The equation indicates that as the product of jet radius and radial temperature gradient increases, the difference between the temperature at the center line and at the surface increases. Therefore, to reduce the discrepancy between the predicted and measured jet profiles, a more accurate prediction of the heat transfer coefficient in the upper jet is needed and two-dimensional temperature effects must be accounted for.

In the region directly adjacent to the nozzle exit, a solution to the transition from internal to free jet flow at low Reynolds numbers must be undertaken. Figs. 15 and 16 illustrate the velocity distribution prediction by the integral analysis presented above. In the immediate vicinity of the nozzle exit the results are certainly in error since the no slip condition at the walls of the nozzle exit are not satisfied.

It may be argued that the actual velocity distribution prevailing in the upper jet is different from the assumed profile, and if a distribution closer to the actual one is used, the agreement between the analytical and experimental results could be improved. Nevertheless, any reasonable velocity profile will at least indicate the correct trend of the two-dimensional effects in the upper jet region if not give quantitatively accurate results. The familiar problem of the development of a boundary layer over a flat plate may be cited as an example of this property of integral techniques. Choices of velocity distribution within the boundary layer ranging from a linear profile to polynomials of varying degrees to trigonometric functions all result in roughly the same value for the drag coefficient and boundary-layer thickness.

Another interesting fact emerged from the experiments with Arochlor. In spinning glass there are certain instabilities observed. One instability is a pulsing of the jet, a swelling and decreasing of the jet size. One possible explanation of the fluctuation assumes that the disturbing force causes a small change in the shape affecting the radiant energy exchange in the upper jet causing the instability [2]. In the case of the Arochlor jet it was also found that a cross current of air near the nozzle exit caused the jet to pulsate. However, as was mentioned earlier, the temperature levels in the case of the Arochlor are only about 150 deg F, where radiation heat transfer is negligible and forced convection is dominant. This would seem to suggest that upper jet instabilities are related primarily to forced convection heat transfer instead of being a radiation controlled phenomenon.

Conclusions and Recommendations

The discrepancies between the predicted and measured jet profiles cannot be accounted for solely by including two-dimensional fluid dynamical effects. It appears likely that errors in the predicted temperature field are a cause of the discrepancy. Further effort should be directed toward the inclusion of two-dimensional temperature variations in the analysis. It is recommended that approximate integral techniques be used for simplicity.

However, before the two-dimensional temperature variations are studied, the prediction of the heat transfer from the upper jet should be refined. The film coefficient of heat transfer from the conical-shaped upper jet must be found. In addition, the properties of glass at high temperatures, notably, the spectral complex index of refraction and the specific heat must be measured.

Upper jet instabilities are not solely due to radiation heat transfer coupled with the fluid dynamics; more likely, forced convection heat transfer is dominant. Further work on the stability of variable viscosity jets at very low Reynolds number is needed.

Acknowledgment

This work was supported in part by a grant from the Beaunit Corporation to the Department of Mechanical Engineering. This support is gratefully acknowledged.

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DISCUSSION

S. I. Pai[2]

The authors study a very interesting problem by considering two dimensional effects on a heated free jet of a hot molten glass including the effects of temperature on viscosity of the liquid. In their analysis, they consider the two dimensional velocity distribution along with a uniform temperature distribution at each section of the jet. Their theoretical results do not agree well with the experimental data. In my opinion, since the Reynolds number of the jet studied is very small, the effect of velocity distribution should not be important. On the other hand, because of high temperature considered and the strong relation between temperature and viscosity of the liquid, the temperature distribution should be very important in the case considered in this paper. I would suggest that similar integral method may be used for a two dimensional temperature distribution along with a uniform velocity distribution at each section of the jet as the next step on investigation of this interesting problem.

B. M. Pinney[3]

In this paper the authors describe the extension of an existing analysis based on constant cross-sectional fluid velocities and temperatures to a two-dimensional treatment of velocity. This refinement is important in the nozzle exit regions of a viscous jet where the diameter is rapidly reducing as the axial velocity increases. The fluid flow equations are developed in cylindrical coordinates to satisfy the tangential boundary conditions. A factor is introduced here to account for the transition from a parabolic velocity distribution due to Poiseuillian flow within the nozzle to uniform velocity at some point downstream in the jet. In this analysis, the jet is being air cooled as it accelerates and, because fluid viscosity is highly dependent on temperature, the Navier Stokes and Energy equations require simultaneous solution. Predictions from the theoretical treatment described are compared with data from glass spinning, and model studies at lower temperature using a chlorinated hydrocarbon.

Problems requiring the type of analysis presented are widely encountered in the synthetic fibre industry where viscous melts of many types are spun (extruded and cooled) to form filaments. In most instances the mechanisms occurring in the quench region of the jet are critical with respect to the development of desired fibre properties. For example, the surface to core effects produced by radial temperature gradients and velocity distribution (differential draw) can have a marked effect on the fibre structure and, hence, the physical properties (such as tenacity) obtained. There is consequently a great incentive to use this type of mathematical model for simulation of fiber-making processes. The extension to viscoelastic fluids, noncircular jets, multiple filament configurations, various quenching techniques, and solution spinning where mass and heat transfer occur simultaneously, could result in significant improvements in fibre-making technology.

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Authors' Closure

The authors would like to thank the discussers for their interest and helpful suggestions.

Regarding Prof. Pai's suggestion, work is currently underway to predict the two-dimensional temperature distribution in the upper jet. The problem is complex since radiation within the glass jet is important and radiation heat flux in the axial direction can not be neglicted. As we pointed out in the discussion, refined measurements of the film coefficient of heat transfer at the glass-air interface must also be included in the analysis of the upper jet.

Although extensions of the analysis are required for application to synthetic fibers, the results we have obtained to date are of value in that field. For example, in melt spinning the film coefficients predicted by various authors differ by factors of two to eight [1]! The correlation of film coefficient used in our previous work for the one-dimensional region [2], which was verified by the agreement between predicted and measured jet shape, should be accurate for similar regions of the melt spinning process.