>
<td>0.19</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>0.06</td>
<td>0.4</td>
<td>5.0</td>
<td>0.6</td>
<td>1.7</td>
<td>-3.0</td>
<td>-1.7</td>
<td>1.7</td>
<td>0.19</td>
<td>11</td>
</tr>
</tbody>
</table>

Figure 1. Best fit for the afterglow 980519 with the SO model. The data set contains measurements at 1.4, 4.9, 8.5, 100 GHz (Frail et al. 2000b), I, R, V, B, U bands (Halpern et al. 1999; Vrba et al. 2000; Jaunsen et al. 2001) and 5 keV (inferred from the 2–10 keV fluxes presented by Nicastro et al. 1999). Vertical bars on the model radio light curve indicate the amplitude of Galactic interstellar scintillation. The model X-ray emission is mostly inverse Compton scatterings. For this afterglow, a redshift was not measured. We assumed \(z = 1\).

K-band light curve appears to be a single power law, unlike the R-band emission, which exhibits a steepening at \(\sim 2\) d), and other optical measurements previously left out of our modelling, we find a poorer fit \((\chi^2 = 2.4\) for a uniform CBM, \(\chi^2 = 5.1\) for a wind\), but with the same afterglow parameters as before.

We have also tested the ability of the jet model including the emission from the RS crossing the GRB ejecta to explain the optical flash observed at 20–600 s (Akerlof et al. 1999), which peaked 50 s after the burst at about 0.8 Jy, and the 1-d radio flare, as proposed by Sari & Piran (1999). Given that the observer injection time-scale \((\sim 100\) s) is comparable to the burst duration, the approximation of instantaneous ejecta release and its consequence given in equation (13) may be invalid. For this reason, we let the ejecta Lorentz factor, \(\Gamma_i\), to be a free parameter of the fit. Together with the injected energy, \(E_i\), it determines the number of the radiating electrons in the GRB ejecta and the Lorentz factor of the RS, i.e. the peak flux and frequency of the synchrotron RS emission.

The best fits obtained with the jet model interacting with a tenuous medium (so that the FS emission accommodates the late radio measurements) and emission from the GRB ejecta are shown in Fig. 2. We find that, for the same microphysical parameters \(\varepsilon_i\) and \(\varepsilon_B\) behind both shocks, the peak RS optical emission is dimmer.
than observed by a factor 50 for a uniform CBM and a factor 5 for a wind, the latter yielding a good fit to the remainder of the early optical measurements (but not as good as a uniform medium to the 1-d optical break). We also find that, under the same assumption of equal microphysical parameters, the RS emission peaks at 8 GHz at 0.1 d, i.e. a factor of 10 too early. In other words, taking into account the cooling of the RS electrons accelerated at ~100 s and the decrease on the synchrotron self-absorption optical thickness of the RS at radio frequencies, we do not find a set of afterglow parameters for which the RS emission can peak in the radio as late as 1 d, while, for the same microphysical parameters, the FS emission can accommodate the rest of afterglow observations. To explain the optical flash of the afterglow 990123, the GRB ejecta must have a larger parameter $\varepsilon$ than for the FS, indicating that the ejecta is magnetized (Fan et al. 2002; Zhang, Kobayashi & Mészáros 2003; Panaitescu & Kumar 2004), a feature that may be required for by the early optical emission of the afterglow 021211 as well (Kumar & Panaitescu 2003; Zhang et al. 2003).

However, for the low-density jet model, an injection of ejecta in the FS at about the same time when the radio flare is seen allows this model to accommodate that flare, as shown in Fig. 3. Note that the injected energy is less than that required for the FS to explain the rest of the observations, i.e. the incoming ejecta provide only the fresh electrons to radiate at radio frequencies, but do not alter the dynamics of the FS.

After including the 1-d radio flare measurements, as well as the millimetre measurements at 1–10 d, we also find with the jet model a higher density solution (Fig. 4), which is close to that obtained for other afterglows. This higher density $^5$

\[ n \lesssim 1 \text{ cm}^{-3}, \]

\[ \text{which is close to that obtained for other afterglows. This higher density} \]

\[ ^5 \text{It also lowers the cooling frequency, } v_c, \text{ to just slightly above the (blueward of) optical domain, which requires a hard electron distribution with index } p \gtrsim 2p_{\text{on}} \approx 4/3. \text{ Furthermore, a low electron parameter } \varepsilon, \text{ is required for the FS peak frequency to reach 10 GHz as early as 1 d after the burst, and to explain the radio flare.} \]
in the optical emission, have been left out. The best-fitting parameters for
the 250 GHz fluxes contains measurements at 1.4, 4.9, 8.5, 15, 22, 99 GHz (Berger
et al. 2000; Smith et al. 2001; Frail et al. 2003) and K, J, I, R, V, B, U bands
(Jensen et al. 2001; Rhoads & Fruchter 2001). The 250 GHz fluxes and their 2σ
limits (triangles) have been shifted downward by a factor 20, for clarity. Measurements between 3.0 and 4.5 d, when there is a fluctuation in
the optical emission, have been left out. The best-fitting parameters for
the uniform medium are given in Table 3, those for the wind medium are
$E_{\text{0}} = 1.5 \times 10^{53}$ erg s$^{-1}$, $\theta_{\text{core}} = 0.9$, $\theta_{\text{obs}}/\theta_{\text{core}} = 2.6$, $q = 1.4$, $A_{V} = 0.6$,
$\epsilon_{B} = 3 \times 10^{-3}$, $\epsilon_{e} = 0.018$, $p = 2.4$, $A_{V} = 0.06$.
is required by the larger FS peak flux necessary to accommodate
the 1-d flare, when $F_{5\text{GHz}} = 0.36$ mJy. For a uniform CBM,
the high-density fit is slightly better than the low-density solution,
the improvement being more substantial for a wind. Note, however,
that the former underestimates the radio measurements at 30 d.

The SO model yields fits of the same quality as the jet model
(Table 1). The best-fitting structural parameter satisfies $q > \tilde{q}(p)$
for a uniform medium and $q < \tilde{q}(p)$ for a wind (Table 3),
thus the post-break afterglow emission arises mostly from the
outflow core in the former case and from the envelope for the
latter.

The best fit obtained with the EI model for a wind medium is shown in Fig. 5. $\chi^2$-wise, this low-density model provides a better fit
than the jet and SO models, although it overestimates the early radio
measurements. Also, this solution has an extremely low wind density
(Table 4), about 100 times smaller than that known for Galactic
WR stars. For such a tenuous wind, equation (5) implies that, if
the outflow opening $\theta_{\text{jet}}$ is wider than $2^\circ$, then the jet break would
appear later than about 30 d (i.e. after the latest measurement),
and the corresponding outflow energy would be larger than $2 \times 10^{50}$ erg, which is less than that of the jet shown in Fig. 4. The best fit obtained for a uniform medium is very poor (it either overestimates
the radio emission or underproduces X-rays) and is not shown.
shown in Panaitescu & Kumar (2001). Because the radio decay of
3.4 991216
and of this afterglow. SOs yield poorer fits for either type of medium, the
Panaitescu & Kumar (2001). A wind provides a poorer fit (Table 1),
the best fit with the jet model for a uniform CBM is shown in
Figure 11. Best fit for the afterglow 010222 with the jet+EI model. For
a uniform medium the FS component accommodates better the radio data
after 10 d than for a wind. The 1-d hump in the radio light curve is the reverse
shock emission, the FS overtaking at 10 d.

Figure 12. Best fit for the afterglow 011211 with the jet model. The
data set contains measurements at 8.5, 22 GHz (http://www.aoc.nrao.edu/
dffrail/011211.dat), K, J, I, R, V, B, U bands (Holland et al. 2002; Jakobsson et al. 2003); and 2 keV (inferred from the 0.2–5 keV fluxes measured
by Borozdin & Trudolyubov 2003). The best-fitting parameters are

\[ E_0 = 2 \times 10^{52} \text{ erg sr}^{-1}, \theta_{\text{jet}} = 5:5, n = 1.0 \text{ cm}^{-3}, \epsilon_B = 6 \times 10^{-4}, \epsilon_i = 0.047, p = 2.3, A_V = 0 \text{ for a uniform medium and } E_0 = 4 \times 10^{52} \text{ erg sr}^{-1}, \]
\[ \theta_{\text{jet}} = 4:0, A_\text{*} = 0.6, \epsilon_B = 5 \times 10^{-4}, \epsilon_i = 0.030, p = 2.2, A_V = 0 \text{ for a wind.} \]

3.3 990510

The best fit with the jet model for a uniform CBM is shown in
Panaitescu & Kumar (2001). A wind provides a poorer fit (Table 1),
as it fails to accommodate the strong break of the optical light curves of this afterglow. SOs yield poorer fits for either type of medium, the
best-fitting structural parameter being \( q = 1.8 \) for a uniform CBM and \( q = 1.4 \) for a wind.

3.4 991216

The best fit obtained with the jet model and a uniform CBM is shown in Panaitescu & Kumar (2001). Because the radio decay of
this afterglow, \( F_{8.5 \text{ GHz}} \propto t^{-0.77_{-0.06}^{+0.06}} \), is significantly slower than that measured at optical wavelengths after the break, \( F_{\lambda} \propto t^{-1.65_{-0.12}^{+0.12}} \),
in our previous modelling we have employed a double power-law electron distribution, harder (smaller index \( p \)) at low energies, to accommodate the shallower radio decay, and a softer (larger index \( p \)) at high energies, to explain the post-break optical light-curve decay.

Here we use a single power-law electron distribution, to assess the ability of each model to explain all the data without recourse to a
light-curve decay steepening originating in a break in the electron distribution. Consequently, the jet model (as well as any other single emission component model) will not accommodate the decays of both the radio and optical emissions of the afterglow 991216, as illustrated by the higher \( \chi^2_{\nu} \) given in Table 1 and the best fit shown in Fig. 6.
This requires a tenuous CBM, which increases the cooling frequency $(\nu_c \propto n^{-3/6}$ before the jet break and $\nu_c \propto n^{-1}$ after that). Going from the jet fit to the jet+EI fit increases the cooling frequency from about $10^{38}$ Hz to $3 \times 10^{39}$ Hz. Then, to accommodate the optical to X-ray SED slope, the jet+EI model requires a softer electron distribution than the jet model, which in turn yields a faster post-break decay of the afterglow light curve, as shown in Fig. 7, and a poorer fit to the optical emission after 10 d.

The addition of emission from the RS, which energizes the ejecta catching up with the FS at about 1 d, improves the radio fit, as shown in Fig. 7. A slower decay radio light curve is obtained as the FS emission overtakes that from the RS at about 10 d. As for the afterglow 990123 (Fig. 3), the energy injected is small enough that it does not alter the dynamics of the FS. However, the afterglow parameters of the jet+EI model for 991216 are different than those obtained with the jet model (Fig. 6) because we require now that the FS peaks in the radio at a later time and at a lower flux ($\sim 0.1$ mJy).\footnote{This requires a tenuous CBM, which increases the cooling frequency $(\nu_c \propto n^{-3/6}$ before the jet break and $\nu_c \propto n^{-1}$ after that). Going from the jet fit to the jet+EI fit increases the cooling frequency from about $10^{38}$ Hz to $3 \times 10^{39}$ Hz. Then, to accommodate the optical to X-ray SED slope, the jet+EI model requires a softer electron distribution than the jet model, which in turn yields a faster post-break decay of the afterglow light curve, as shown in Fig. 7, and a poorer fit to the optical emission after 10 d.}

The SO model yields a best fit which is very similar to that shown in Fig. 6 for the jet model. The smaller intensity-averaged source size resulting in the SO model leads to a larger amplitude for the interstellar scintillation and, thus, to a better $\chi^2$-wise fit (Table 1), but this model also fails to accommodate the slow radio decay. The best-fitting structural parameter satisfies $q \lesssim 1.6 - 1.8$ (Table 3), i.e. the outflow envelope emission dominates over that from the core after the optical break.

### 3.5 000301c

The best fit for the jet model is discussed in Panaitescu (2001). As for the afterglow 991216, the slower radio decay of 000301c, $F_{\nu} \propto t^{-1.4 \pm 0.3}$ and its faster optical fall-off, $F_o \propto t^{-2.83 \pm 0.12}$, prompted us to consider a double power-law electron distribution. Furthermore, the large break magnitude $\Delta \nu = 1.8 \pm 0.2$ exhibited by the optical light curve 000301c (the largest in the entire sample – Table 1) exceeds that allowed by the jet model for the electron distribution index $p$ required by the pre-break decay index and optical SED slope (Panaitescu 2005). Such a strong break is also better explained if there is a contribution from the passage of a spectral break. However, since we want to test the three models for light-curve break without any contribution from another mechanism, we list in Table 1 the best fit obtained previously with a soft electron distribution.

Fig. 8 shows the best fit obtained with the SO model, which is only slightly better than that of the jet model, has similar parameters and shares the same deficiencies: it fails to explain the sharpness of the optical light-curve break and yields a radio emission decaying faster than observed. We note that optical measurements between 3.0 and 4.5 d, when a light-curve bump is seen, have been excluded from the fit.

### 3.6 000926

The best fit obtained with the jet model is shown in Panaitescu & Kumar (2002). After including in the data set the near-infrared
measurements, the noisy $I$-band, and some optical outliers that were previously excluded, the best fit is $\chi^2$-wise poorer, but the fit parameters are the same as before (within their uncertainty). An equally good fit can be obtained with the SO model (Fig. 9). We note that for both jet and SO models, the X-ray emission is mostly inverse Compton scatterings.

3.7 010222

The best fit obtained with the jet model is shown in Fig. 10. Note that the model radio light curve decays after 1 d faster than observed. For a uniform CBM, the best fit obtained with an SO is slightly better in the radio, but it too fails to explain the slow radio decay. For a wind, the best fit obtained with the SO model overestimates the millimetre emission $[F_{\nu}(100 \text{ d}) \simeq 1.2 \text{ mJy}, F_{\nu}(20 \text{ d}) \simeq 3.9 \text{ mJy}]$ attributed to the host galaxy by Frail et al. (2002), who also suggest a possible host synchrotron emission at 8.5 GHz of $\sim 20 \mu$Jy. This is only marginally consistent with the $F_{\nu}(447 \text{ d}) = -1 \pm 35 \mu$Jy reported by Galama et al. (2003) and the host synchrotron SED, $F_{\nu} \propto \nu^{-0.75}$, adopted by Frail et al. (2002), which imply a host flux $F_{\nu,8 \text{ GHz}} = 0 \pm 9 \mu$Jy.

If we subtract a radio host contribution of $F_{\nu}^{\text{host}} = 20 (\nu/8 \text{ GHz})^{-0.75} \mu$Jy from the 44 radio measurements, then, within the jet model, the contribution to $\chi^2$ of the radio data decreases from 107 to 72 for a uniform medium, and from 103 to 53 for a wind, with similar changes for the SO model. Though these are significant improvements, the best fit still underestimates the radio flux measured after 10 d, because the optical measurements have a smaller uncertainty and determine the electron index $p$ and, implicitly, the decay of the model radio emission. In general, one-component models cannot accommodate both the host-subtracted radio emission, $F_{\nu,8 \text{ GHz}} \propto t^{-0.76 \pm 0.12}$, of the afterglow 010222 at 1–200 d, and the optical decay, $F_{\nu} \propto t^{-1.78 \pm 0.08}$, observed at 10–100 d.

A better fit to the radio emission of the afterglow 010222 may be obtained with the jet model if, in addition to the FS emission, there is a RS contribution to the early radio afterglow, as illustrated in Fig. 11 for a uniform CBM. Just as for the afterglows 990123 and 991216, the incoming ejecta carry less energy than that in the FS (and do not alter its dynamics) and the medium density is lower than for the jet model because the FS peak flux must be smaller, to match the radio flux measured after 10 d, when the FS synchrotron peak frequency crosses the radio domain.

3.8 011211

As illustrated in Fig. 12, even for a uniform CBM, the jet model has difficulty in accommodating the sharp break exhibited by the optical light curve of this afterglow, assuming that the 1–2 d optical emission is not a fluctuation. Such fluctuations have been seen in the afterglows 000301c, 021004 and 030329, but the lack of a continuous monitoring for 011211 prevents us to determine if its optical emission at 1 d is indeed a fluctuation. A stronger break can be obtained with the SO model when the brighter outflow core becomes visible to an observer located outside it, hence the better fit that the SO model yields (Fig. 13).

3.9 020813

The best fit obtained with the jet model is shown in Fig. 14. SOs yield fits with $q > \bar{q}(p)$ for a uniform CBM and $q < \bar{q}(p)$ for a wind, i.e. with the post-break emission arising mostly in the outflow core and envelope, respectively. A significant improvement over the jet model is obtained only for a uniform CBM (Table 1).

A fit of comparable quality is also obtained with the EI model for a wind medium (Fig. 15), but not for a uniform CBM, which overproduces radio emission and largely underestimates the observed X-ray fluxes. For the best-fitting parameters given in Table 4, equation (5) shows that a jet break would occur after the last measurement (40 d) if the outflow opening were larger than $5^{\circ}$, which implies a minimum collimated energy of $2 \times 10^{51}$ ergs, which is three times larger than that of the jet shown in Fig. 14. Therefore, just as for 990123, the EI model may not involve significantly larger energies than the jet model, if allowance is made for a possible outflow collimation.

3.10 030226

As for 011211, the strong optical light-curve break of this afterglow is better accommodated by the SO model than the jet model (Figs 16 and 17), though an acceptable fit to the optical break is obtained only for a uniform CBM (in addition, for a wind medium, the SO model overproduces radio emission). However, given the poor coverage before 0.3 d, it is possible that the measured optical emission at that time is a fluctuation below that expected in the jet model.

3.11 970508

Although the decay of the optical emission of this afterglow does not exhibit a steepening, being so far the longest-monitored power-law fall-off (2–300 d), it can be explained with the jet model if its unusual rise at 1–2 d is interpreted as a jet becoming visible to an observer located outside the initial jet opening (see fig. 1 in Panaitescu & Kumar 2002). This requires a rather wide jet, with $\theta_{\text{jet}} = 18^{\circ}$, and yields a poor fit to the X-ray emission ($\chi^2 = 2.8$). A less energetic envelope surrounding this jet would produce the prompt $\gamma$-ray emission and would account for the flat optical emission prior to the 2 d optical rise, hence this model is similar to an SO with a wide uniform core and an envelope whose structure is poorly constrained by observations.

Another possible reason for the 2-d sharp rise of this afterglow is a powerful EI episode in which the incoming ejecta carry an energy larger than that already existing in the FS (Panaitescu et al. 1998), i.e. the blast wave dynamics is altered drastically by the EI. Fig. 18 shows the best with the EI model. Note that the light-curve rise is achromatic, as it arises from the blast wave dynamics, which explains why the X-ray light curve cannot be accommodated.

4 CONCLUSIONS

In this work we expand our previous investigation (Panaitescu & Kumar 2003) of the jet model to four other GRB afterglows for which an optical light-curve break was observed: 010222 (for which radio observations were not available at the time of our previous modelling), 011211, 020813 and 030226. We also note that, here, we have not employed a broken power-law electron distribution to explain both the slower decaying radio emission and the faster falling-off optical light curve observed for some afterglows (e.g. 991216 and 000301c).

Our prior conclusion that a homogeneous CBM provides a better fit to the broadband afterglow emission than environments with a wind-like stratification stands (Table 1). For afterglows with a sharp steepening $\Delta z$ of the optical light-curve decay (990510, 000301c,
011211 and 030226), this is due to the transition between the jet asymptotic dynamical regimes (spherical expansion at early times and lateral spreading afterward) occurs faster for a homogeneous medium than for a wind (Kumar & Panaitescu 2000). For the afterglows with shallower breaks, a wind medium (as expected if the GRB progenitor is a massive star) provides an equally good fit as (or, at least, not much worse than) a homogeneous medium.

For the acceptable fits (Table 2) obtained with the jet model and for a uniform medium, the jet initial kinetic energy, $E_\text{jet}$, is between $2 \times 10^{50}$ and $6 \times 10^{59}$ ergs, the initial jet opening angle, $\theta_\text{jet}$, is between $2^\circ$ and $3^\circ$ (with 000926 an outlier at $8^\circ$) and the medium density, $n$, between 0.05 and 1 cm$^{-3}$ (with 000926 an outlier, again, at 20 cm$^{-3}$). For a wind, $E_\text{jet}$ has about the same range, $\theta_\text{jet}$ ranges from $2^\circ$ to $6^\circ$, while the wind parameter $A_\text{w}$ is between 0.1 and 2. Seventy per cent of the 64 Galactic WR stars analysed by Nugis & Lamers (2000) have an $A_\text{w}$ parameter in this range; for the rest $A_\text{w} \in (2, 5)$. We note that the $E_\text{jet}$ resulting from our numerical fits is uncertain by a factor 2, $\theta_\text{jet}$ by about 30 per cent, $n$ by almost one order of magnitude, and $A_\text{w}$ by a factor of 2.

Although the inferred wind density parameter $A_\text{w}$ is in accord with the measurements for WR stars, a uniform medium is instead favoured by the fits to the afterglow data. This is an interesting issue for the established origin of long bursts in the collapse of massive stars. Wijers (2001) has proposed that the termination shock resulting from the interaction between the wind and the circumstellar medium could homogenize the wind. The numerical hydrodynamical calculations of Garcia-Segura, Langer & Mac Low (1996) show that, for a negligible pressure, $nT_\text{w}$, of the circumstellar medium (about $10^4$ cm$^{-3}$ K), the termination shock of the RSG wind has a radius of 10 pc, outside of which a shell of uniform density is formed. However, this radius is much larger than the distance of about 1 pc where the afterglow emission is produced. Chevalier, Li & Fransson (2004) made the case that, if the burst occurred in an intense starburst region, where the interstellar pressure is about $10^7$ cm$^{-3}$ K, then the shocked RSG wind would form a bubble of uniform density of about 1 cm$^{-3}$ extending from 0.4 to 1.6 pc, in accord with the fit densities and the blast wave radius $r = 8 c \Gamma^2 / (1 + z) = 0.5 (E_{0.55}/n_0)^{1/4} [1/(1+z)]^{1/4}$ pc, resulting from equation (2).

The SO model involving an energetic, uniform core, surrounded by a power-law envelope (equation 7) that impedes the lateral spreading of the core, retains most of the ability of the jet model to yield a steeply decaying optical light curve after the core (or its boundary) becomes visible to an observer located outside the core opening (or within it), while accommodating, at the same time, the observed slope of the optical SED. This is so because, in both models, more than half of the increase, $\Delta \nu$, in the light-curve power-law decay index is due to the finite opening of the bright(en) ejecta. For a jet, the remainder of the steepening $\Delta \nu$ is caused by the lateral spreading. For an SO model, an extra contribution to the steepening, which can lead to a $\Delta \nu$ even larger than that produced by a jet, results if the observer is located outside the core opening.

For a uniform medium, we find that the SO model provides a better fit than the jet model for 6 of the 10 afterglows analysed here (Table 1). For a wind, the former model works better for three afterglows, fits of comparable quality being obtained for the other six. Only the afterglow 990510 is better explained with a jet, for either type of CBM. Within the framework of SOs, eight out of 10 afterglows are better fitted with a uniform medium than with a wind, which is partly due to that, just as for a jet, the light-curve steepening is sharper if the medium is uniform (Panaitescu & Kumar 2003).

For the “acceptable” fits ($\chi^2 < 4$) obtained with the SO model, the ejecta kinetic energy per solid angle in the core, $E_\text{e}$, ranges from $0.5 \times 10^{51}$ to $3 \times 10^{51}$ erg sr$^{-1}$ for either type of medium, with an uncertainty of a factor 3. The angular opening of the core, $\theta_\text{core}$, is between 0.5 and 1$^\circ$ for a homogeneous medium, and about 1$^\circ$ for a wind (991216 being an outlier at 0.5), with an uncertainty less than 50 per cent. The structural parameter $q$ (equation 7), which characterizes the energy distribution in the outflow envelope, is found to be between 1.5 and 2.7 for a uniform medium, and between 1.0 and 2.3 for a wind (with 000926 being an outlier at 2.9). These values are consistent with that proposed by Rossi et al. (2002), $q = 2$, to explain the quasi-universal jet energy resulting in the jet model. For SOs, the ejecta kinetic energy contained within an opening of twice the location of the observer (relative to the outflow symmetry axis), $\theta_\text{obs}$, ranges from $1 \times 10^{50}$ to $4 \times 10^{50}$ erg, for either type of medium. Hence, the energy budget required by the SO model is very similar to that of the jet model. The best-fitting medium densities obtained with the SO model are similar to those resulting for a jet.

We find that the EI model, where a light-curve decay steepening is attributed to the cessation of EI in the FS, can be at work only in two afterglows, 990123 and 020813. The major shortcoming of the EI model is that, in order to explain the steep fall-off of the optical light curve after the break with a spherical outflow, an electron distribution that is too soft (i.e. an index $p$ too large) is required to accommodate the relative intensities of the radio and X-ray emissions. Low-density solutions ($n < 1$ cm$^{-3}$, $A_\text{w} < 0.1$), for which the X-ray emission is mostly synchrotron, yield a cooling frequency that is too low, resulting in model X-ray fluxes overestimating the observations by a factor of at least 10. High-density solutions ($n > 10$ cm$^{-3}$, $A_\text{w} > 1$), for which the X-ray is mostly inverse Compton scatterings, produce a FS peak flux that is too large, which leads to model radio fluxes overestimating the data by a factor of 10.

Figure 18. Best fit for the afterglow 970508 with the EI model. The data set contains 309 measurements at 1.4, 4.9 GHz (Frail et al. 2000a), 86 GHz (Bremer et al. 1998), K, I, R, V, B bands (Chary et al. 1998; Galama et al. 1998; Garcia et al. 1998; Pedersen et al. 1998; Sahu et al. 1997; Sokolov et al. 1998) and 5 keV (inferred from 2–10 keV fluxes and spectral slope given in Piro et al. 1998). The fit parameters are $E_\text{jet} = 9 \times 10^{51}$ erg s$^{-1}$, $e = 2.4$, $t_{\text{off}} = 1.6$ d, $n = 0.9$ cm$^{-3}$, $\epsilon_B = 10^{-3}$, $\epsilon_i = 0.11$, $p = 2.3$, $A_V = 0.15$ for a uniform medium ($\chi^2 = 4.5$) and $E_\text{jet} = 7 \times 10^{51}$ erg s$^{-1}$, $e = 2.7$, $t_{\text{off}} = 1.8$ d, $A_\text{w} = 0.7$ cm$^{-3}$, $\epsilon_B = 2 \times 10^{-2}$, $\epsilon_i = 0.063$, $p = 2.0$, $A_V = 0.18$ for a wind ($\chi^2 = 5.9$).
We note that, due to adiabatic cooling, the GRB ejecta electrons, which were energized by the reverse shock during the burst, do not contribute significantly to the afterglow radio emission at days after the burst. Numerically, we find that, if the microphysical parameters are the same for both the reverse and FSs, then the 1 d radio flare of the afterglow 990123 cannot be explained with the emission from the GRB ejecta electrons, as proposed by Sari & Piran (1999), as the synchrotron emission from these electrons peaks in the radio earlier than 0.1 d, regardless of how relativistic is the reverse shock. For the reverse shock emission to be significant at 1 d after the burst, there must be some other ejecta catching up with the decelerating outflow at that time, or the assumption of equal microphysical parameters for both shocks must be invalid (Panaitescu & Kumar 2004). Furthermore, the optical emission from the ejecta electrons is below that observed at 100 s (Akerlof et al. 1999) by a factor 5 for a wind and a factor 50 for a uniform medium.

A delayed injection of ejecta, carrying less kinetic energy than that already existing in the FS (i.e. not altering the FS dynamics), may be at work in the afterglows 990123, 991216 and 010222, whose radio decay is substantially slower than that observed in the optical after the break. As shown by us (Panaitescu & Kumar 2004), such a decoupling of the radio and optical emissions cannot be explained by models based on a single emission component (FS) and require the contribution of the reverse shock. Figs 3, 7 and 11, illustrate that the sum of the reverse and FS emissions yields a slower decaying radio emission, consistent with the observations. However, for the FS peak flux to be as dim as observed in the radio after 10 d, when the FS peak frequency decreases to 10 GHz, an extremely tenuous medium is required: \( n \sim 10^{-3} \text{ cm}^{-3} \) or \( A_e \sim 0.05 \). The former might be compatible with a massive star progenitor if GRBs occur in the ‘superbubble’ blown by preceding supernovae exploding in the same molecular cloud (Scalo & Wheeler 2001), while the latter points to a progenitor with a low mass and low metallicity or, otherwise, to a small mass-loss rate and high wind speed during the last few thousand years before the collapse, when the environment within 1 pc is shaped by the stellar wind.

As a final conclusion, we note that the SO model is a serious contender to the jet model in accommodating the broadband emission of GRB afterglows with optical light-curve breaks. In both models, the best-fitting parameters describing the ejecta kinetic energy – jet opening energy or core opening energy density along the outflow symmetry axis – have narrow distributions, hinting to a possible universality of these parameters.

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