Table 1

Input values, for example

w	W	Q	P_0	L	t_0	V_0	D
lb/ft³	lb-sec²/ft⁴	ft²/lb	lb/ft² abs	ft	sec	ft/sec	ft
71.4	2.219	0.60234×10^{-7}	8640.0	137	0.0	27.4	0.53

Output values, instantaneous valve closure

x	F	t	P_1	P_2	V_1	V_2	$P_1 + P_2$
ft	$\frac{\text{lb-sec}}{\text{ft}^4}$	sec	lb/ft²	lb/ft²	ft/sec	ft/sec	lb/ft²
1	30.9	0.050	125,452.1	0.0	8.0	0.0	125,452.1
		0.150	9,155.7	−49,424.2	−14.8	−5.1	−40,268.5
		0.250	37,499.4	8,333.6	3.0	7.9	45,833.0
		0.350	8,818.7	−5,702.3	−4.1	−1.7	damped out

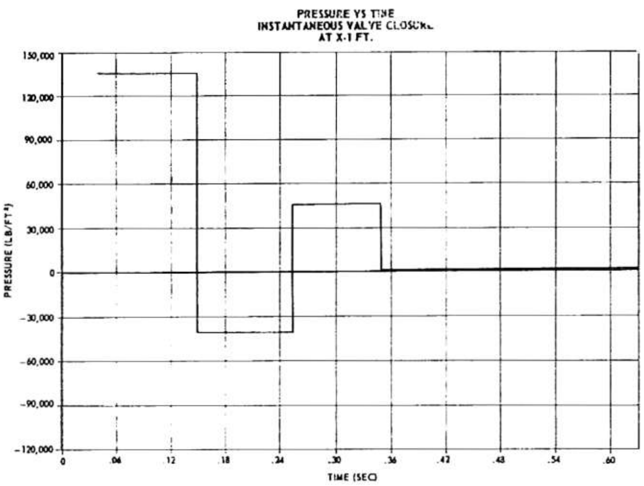


Fig. 2 Instantaneous valve closure

$$+ \frac{C_6}{(F/2W)^7}\left[\left(\frac{Ft}{2W}\right)^6 + 6\left(\frac{Ft}{2W}\right)^5 + 30\left(\frac{Ft}{2W}\right)^4 + 120\left(\frac{Ft}{2W}\right)^3 + 360\left(\frac{Ft}{2W}\right)^2 + 720\left(\frac{Ft}{2W}\right) + 720\right]\Bigg\} \quad (39)$$

If

$$t < t_2, \quad E_4 = 0 \quad (40)$$

If

$$t > t_2,\ E_4 = \frac{Ft_2}{2W}\int_{t_2}^{t} \frac{e^{-\frac{Ft}{2W}} I_1\left(\frac{Ft}{2W}\sqrt{t^2 - t_2^2}\right)}{\sqrt{t^2 - t_2^2}}\, dt \quad (41a)$$

$$E_4 = \frac{F}{2W} t_2 \left[f_4(t) - f_4(t_2)\right] \quad (41b)$$

Where $f_4(t)$ is the same as $f_3(t)$ with the Di's substituted for the respective Ci's. Then, compute:

$$V = V_0 - V_0 \sum_{n=0}^{\infty} (-1)^n \left[h(t_1) + h(t_2)\right] \quad (42)$$

where

$$h(t_1) = e^{-\frac{Ft_1}{2W}} + E_3 \text{ for } t \geq t_1 \quad (43)$$

$$h(t_1) = 0 \quad \text{for} \quad t < t_1 \quad (44)$$

$$h(t_2) = e^{-\frac{Ft_2}{2W}} + E_4 \quad \text{for} \quad t \geq t_2 \quad (45)$$

$$h(t_2) = 0 \quad \text{for} \quad t < t_2 \quad (46)$$

Starting with $x = x_0$, the pressure and velocity are computed for each time point starting at $t = (L - x)/a$ at intervals of $2L/a$. Since pressure and velocity are discontinuous at these points, their value is computed first by approaching the time point from the right and then from the left. When the computations are completed to $t = t_{\max}$ or until $|P| \leq P_0 + \Delta P$, x is increased by Δx and the same calculations are made for this new x. This process is continued till x equals $x_{\max}$.

The use of the above computer method enables the investigation of a wide range of parameters in the determination of critical design points along the length of a missile transfer line. For examples, see Table 1 and Figs. 1 and 2.

References

1 G. R. Rich, "Hydraulic Transients," McGraw-Hill Book Company, Inc., New York, N. Y., first edition, 1951, p. 7.

2 G. R. Rich, "Water Hammer Analysis by the LaPlace-Mellin Transformation," Trans. ASME, vol. 67, 1945, p. 366.

3 F. S. Woods, "Advanced Calculus," Ginn and Co., New York, N. Y., third edition, 1934, chapter XII, p. 275 ff.

4 F. D. Ezekiel and H. M. Paynter, "Computer Representation of Engineering Systems Involving Fluid Transients," Trans. ASME, vol. 79, 1957, pp. 1840–1850.

5 John Parmakian, "Waterhammer Analysis," Prentice-Hall, Inc., New York, N. Y., first edition, 1955, p. 13 ff.

DISCUSSION

F. T. Brown[2]

The authors' use of the digital computer has value for determining the low frequency portion of the transient response of a line. For frequencies

$$\omega > \frac{2F}{W} \text{ sec}^{-1}$$

Equations (1) and (2) may be adequately characterized by the propagation operator

$$\Gamma(s) = L\sqrt{QWs}\sqrt{1 + \frac{F}{Ws}} \approx L\sqrt{QWs}\left(1 + \frac{F}{2Ws}\right)$$

and the characteristic impedance per unit length,

$$Z_c(s) = \sqrt{\frac{W}{Q}}\sqrt{1 + \frac{F}{Ws}} \approx \sqrt{\frac{W}{Q}}\left(1 + \frac{F}{2Ws}\right),$$

so that the problem becomes straightforward and machine com-

[2] Instructor of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

putation is unnecessary.[3] However, it has been demonstrated[4,5,6] that equations (1) and (2) themselves are incorrect for frequency components

$$\omega > \frac{8F}{W}$$

because the parabolic velocity distribution is no longer valid. The pressure response 1 foot from the end of the line in the authors' example is certainly in this region, but easily may be found from results given by the writer.[6] The correct response is given below along with the pressure at the valve end of the line:

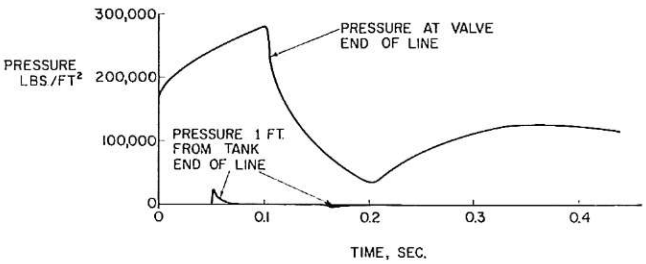


Fig. 3

The pressure 1 foot from the end of the line *does not have extremely sharp narrow peaks* as is often assumed. Further details on problems such as the computation of the pressure at the valve end of the line, where both low and high frequency components are present, will be presented by the writer in a future paper.

The writer understands that the authors have noticed the computational errors reflected in Table 1 and Fig. 1.

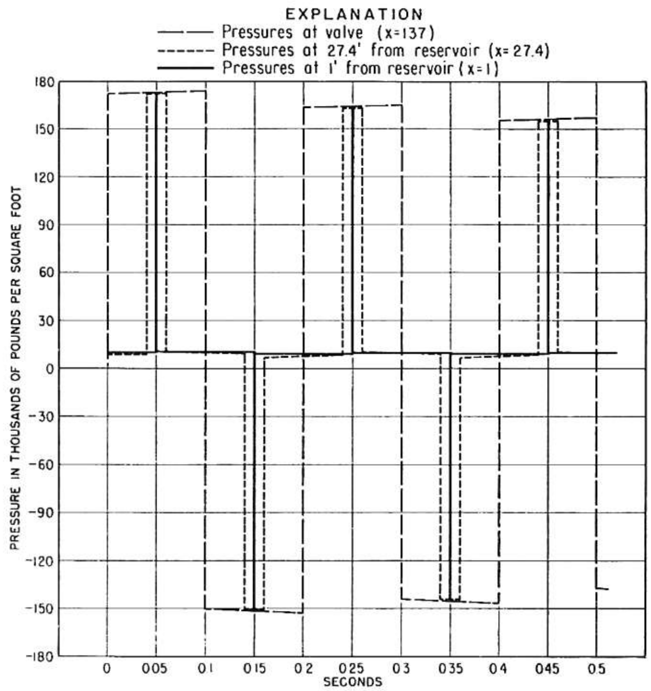
Benjamin Donsky[7]

A computer method by which the pressures in a liquid transfer line can be computed at any point and at any time after a valve closure is of value to the engineers. The authors have presented a computer method for determining pressures in a pipeline after instantaneous valve closure in which the hydraulic losses are assumed to vary linearly with velocity.

The writer, however, believes that the pressure-time history given in Fig. 2 is incorrect for the data of Table 1. Fig. 2 does not represent the pressure-time history at any point along the pipeline. The more nearly correct pressure-time history is shown in Fig. 4 for a point 1 foot from the reservoir ($x = 1$), a point about 27.4 ft from the reservoir, and at the valve. Fig. 4 was computed by methods of Reference [5] and the hydraulic losses were taken as varying linearly with velocity.

As shown in Fig. 4 the pressure at $x = 1$ will not rise until a pressure wave formed at the valve reaches point $x = 1$ (about L/a sec). This pressure will be about the same magnitude as that produced at the valve. It will remain at the same magnitude for the time that it takes the passing wave to go to the reservoir and back $\left(\frac{1}{137}\frac{2L}{a}\text{ sec}\right)$.

The pressure at the valve immediately after valve closure is about 172,200 lb/ft². This value corresponds to an initial pressure before valve closure plus the pressure rise determined by $\Delta P = \frac{aw\Delta V}{g}$ for instantaneous valve closure. The pressure at the valve increases during the time of $\frac{2L}{a}$ seconds to a value of about 175,000 lb/ft². This is due to the stopping of the flow in the line as the initial pressure wave travels up the pipeline and as the flow is stopped the losses in the line are recovered.



PRESSURES IN PIPE LINE OF FIG. I OF ASME PAPER NO. 61-WA-79

INSTANTANEOUS VALVE CLOSURE

Fig. 4

The above type of wave shape will be repeated with some damping as shown in Fig. 4, but because the losses are only about 1.6 per cent of the maximum pressure, the damping will be small. The pressure-time histories as shown in Fig. 4 are, of course, valid only up to the time a water-column separation occurs.

In the problem of Table 1, the hydraulic losses are small compared to the pressure rise. However, there are many problems in which the losses may be large. A problem was studied by the writer in which the losses were 80 per cent of the static pressure and the immediate pressure rise at the valve after an instantaneous valve closure was 100 per cent of static pressure. For the case where the losses were assumed to vary linearly with velocity the pressure rise above static was about 12 per cent less than that for the case when the losses were taken as varying with the square of the velocity. Therefore, the authors' equations may give nonconservative results when the hydraulic losses are large.

Authors' Closure

The authors thank Mr. Brown and Mr. Donsky for their penetrating discussions and additional contributions.

The advantage of the digital computer method is the fast, complete coverage of every desired point along the system for

[3] H. M. Paynter, discussion of paper by W. T. Rouleau, "Pressure Surges in Pipelines Carrying Viscous Liquids," Journal of Basic Engineering, Trans. ASME, Series D, vol. 82, 1960.

[4] A. S. Iberall, "Attenuation of Oscillatory Pressures in Instrument Lines," *Journal of Research*, National Bureau of Standards, vol. 45, July, 1950, R.P. 2115.

[5] N. B. Nichols, "The Linear Properties of Pneumatic Transmission Lines," presented at Joint Automatic Control Conference, June, 1961, Boulder, Colo., to be published by Instrument Society of America.

[6] F. T. Brown, "The Transient Response of Fluid Lines," ASME Paper No. 61—WA-143, to be published, Journal of Basic Engineering, Trans. ASME.

[7] Engineer, Technical Engineering Analysis Branch, Division of Design, Bureau of Reclamation, U. S. Department of the Interior, Denver, Colo.

every desired time interval (including the high frequency and low frequency transient responses).

When the pressure-time history for typical layout Fig. 1 was computed from the data of Table 1, it checked with Mr. Donsky's contribution of Fig. 4. The pressure-time history given in Fig. 2 was for a different point in vertical line.

Mr. Brown refers to another analytical solution for practical problems presented by Professor Rouleau, of Carnegie Institute of Technology.[3] To obtain the solution, Professor Rouleau assumed his $k^2 \ll \pi^2$. The operational solution presented by Mr. Brown assumes rigid walls (or $k \to 0$). The digital computer solution is not limited in this way, particularly for shorter, thin wall missile lines where the low frequency portion of the transient response is important.

The authors believe the specific application would determine the most advantageous method to use in any case.