Particle-in-cell simulation studies of the non-linear evolution of ultrarelativistic two-stream instabilities

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ABSTRACT
Gamma-ray bursts are associated with relativistic plasma flow and intense X-ray and soft gamma-ray emissions. We perform particle-in-cell simulations to explore the growth and saturation of waves driven by the electrostatic two-stream instability that may contribute to the thermalization of the relativistic plasma flows and to the electromagnetic emissions. We evolve self-consistently the instability driven by two charge-neutral and cool interpenetrating beams of electrons and protons that move at a relative Lorentz factor of 100. We perform three simulations with the beam density ratios of 1, 2 and 10. The simulations show that the electrostatic waves saturate by trapping the electrons of only one beam and that the saturated electrostatic wave fields spatially modulate the mean momentum of the second beam, while retaining its temperature. Cavities form in the charge density of the latter beam which, in turn, compress the electrostatic waves to higher intensities. A runaway process develops that terminates with the collapse of the waves and the development of an exponential electron high-energy tail. We bring forward evidence that this energetic tail interacts stochastically with the charge density fluctuations of the relativistic proton beam. In response, an electron momentum distribution develops that follows an inverse power law up to a spectral break at four times the beam Lorentz factor.

Key words: acceleration of particles – instabilities – waves – methods: numerical.

1 INTRODUCTION
Gamma-ray bursts (GRBs) are eruptions of cosmic ray radiation at cosmological distances. Increasing evidence shows that some of the sources are massive stars that collapse under their own gravity. The progenitor star implodes, by which a compact central object is formed, e.g., a neutron star or a black hole. A fraction of the released gravitational energy is converted into a relativistic outward flow of plasma. It is thought that this outflow is in form of a collimated jet that moves at Lorentz factors $10^2 < \Gamma < 10^3$. Here, the Lorentz factor is $\Gamma(v_b) = (1 - v_b^2/c^2)^{-0.5}$ and $v_b$ is the relative speed between the colliding jet and the ambient plasma, and $c$ is the speed of light.

Any initially inhomogeneous plasma flow speeds and densities in this jet lead to the development of instabilities. Such instabilities may lead to shocks inside the jet (internal shocks) that eventually remove these inhomogeneities (Fender, Belloni & Gallo 2004; Piran 2004). It is however not clear whether shocks can form at all. The plasma flow may also relax through plasma instabilities without the formation of a shock (Brainerd 2000; Schlickeiser 2003). The jet expands into the surrounding plasma, the stellar wind of the progenitor star, and it is slowed down by it. Again this may be by the formation of a shock at the leading edge of the jet (external shock) or by other plasma thermalization processes. A recent review of the physics of GRBs and an extensive collection of experimental observations is given by Piran (2004).

Plasma particles, that are accelerated by streaming instabilities or by shocks, are thought to be the source of the electromagnetic emissions of GRBs with X-ray and soft gamma-ray energies. This electromagnetic radiation is emitted throughout two phases that have been explained with the help of the internal/external shock scenario. The initial prompt emission is associated with particle acceleration processes at the internal shocks of the jet. These shocks form at the interface of plasmas colliding at speeds $v_b$ with $\Gamma(v_b)$ of a few. After the initial prompt emissions, the radiation is dominated by the afterglow. The afterglow has been attributed to particle acceleration at the external shock of the GRB. The external shock develops between plasmas colliding with $\Gamma(v_b) \approx 10^3$. A clear distinction between the prompt emission and the afterglow is possible, since we can not extrapolate the radiation spectrum and intensity from one emission on to the other (Piran 2004).

Insight into the processes by which the jets thermalize is provided by the rapidly fluctuating intensities of the gamma-ray emissions during the prompt phase, and by the spectrum of the

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Electromagnetic emissions, both during the prompt phase and the afterglow. The prompt emissions show a power spectrum that frequently resembles a broken power law, whereas the afterglow spectrum typically shows a single power law (Piran 2004). If the electromagnetic gamma-ray emission is due to synchrotron radiation, it is likely that the momentum distribution of the relativistic electrons also follows a power law or a broken power law. This, together with strong local magnetic fields, may produce synchrotron radiation with the observed characteristics. The synchrotron cooling takes, however, place on time-scales that do not agree with neither the rising time nor the decay time of the prompt emissions of GRBs (Piran 2004). This suggests that these characteristic times must be provided by other mechanisms. The time-scale, during which the emission intensity rises, is probably set by the electron acceleration mechanism (Piran 2004). The decay of the emission is slower than expected from synchrotron cooling. The increased decay time may be due to, for example, turbulent magnetic fields (Dermer & Humi 2001), contributions from inverse Compton scattering or thermal emissions (Ryde 2004, 2005) or from interactions between strong electrostatic waves and electrons (Schlickeiser 2003). To develop a self-consistent description of the plasma physics involved in the relaxation of the relativistic GRB flows, the key particle acceleration processes have to be further examined, and the particle distribution functions have to be identified that develop out of the acceleration.

The present computer performance allows us to extend substantially, in a fully kinetic treatment of the plasma, the initial particle-in-cell (PIC) simulations of instabilities driven by relativistic beams, such as those in Thode & Sudan (1973). Colliding $e^+e^-$ beams have been examined in three dimensions (Silva et al. 2003; Jaroschek, Lesch & Treumann 2005). Collisions between non-relativistic charge-neutral $e^+p$ beams have been examined in one dimension (Shimada & Hoshino 2000; Schmitz, Chapman & Dendy 2002; Scholer & Matsukiyo 2004; Lee, Chapman & Dendy 2005a,b) and in three dimensions (Frederiksen et al. 2004; Hededal et al. 2004). PIC simulations show, for example, that the thermalization of streaming instabilities can be the origin of power-law distributions in the electron energy spectrum (Frederiksen et al. 2004; Hededal et al. 2004; Dieckmann 2005). In addition, the interaction of electrons with high-frequency Langmuir waves, that move parallel to the magnetic field or through unmagnetized plasma, or with upper hybrid waves, that move across a magnetic field, could accelerate the electrons to ultrarelativistic energies during the required short time-scales (Dieckmann, Eliasson & Shukla 2004a; Dieckmann et al. 2005). Such streaming instabilities may also amplify the local magnetic fields to the values that are required to yield the synchrotron emissions (Weibel 1959; Schlickeiser & Shukla 2003; Dieckmann et al. 2004a; Frederiksen et al. 2004; Hededal et al. 2004). This emphasizes the need for further examining the relaxation of relativistic plasma flows with PIC simulations.

Typically the large difference in the spatio-temporal scales associated with protons and with electrons requires the usage of a reduced proton-to-electron mass ratio in PIC simulations, e.g., 16 in the works by Frederiksen et al. (2004) and Hededal et al. (2004) for moderately relativistic beams. The relativistic mass increase associated with the plasma flows in GRB settings will further expand the spatio-temporal range of the plasma processes. Even with reduced proton-to-electron mass ratios or with leptonic beams, the beam Lorentz factors in the large-scale three-dimensional (3D) simulations by Silva et al. (2003), Frederiksen et al. (2004) and Hededal et al. (2004) have been limited to about 10. Typical flow speeds at GRBs are, however, at least an order of magnitude larger.

It is possible though to achieve a full mass ratio for highly relativistic streaming plasma by choosing spatially homogeneous particle beams in one dimension, as in the work by Thode & Sudan (1973). This simplification is similar also to that in Pohl & Schlickeiser (2000) and Pohl, Lerro & Schlickeiser (2002), where electrostatic and electromagnetic streaming instabilities have been examined that develop close to the shock front of the relativistic jets ejected by active galactic nuclei (AGNs). This approach has also been pursued by Dieckmann (2005) with a self-consistent and fully relativistic and electromagnetic PIC simulation code.

In the paper by Dieckmann (2005), the one-dimensional (1D) PIC simulation box is aligned with the flow direction of a $e^+p$ beam that moves through a second $e^-p$ plasma that is at rest in the simulation box. We further examine this system in this work. Aligning the simulation box with the beam flow direction excludes wavevectors that are not parallel to the flow direction. In what follows, we refer with electrostatic modes or electrostatic waves to the waves that have wavevectors and electric field vectors that are parallel to the beam flow direction. Mixed modes and filamentation modes denote waves with oblique and perpendicular wavevectors, respectively. The linear growth rates of the mixed modes and the filamentation modes are considerably larger than those of the electrostatic modes for relativistic flow speeds and cold beams (Brainerd 2000; Bret, Firpo & Deutsch 2004). A restriction to one spatial dimension is thus not physically accurate, if one considers the wave growth in cold homogeneous plasma from thermal noise levels to non-linear saturation. The plasma conditions at GRBs prior to the wave growth are, however, not well understood. In principle, the GRB jets may be so hot that no plasma instability develops. If the plasma beams are unstable, then thermal effects, initial charge density fluctuations, plasma inhomogeneities or beam aligned magnetic fields may favour the growth of electrostatic modes over mixed and filamentation modes. The growth rate of filamentation modes is, for example, reduced if the plasma beams are hot (Silva et al. 2003) and the growth rate may scale differently with the temperature, in comparison to that of the electrostatic modes. Electrostatic modes may also develop in the current channels (Frederiksen et al. 2004; Hededal et al. 2004) that constitute the final state of the filamentation instability. The electrostatic modes may thus be important for the plasma physics of GRBs, despite their comparatively low linear growth rate.

This work expands that by Dieckmann (2005) by comparing the growth and the saturation of the electrostatic modes and the turbulence, that develops out of their collapse, for a wider range of beam density ratios, with a substantially higher simulation resolution and for a relative streaming speed $v_b/v_e$ with $\Gamma(v_b) = 100$. We model numerically the two-stream instability for the beam density ratios 1, 2 and 10. In Frederiksen et al. (2004) and Hededal et al. (2004), beam density ratios of 3 have been investigated. In Section 2, we discuss our simulation model, the initial simulation conditions and the equations that are solved by the PIC code. In Section 3, we examine the electrostatic fields evolving in the three simulations and their properties, as well as their non-linear saturation mechanism, which is electron trapping (Rosenzweig 1988; Eliasson & Shukla 2005; Luque & Schamel 2005). In Section 4, we find developing spikes in the electron phase-space distribution in all three simulations. Here we associate these spikes, that have also been reported by Dieckmann (2005), with plasma cavities (Zakharov 1972; Goldman & Newman 1993). These cavities develop due to the reaction of a cold electron beam to the turbulent electric fields associated with the hot electron beam. This cavity formation is thus probably not dependent on the exact nature of the initial instability, i.e., the orientation of the wavevector. It may even develop in response to the noise.
electric fields (Dieckmann et al. 2004b) that are advected by a hot turbulent plasma jet into a cool interstellar medium. These spikes can, however, only form if the electrons of the jet and the ambient plasma are well separated in momentum space. Beam Lorentz factors well in excess of the $3 < \Gamma(v_b) < 15$ in Silva et al. (2003), Frederiksen et al. (2004) and Hededal et al. (2004) are probably necessary. Section 5 examines the electron thermalization by strong electrostatic turbulence (Schlickeiser 2003) that is driven by the relativistically hot electrons and by charge density fluctuations of the beam ions. In response to the fluctuating electrostatic fields, the electrons develop an inverse power-law distribution with a hardness that is similar to those found in Hededal et al. (2004) and Dieckmann (2005). We discuss in Section 6 the relevance of our findings for the understanding of electron acceleration in astrophysical environments and the radiation emitted by relativistic jets, e.g., AGN and GRB jets.

2 SIMULATION MODEL AND INITIAL CONDITIONS

We consider an unmagnetized cylindrical jet that expands into the ambient plasma as in Pohl et al. (2002). This model is shown schematically in Fig. 1. The ambient plasma could correspond to the source plasma during the initial phase of the jet evolution, for example, the accretion disc plasma. It could also correspond to the stellar wind of the progenitor star during the later stages of the jet evolution. We assume here that the jet plasma and the ambient plasma have a thermal speed that is much less than the jet speed $v_b$. We model in the simulations both the ions (protons) and the electrons as mobile charges. We define $\omega_{e1}$ and $\omega_{e2}$ as the plasma frequencies of the jet electrons and ambient electrons in the respective rest frame of the species. The plasma frequencies of the jet protons and of the ambient plasma protons in their respective rest frame are $\omega_{p1}$ and $\omega_{p2}$. In what follows we refer to the jet plasma as beam 1 and to the ambient plasma as beam 2. The plasma frequency is defined as $\omega_j = (e^2 n_j/m_e \epsilon_0)^{1/2}$, where $e$, $n_j$, $m_j$, and $\epsilon_0$ are the magnitude of the elementary charge, the number density and particle mass of species $j$, the particle charge and the dielectric constant, respectively. We use a proton to electron mass ratio $m_p/m_e = 1836$. Initially the much larger proton inertia implies that the two-stream instability can be described by the cold linear dispersion relation of the electrons of both interpenetrating plasmas as discussed by Bret et al. (2004). The restriction to one spatial dimension excludes wavevectors that are not parallel to the simulation direction and we introduce a scalar wavenumber $k$. Since we align the simulation box with the beam flow direction, we exclude the filamentation modes and the mixed modes. The electromagnetic Weibel modes, that have a wavevector parallel to the beam flow direction (Bret et al. 2004), are resolved by the simulation but we exclude these from the discussion, because in all simulations the magnetic energy is at least four orders of magnitude less than the electrostatic energy. The contribution of Weibel modes to the electron dynamics, on which we focus in this work, is thus negligible. With these restrictions and in the reference frame of beam 1, the relativistic linear dispersion relation for electrostatic modes with frequency $\omega$ in a cold streaming plasma is

$$1 - \frac{\omega^2}{\omega_{e1}^2} = \frac{\omega^2}{\Gamma^2(v_b)\omega - k v_b} = 0. \quad (1)$$

Note that $\omega_{e2}$ is defined in the rest frame of beam 2. The most unstable wavenumber is $k_u \approx \omega_{e1}/v_b$, the real part of the most unstable frequency is $\omega_u \approx \omega_{e1}$, and this wave grows at the rate $\omega_j/\omega_{p1} \approx (3^2/5 \omega_{e1}/16 \omega_{p1}^2)^{1/3} / \Gamma^{2/3}(v_b)$ as discussed by Thode & Sudan (1973) for the relativistic two-stream instability and reviewed recently for the equivalent non-relativistic plasma by Lapuerta & Ahedo (2002). The PIC code is based on the virtual particle scheme (Eastwood 1991) and it solves the full set of Maxwell’s equations on a discrete grid, together with the relativistic Lorentz equation for an ensemble of computational superparticles. Each superparticle with index $i$ corresponds to a cluster of physical particles. For our chosen initial conditions, between $10^8$ and $5 \times 10^8$ physical particles are grouped to one superparticle. The scheme satisfies $\nabla \cdot \mathbf{B} = 0$ and $\nabla \times \mathbf{E} = \rho/\epsilon_0$ within round-off precision and it dynamically updates $\nabla \times \mathbf{E} = \mu_0 \mathbf{j} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$, $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$, $\frac{\partial \mathbf{p}_i}{\partial t} = q(E + \mathbf{v}_i \times \mathbf{B})$.

Since we restrict our simulations to one dimension, the spatial position $x$ is a scalar. The PIC code solves the relativistic Lorentz equation for all momentum components. We represent the spatial x direction by $4 \times 10^6$ grid cells, each with the physical length $\Delta_x = 10$ m, and we use periodic boundary conditions for our simulation box. The simulation box length is $L = 4 \times 10^5$ m $\approx 130 \times 2\pi/k_u$. Each cell is 30 per cent shorter than that in Dieckmann (2005), and $L$ is 30 per cent longer. We set the simulation time-step to $\Delta_t = 1.18 \times 10^{-8}$ s, by which we obtain the grid speed $\Delta_x/\Delta_t \approx 2.8c$. This grid speed resolves well the developing electrostatic waves and the phase-space paths of the simulation particles. In physical units, we set $\omega_{e1} = 2\pi \times 10^5$ s$^{-1}$ and $\omega_{p1} = \omega_{e1}(m_e/m_p)^{0.5}$. We perform three simulations. Simulation 1 uses $\omega_{e1} = 0.1 \omega_{e1}$. Simulation 2 uses $\omega_{e1} = 0.5 \omega_{e1}$, which is the case considered by Dieckmann (2005). This density ratio is close to that in Frederiksen et al. (2004) and Hededal et al. (2004). Simulation 3 uses $\omega_{e1} = \omega_{e1}$. We set $\omega_{e2} = \omega_{e1}(m_e/m_p)^{0.5}$. Note that we compare the plasma frequencies in the respective rest frame of the species. These simulations cover the range of densities for which the growth rate $\omega_i$ of the electrostatic wave is large enough to allow for a reasonable simulation time.


Figure 1. Our jet model: a relativistic jet expands into the ambient plasma with a Lorentz factor $\Gamma = 10^2$. The jet and the ambient medium are constituted of almost cold and unmagnetized $e^\pm$ plasmas. Close to the leading edge, the jet plasma and the ambient plasma co-exist and two-stream instabilities can develop. We place our 1D simulation box in this region, sufficiently far from the jet boundary to justify the 1D simulation with periodic boundary conditions, and we align it with the jet axis.
The densities of the blast shell of the jet and the ambient plasma differ by many orders of magnitude (Panaitescu & Meszaros 1998; Pohl & Schlickeiser 2000). However, we may find thin plasma shell fragments that can outrun the main jet. Such fragments would have a density that is much closer to that of the ambient plasma and the ratio may become similar to that in our simulations. We may also consider an interaction between the jet and the much denser stellar material of the progenitor star, for example, that of the accretion disc of the forming compact object. Hence the densities of both beams may be comparable.

In all simulations, we take a temperature of 400 eV for all plasma species, specified in the rest frames of the respective plasma species. We refer to the electron thermal speed as $v_e$ and to the proton thermal speed as $v_p$. Naturally, $v_e^2/v_p^2 = m_p/m_e$. This temperature allows us to consider the plasma as cold, since $v_b \gg v_e$, and the solution of equation (1) should approximate well the initial growth phase of the instability. This temperature yields an electron Debye length $\lambda_e = v_e/\omega_{pe} \approx 13.4 \mu m$ which is well resolved by our $\Delta x$. The Lorentz contracted Debye length of the electrons of beam 2 is less than $\Delta_x$, however the instability initially develops on scales that are much larger than $\lambda_e$ and the plasma temperature, and thus the Debye length, rapidly increase. Each wave period $\pi/v \approx 2\pi/k_\parallel$ is resolved by more than 300 grid cells. To provide an adequate resolution of the distribution functions, that develop out of the instability and the consecutive electrostatic turbulence, and to obtain a similar weight for the simulation particles that represent the electrons of both beams, we initialize the electrons and protons of beam 1 with 320 particles per cell (ppc) each. The electrons and protons of beam 2 are represented by 8000 and 1280 ppc, respectively. We thus obtain a total of $3.3 \times 10^7$ computational electrons, which exceeds the number used by Dieckmann (2005) by an order of magnitude.

3 THE LINEAR WAVE GROWTH
AND ITS SATURATION

In this section, we compare the electric fields that grow in the three simulations from noise levels to their non-linear saturation. Because of the exclusion of wavevectors that are not parallel to the beam flow direction, the dominant wave modes are electrostatic and their growth rate is given by the solution of equation (1). The electromagnetic Weibel modes are polarized orthogonally to the $x$-direction (Bret et al. 2004) but will not be considered further since they are weak.

During the growth phase of the electrostatic modes, the evolution of the wave fields should be similar in all simulations, except for the different linear growth rate. To confirm this, we analyse the electrostatic field components $E_j(x,t)$ by performing a Fourier transform over space. The number of grid cells along the $x$ direction and the duration between two consecutive field samples are $N = 4 \times 10^4$ and $\Delta T = 16 \Delta x$. In what follows, we neglect the discreteness of the time in our simulations and we refer to the time in physical units as $t = m \Delta x$, with an integer $m$. With $\Delta k = 2\pi/L$ being the minimum resolved wavenumber, the Fourier transform reads

$$ E_j(k \Delta k, t) = N^{-1} \sum_{p=1}^{N} E_j(p \Delta k, t) \exp(-2\pi i p k/N). $$

The electric field growth and possibly the saturation should be similar in the three simulations with $R = \omega_{pe}^2/\omega_{pe}^2 = 1, 2$ and 10, apart from the scaling factor $R^{1/3}$ that results from the growth rate of the most unstable wave, obtained from solving equation (1). This scaling is confirmed by Figs. 2, 3 and 4 that show the power spectra

![Figure 2](https://academic.oup.com/mnras/article-fig/367/3/1072/1040044/1075)

**Figure 2.** The power spectrum log$_{10} E_j^2(k \Delta k, t)$ for $R = 10$ in units of $V^2/m^2$. A quasi-monochromatic electrostatic mode grows at $k \approx k_\parallel$ and collapses at $t \approx 240 \times (2\pi/\omega_{pe})$. The wave growth is accompanied by broad-band electrostatic noise covering the shown $k$ interval.

![Figure 3](https://academic.oup.com/mnras/article-fig/367/3/1072/1040044/1076)

**Figure 3.** The power spectrum log$_{10} E_j^2(k \Delta k, t)$ for $R = 2$ in units of $V^2/m^2$. A quasi-monochromatic electrostatic mode grows at $k \approx k_\parallel$ and collapses at $t \approx 135 \times (2\pi/\omega_{pe})$. The wave growth is accompanied by broad-band electrostatic noise covering the shown $k$ interval.

![Figure 4](https://academic.oup.com/mnras/article-fig/367/3/1072/1040044/1077)

**Figure 4.** The power spectrum log$_{10} E_j^2(k \Delta k, t)$ for $R = 1$ in units of $V^2/m^2$. A quasi-monochromatic electrostatic mode grows at $k \approx k_\parallel$ and collapses at $t \approx 105 \times (2\pi/\omega_{pe})$. The wave growth is accompanied by broad-band electrostatic noise covering the shown $k$ interval.
where the summation is over the electrons of the respective species. \( \varepsilon \) for the protons of beam 1, and when an equilibrium has been reached, the simulation with accuracy has been lower and a significant net current with contrast to the paper by Dieckmann (2005). Here, the current carried these simulations no significant perturbation is found at in the paper by Dieckmann (2005), i.e., the results are robust against the perturbation with \( k = 0 \).

The power spectrum further shows that for all simulations the electrostatic fields have comparable strengths. This is also reflected by the total electric field energy density \( \varepsilon_E(t) = \sum_{j=1}^{R} \epsilon_j E_j^2(j \Delta \omega_j, t) / E_0 \) shown in Fig. 5. The total kinetic energy densities of the electron beam 1 and beam 2 are \( \varepsilon_{e1}(t) = m_e c^2 \sum_{j=1}^{N_{e1}} [\Gamma_j(t) - 1] \) and \( \varepsilon_{e2}(t) = m_e c^2 \sum_{j=1}^{N_{e2}} [\Gamma_j(t) - 1] \), where the summation is over the electrons of the respective species. We set \( E_0 = \varepsilon_{e1}(t = 0) \). The numbers of simulation electrons are \( N_{e1} = 1.28 \times 10^7 \) and \( N_{e2} = 3.2 \times 10^8 \). Fig. 5 confirms that the electric fields grow in accordance with the linear growth rate in the interval \( t / T_p R^{1/3} = 30 \) to \( 10^2 \) times the initial thermal energy density \( E_0 \) of the electrons of beam 1. A multidimensional PIC simulation would also show a substantial growth of the magnetic field energy density. In the case considered by Hededal et al. (2004), this energy density is a few times larger than \( \varepsilon_E \), whereas in our 1D simulations the magnetic field energy density is four orders of magnitude lower than \( \varepsilon_E \), emphasizing the importance of mixed modes and filamentation modes. Over a short time interval, the energy densities \( \varepsilon_{e1} \) and \( \varepsilon_{e2} \) are identical for \( R = 1, 2 \) and comparable for \( R = 10 \). For all considered \( R \) the electron kinetic energy increases significantly, even after the electrostatic wave has saturated. The final energy density \( \varepsilon_{e1} \) of species 1 is in all simulations between \( 4 \times 10^4 E_0 \) and \( 5 \times 10^4 E_0 \).
The subinterval is chosen such that it covers two wavelengths of the proton energy density higher by a factor of 4. As in the simulation by Dieckmann (2005), the most unstable electrostatic mode with length \( L \) is \( R \) times the simulation box and for \( R = 2 \) at the time \( t = \frac{65}{R^{1/3}} T_p \). The grey-scale shows the logarithm of the number of computational electrons normalized to the peak value in the entire box. The electron beam with \( p_u \approx 0 \) represents the bulk of the electrons of beam 1. The trapped electrons of beam 1 are accelerated at \( x/x_u \approx 0 \) and at \( x/x_u \approx 1 \). The oscillating beam with the mean momentum \( (p_u/m_u c) \approx 100 \) corresponds to the electrons of beam 2.

The electron distributions, that correspond to the electrostatic field energies in Fig. 5 at the times \( t = \frac{65}{R^{1/3}} T_p \), are shown in Fig. 9(a) for \( R = 10 \), in Fig. 10 for \( R = 2 \) and in Fig. 11 for \( R = 1 \). The subinterval is chosen such that it covers two wavelengths \( x_u \) of the most unstable electrostatic mode with \( k = k_u \), while the total box length is \( L = 130 x_u \). The offset of the subinterval is chosen such that the phase-space distribution of beam 1 bifurcates at \( x \approx 0 \). All plots show the electron distributions in the reference frame of beam 1 that is the box frame of reference. By comparing the Figs 9, 10 and 11, we find that the quasi-monochromatic electrostatic waves with \( k \approx k_u \) always saturate by trapping the electrons of beam 1. Trapping implies that electrons gyrate in the electrostatic wave potential, by which their phase-space distribution becomes multivalued as a function of space. We find bifurcations of the phase-space distributions of the electrons of beam 1 in the Figs 9(a), 10 and 11. The electron distributions of beam 2 do not bifurcate in any of these figures and the electrons are thus not trapped. Fig. 9 furthermore shows the correlation between the islands of trapped electrons (Fig. 9a) and the electrostatic field measured at the same time and place (Fig. 9b). Large positive values of the electric field are correlated with low positive momenta of the electrons of beam 2, as expected. The bulk electrons of beam 1 are overtaken by the electrostatic wave that moves at a relativistic phase speed toward positive \( x \). Electrons are accelerated toward negative momenta by the positive electric field between \( 1.6 < x/x_u < 2 \). Eventually these electrons enter an interval in which the electric field value is negative and are accelerated.
toward positive $p_x$. The wave potential can trap electrons if their kinetic energy is sufficiently low in the reference frame moving with the wave. This is the case for some electrons of beam 1. In contrast, the electrons of beam 1, that have a large kinetic energy in the wave frame, remain untrapped. Beam 1 thus bifurcates close to that zero-crossing of the electric field that corresponds to an unstable equilibrium point, e.g., at $x \approx 0, x_u$ in Fig. 9. This correlation between the electric field and the electron distribution also holds for $R = 1$ and 2.

The strong currents, that are associated with the saturated electrostatic wave at $t \approx 50R^{3/3}T_p$, will give rise to a magnetic field that is oriented perpendicularly to the current flow direction and that has a significant strength. Spatial inhomogeneities perpendicular to the beam flow direction, which are not represented by our simulation box, will thus yield magnetic field gradients. These magnetic field gradients will, in turn, exert a magnetic pressure on the current filaments, leading to their increasing spatial localization. For sufficiently weak spatial inhomogeneities though, the timescale during which the filaments form will be large compared to the simulation time in this work. Our simulations may then correctly reproduce the further non-linear evolution of the electrostatic instability.

4 DEVELOPMENT OF SPIKES IN THE ELECTRON DISTRIBUTION

At the time $t = 131T_p$, the electron phase-space distribution for $R = 2$ in Fig. 12 still shows structures on the scale of the initial wavelength $x_u$. This accounts for the electrostatic quasi-monochromatic wave in Fig. 3. The electrons of species 1 and 2 will eventually mix due to the strong broad-band fluctuations of $E_z$, for example, for $R = 2$ after $t \approx 60T_p$ (Fig. 3). The electrons of beam 1 are thus gradually filling up the phase space are however confined by the beam 2. At this stage, the electrons of beam 2 have not been scattered and they have not mixed with the electrons of beam 1. The clear separation in phase space between the scattered electrons of beam 1 and the electrons of beam 2 is preserved, even after the collapse of the electrostatic wave. Fig. 13 confirms the observation by Dieckmann (2005) that spikes form in the phase-space distribution of beam 2 that are oriented along the $p_z$ direction. These spikes form for all $R$ but we consider only the case with $R = 2$ as a placeholder.

Comparing Figs 12 and 13 may reveal the origin of these spikes. In the absence of strong electromagnetic waves, the extreme acceleration of the electrons of beam 2, for example, at $x/x_u \approx 0.45$ in Fig. 13, can only be accomplished by a strong electrostatic potential. The formation and expansion along $p_z$ of the spikes is a process in which the electron acceleration and thus the electrostatic energy density steadily increase. The Figs 12 and 13 reveal that the fluid-like electrons of beam 2 slowly develop a strong oscillation along $p_z$, which is related to electron charge density fluctuations of the order of the electron mean charge density (not shown).

These observations may be related to cavity formation in a fluid plasma, as introduced by Zakharov (1972) and expanded by Goldman & Newman (1993) to include dissipative effects. In the Zakharov model, an electrostatic wave is trapped in an ion density depletion and its ponderomotive force further deepens the ion density cavity, which in turn compresses the wave to higher energy densities.

The scattered electrons of beam 1 provide high-frequency oscillations in the frame of reference of beam 2 by which they exert a ponderomotive force. Interesting in this aspect is that $\varepsilon_R(t)$ saturates for all $R$ at about the same value (Fig. 5). This indicates that this energy density depends primarily on beam 1, since this beam is identical for all $R$. This is probably because it is the electrons of beam 1 that initially interact non-linearly with the electrostatic wave. In contrast to the work by Zakharov (1972), the relativistic electrons of beam 2 replace the ions as the plasma species with the large inertia. This process is halted in a physical plasma by dissipative processes (Goldman & Newman 1993) that lead to a collapse of the waves in the cavities (burnout).

In the PIC simulations in this work and in Dieckmann (2005), the spatial scale of the spikes is comparable to $10\Delta_x$, and this may lead to numerical diffusion. The values for $\Delta_x$ and for $R = 2$ here and in Dieckmann (2005) have been significantly different. This has not led to a quantitative change in the spike evolution, neither in the spatial width nor in the peak $p_z$ the electrons reach. This suggests that numerical artifacts, if present, are weak.

Figure 12. The electron phase-space distribution in a subinterval of the simulation box and for $R = 2$ at $t = 131T_p$. The grey-scale shows the logarithm of the number of computational electrons, normalized to the peak value in the entire box. The electrons of beam 2 form the dense beam that is the upper limit on $p_x$ accessible to the electrons. The electrons of beam 1 have been scattered further.

Figure 13. The electron phase-space distribution in a subinterval of the simulation box and for $R = 2$ at $t = 150T_p$. The grey-scale shows the logarithm of the number of computational electrons, normalized to the peak value in the entire box. The electrons of beam 2 still form a well-defined beam that is the upper limit on $p_z$ accessible to the electrons. The electrons of beam 1 have been scattered further.
that is the increase of $\varepsilon$ exponential drop, up to a cut-off at $10^7$. The width of each momentum bin is $\hbar c$. The exponential tail in the electron distribution have developed for the other hand, a pronounced maximum for $N_e(p_x)$ responding to Fig. 13 for $R = 1, 2$ with $\Gamma(v_b)\sigma_1^2 > \sigma_2^2$ show identical $N_e(p_x)$ outside the interval $0 < p_x/m_c < 150$ and a density maximum in between, as we find from Fig. 14. For $R = 1$ this maximum is flat over this interval. The curves for $N_e(p_x)$ and for $R = 1, 2$ in Fig. 14 drop by more than an order of magnitude at $p_x = 0$ and decrease faster than exponential below $p_x = 0$. The high-energy tails of $N_e(p_x)$ show an exponential drop, up to a cut-off at $p_x/m_c \approx 500$ for $R = 1$ and $p_x/m_c \approx 400$ for $R = 2$. The simulation with $R = 10$ shows, on the other hand, a pronounced maximum for $N_e(p_x)$ and exponentially decreasing momentum densities to both sides of the maximum, in the interval $0 < p_x/m_c < 150$. The momentum density for $R = 10$ rapidly drops for $p_x/m_c < -50$.

The spikes in the electron phase-space distributions and the exponential tail in the electron distribution have developed for the simulations with $R = 1, 2$ in the time interval during which $\varepsilon_x(t)$ has increased in value beyond its initial saturation value in Fig. 5 and while the quasi-monochromatic wave has been present, that is, between $80 T_p < t < 150 T_p$ for $R = 2$. The spikes in the simulation with $R = 10$ (not shown) are still expanding along $p_x$ at the simulation’s end. We may thus associate the oscillation in $\varepsilon_x(t)$, that is the increase of $\varepsilon_x(t)$ beyond its initial saturation value at $t \approx 55 T_p R^{1/3}$, for all $R$ with the compressed electrostatic wave potentials during the cavity formation.

The simulations suggest that the cavity formation has developed due to the reaction of the cool electrons of beam 2 to the electrostatic waves carried by the beams of electron 1. The mixed modes and the filamentation modes driven by the two-stream instability or simply thermal noise of a hot electron component may also trigger this cavitation. Cavitation may thus be important for a much wider range of plasma conditions than assumed in this work. This cavitation will accelerate electrons to energies beyond the proton beam speed and increase the proton charge density fluctuation levels and thus the possibility for collisionless interaction between the electrons and the beam protons.

The initial quasi-monochromatic electrostatic waves have just collapsed in all three simulations at the times $t = 120 R^{1/3} T_p$, corresponding to Fig. 13 for $R = 2$. It is informative to examine the electron momentum distributions $N_e(p_x) = N_e^1(p_x) + N_e^2(p_x)$ for the individual $R$ after this collapse. The electrons in the simulations for $R = 1, 2$ with $\Gamma(v_b)\sigma_1^2 > \sigma_2^2$ show identical $N_e(p_x)$ outside the interval $0 < p_x/m_c < 150$ and a density maximum in between, as we find from Fig. 14. For $R = 1$ this maximum is flat over this interval. The curves for $N_e(p_x)$ and for $R = 1, 2$ in Fig. 14 drop by more than an order of magnitude at $p_x = 0$ and decrease faster than exponential below $p_x = 0$. The high-energy tails of $N_e(p_x)$ show an exponential drop, up to a cut-off at $p_x/m_c \approx 500$ for $R = 1$ and $p_x/m_c \approx 400$ for $R = 2$. The simulation with $R = 10$ shows, on the other hand, a pronounced maximum for $N_e(p_x)$ and exponentially decreasing momentum densities to both sides of the maximum, in the interval $0 < p_x/m_c < 150$. The momentum density for $R = 10$ rapidly drops for $p_x/m_c < -50$.

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5 DEVELOPMENT OF A POWER-LAW DISTRIBUTION

From Dieckmann (2005) we expect that the exponential tail of $N_e(p_x)$ for the case $R = 2$ will further develop into a final distribution that resembles an inverse power law with a spectral break. This is confirmed by the results from the simulations with $R = 1, 2$ as Fig. 15 shows. The momentum distributions for the simulations with $R = 1, 2$ decrease slower than exponential for Lorentz factors between $50 < p_x/m_c < 400$. Both curve can be well approximated by a power law $N_e(p_x) \sim p_x^{-2.5}$. At $p_x \approx 400 m_c$, the inverse power law undergoes a break. The dynamical range in $p_x$ is not sufficient to identify $N_e(p_x)$ above $p_x \approx 400 m_c$. Fig. 15 suggests that $N_e(p_x)$ may either be a steeper inverse power-law drop or an exponential drop.

The high-energy tail for the simulation with $R = 10$ in Fig. 15 apparently drops exponentially for $\Gamma(v_b)/2 < p_x/m_c < 3\Gamma(v_b)$ and less rapidly for $p_x/m_c > 3\Gamma(v_b)$. Comparing the curves for $R = 10$ in the Figs 14 and 15 indicates that the momentum density at large positive $\Gamma$ is steadily increasing with time and that it converges to an exponential tail. The formation of the spikes along $p_x$ has not been finished at the simulation’s end, for the simulation with $R = 10$. This suggests that the final high-energy tail of $N_e(p_x)$, resulting from the spike formation, decreases exponentially.

To emphasize the power-law drop of $N_e(p_x)$ at $p_x/m_c \approx \Gamma(v_b)$, we show $N_e(p_x)$ in Fig. 16 for the simulation with $R = 2$ at the time $t = 263 T_p$. We find that the electron momentum distribution follows a power law $N_e(p_x) \sim p_x^{-2.7}$ in the interval $90 < p_x/m_c < 400$, that is, 310 momentum bins. Interestingly, this hardness is identical to that found by Hededal et al. (2004) in 3D PIC simulations of $e^-p$ plasmas that collide at relativistic speeds. Here and in Hededal et al. (2004), the power-law distribution is found for momenta $p_x/m_c \approx \Gamma(v_b)$. In the simulation by Hededal et al. (2004), the beam Lorentz factor has been $\Gamma(v_b) = 15$ and the beam density ratio has been 3. The power-law drop $N_e(p_x) \sim p_x^{-1.3}$ close to $p_x/m_c \approx \Gamma(v_b)$ reported by Dieckmann (2005) is due to the logarithmic binning of the simulation particles which has been necessary to reduce the statistical fluctuations of $N_e(p_x)$. There, the momentum density has not been normalized to the varying bin size which overestimates $N_e(p_x)$.
at large \(p_e\). With uniform momentum bins, Dieckmann (2005) would have found \(N_e(p_e) \sim p_e^{-2.5}\). This indicates that the power-law index is not strongly dependent on the plasma resolution, the simulation box size and the grid resolution.

All electron momentum distributions in Fig. 15 show a maximum electron number density at \(p_e/m_e c \approx 50\) and an exponential drop at negative and decreasing values of \(p_e\). The identical exponential tails for \(p_e < 0\) and for all \(R\) suggest that the slope of this tail is determined exclusively by the electron density of the jet plasma, which has been identical for all values of \(R\). Remarkable is here that the electrons for \(R = 1, 2\) reach \(p_e/m_e c \approx -150\). Transforming this peak value into the reference frame of the ambient plasma that moves at \(\Gamma(v_b) = 100\), gives an ultrarelativistic peak momentum of \(p_e/m_e c \approx -3 \times 10^4\) for the electrons.

The high-energy tails of the electron momentum distributions in the simulations with \(R = 1, 2\) have further developed after the wave collapse at \(t \approx 110 T_p R^{1/3}\), that is, in the absence of a coherent monochromatic wave. They have changed from being exponential to resembling a power law above \(p_e/m_e c \approx \Gamma(v_b) / 2\) that ranges up to a spectral break at about \(p_e/m_e c = 4 \Gamma(v_b)\). At the same time, the tail has expanded to higher momenta, i.e., the electrons have gained energy, also seen from the evolution of \(\epsilon_{e1}(t)\) in the Figs 6, 7 and 8 for the electrons of beam 1 and in Dieckmann (2005) also for the electrons of beam 2, while the energy densities \(\epsilon_{e2}\) have not decreased such as to account for the gain in electron energy. It is thus likely that the electrons have gained energy at the expense of the protons of beam 2, which is confirmed by Fig. 17. The kinetic energy density of this beam is defined as \(\epsilon_{p2}(t) = m_p c^2 \sum_{j=1}^{N_p} \Gamma_j(t) - 1\), with \(N_p = 4N_{p1}\), and \(E_{p1} = \epsilon_{p2}(t = 0)\).

Initially, the protons of beam 2 gain energy. As shown in Dieckmann (2005), \(\epsilon_{p2}\) increases at the expense of \(\epsilon_{e2}\). It is during the times when \(\epsilon_{e2}\) has increased beyond its initial saturation level in Fig. 5, when the protons of beam 2 and also of beam 1 are accelerated. Fig. 17 shows that the protons of beam 2 in the simulations with \(R = 1, 2\) gain considerably more kinetic energy than those for \(R = 10\). Note that the normalization constant \(E_{p1}\) for \(R = 10\) in Fig. 17 is less than that for \(R = 1, 2\).

During the evolution of the high-momentum electron tails for \(R = 1, 2\) from an exponential drop at \(t \approx 120 T_p R^{1/3}\) to an inverse power-law tail with a break at the simulation’s end, the protons of beam 2 lose kinetic energy, as seen from Fig. 17. In the absence of coherent waves, this energy transfer is likely to take place via a stochastic interaction between the electrons and the charge density fluctuations of beam 2. Indeed, these broad-band charge density fluctuations reach a significant amplitude, as demonstrated by the distribution in \(\omega_0, k\) space of \(E_n(m \Delta_{se}, t)\) sampled in the time interval \(150 T_p < t < 180 T_p\) over the full simulation box for \(R = 2\). This spectral distribution is shown in Fig. 18. The fluctuations have amplitudes of the order of 100 V/m or, with \(\Delta_{se} \approx 10 \text{ m kV potentials on a grid cell scale. The high-frequency oscillations with } \omega_0 < 0 \text{ are Langmuir waves in relativistic plasma. The high-frequency oscillations with } \omega_0 > 0 \text{ have phase speeds } \omega_0/k \approx c \text{ and are a mixture of Langmuir waves originating from charge density fluctuations of the electrons, while the fluctuations with } \omega_0 > 0 \text{ are dominated by the proton charge density fluctuations of beam 2. The electric field perturbations with } \omega_0 \approx 0 \text{ are due to proton charge density fluctuations of beam 1.}
waves and proton charge density fluctuations of beam 2. The relativistic electron mass increase and the mixing of the electrons has led to a change in the plasma frequency, which corresponds to the high-frequency wave with \( \omega_{0}/\omega_{e1} \approx 0.3 \) at \( k = 0 \). Fig. 18 further shows waves with \( \omega \approx 0 \). These waves have their peak power at \( -1 < k/k_{0} < 1 \) and two sidelobes are found at \( |k|/k_{0} \approx 2 \). The low phase speed of these perturbations indicates their connection to charge density fluctuations of beam 1. This beam has gained energy during the growth and saturation of the electrostatic mode, as the Figs 6, 7 and 8 show. The sidelobes are probably due to wave–wave coupling. Backscattering of the initial Langmuir wave with \( \omega \approx \omega_{e1} \) and \( k \approx k_{0} \) could yield a second Langmuir wave with \( \omega \approx \omega_{e1} \) and \( k \approx -k_{0} \) and a low frequency wave with \( k \approx 2 k_{0} \) and \( |\omega| \ll \omega_{e1} \).

Since the perturbation of the protons of beam 2 is much less for \( R = 10 \) compared to \( R = 1, 2 \), the charge density fluctuations and thus the electrostatic fields are also weaker. This implies that the electron momentum distribution for \( R = 10 \) would need much longer to establish an equilibrium with the electrostatic fields and we do not follow this simulation further in this work.

6 DISCUSSION

In this paper, we have expanded the initial work by Dieckmann (2005) that has examined the linear and non-linear development of the two-stream instability for a relative streaming speed \( v_{0} \) with \( \Gamma(v_{0}) = 100 \). The numerical simulations employed a 1D relativistic and electromagnetic PIC code and the simulation box has been aligned with the beam flow direction. This excludes the important mixed modes and filamentation modes (Bret et al. 2004) that are likely to dominate the beam relaxation. 1D simulations allow us, however, to examine the plasma dynamics of simple systems at a high simulation resolution. By this, we can identify relevant plasma processes and find their signatures, e.g., characteristic phase-space structures. Looking for such signatures in the higher-dimensional data produced by 3D PIC simulations, can then facilitate the interpretation of these much more complex data sets.

The purely electrostatic streaming instability examined by Dieckmann (2005) and in this work has developed between two almost cold charge-neutral plasma beams, each consisting of electrons and protons. The focus in the work by Dieckmann (2005) has been on the electron momentum distribution results from the wave–particle interactions. There only a ratio of 2 between the jet plasma density and the density of the ambient plasma has been investigated. In reality, the jet density is much larger than that of the ambient medium, but thin and fast jet components may outrun the bulk plasma of the jet. These plasma fragments may establish a density ratio relative to the ambient plasma that is closer to unity. The key findings of Dieckmann (2005) have been the electron distribution of the ambient plasma develops spikes in phase space that are oriented along \( p_{x} \) and that the high-energy tail at the simulation’s end could be approximated by an inverse power law with a spectral break at high energies. No explanation has, however, been given for the mechanisms behind the formation of the spikes and the inverse power-law distribution.

Here, we have increased significantly the simulation resolution compared to that in Dieckmann (2005) and we have modelled the electrostatic two-stream instability for ratios \( R \), between the jet plasma density and the density of the ambient plasma, of 1, 2 and 10. The aim has been to identify the saturation mechanism of the electrostatic waves for these beam density ratios, the mechanisms behind the formation of the density filaments (spikes) along the \( p_{x} \)-axis and the development of the inverse power law in the electron momentum distribution with its spectral break.

We have found that the initial quasi-monochromatic electrostatic wave, that is driven by the two-stream instability, saturates by the trapping of electrons. Electron trapping by relativistic waves has been discussed by Rosenzweig (1988). As in Dieckmann (2005), initially only the electrons of the jet plasma have been trapped. The electrons of the ambient plasma have retained their initial low thermal speed and could be considered as a cold fluid-like electron beam during most of the simulation duration. This clear separation in momentum space of the jet electrons and the electrons of the ambient plasma, has resulted in two electron species that differ in their relativistic mass. The simulation data have suggested that the spikes along the \( p_{x} \) direction could develop in response to a process that is similar to cavity formation (Zakharov 1972) in electron–ion plasma.

The electrostatic wave energy, after the saturation of the initial monochromatic electrostatic wave, that has been similar for all beam density ratios, suggests that it is set by the electrons of beam 1. This is because these electrons had the same density for all \( R \). The turbulent electrostatic fields due to these electrons would result in electrostatic fluctuations with a high energy density. In the reference frame of the ambient plasma (beam 2), these fluctuations could be considered as a high-frequency oscillation that exerts a ponderomotive force on the fluid-like electrons of the ambient plasma. This ponderomotive force, in turn, results in the formation of cavities in the electron charge density of the ambient plasma. The electrostatic wave fields are further compressed in this cavity (Zakharov 1972). As for electron–ion plasmas, this process has reinforced itself to form localized electron momentum oscillations to which we refer to as spikes. Eventually this process ceases because the electrostatic wave collapses. The simulations suggest that the electron momentum distributions, that result from this cavitation, develop an exponentially decreasing high-energy tail.

We have further found evidence in this paper that the strong electrostatic perturbations have modulated the protons of the ambient plasma, probably the only source of free energy left after the cavity burnout. The, probably, stochastic interaction between the electrons and the (electrostatic) charge density fluctuations of the proton beam, would allow for an energy transfer between electrons and the beam protons. This energy exchange would be strongest for electrons moving at the Lorentz factor \( \Gamma \approx \Gamma(v_{0}) \). This could correspond to the momentum space interval \( 0.9 < p_{x}/m_{e}c < 4 \Gamma(v_{0}) \), in which the electrons have developed the inverse power-law distribution with the slope \( N_{e}(p_{x}) \sim p_{x}^{-2.7} \). The break in the electron momentum distribution at \( p_{x}/m_{e}c \approx 4 \Gamma(v_{0}) \) may thus indicate the maximum momentum at which electrons can strongly interact with the proton beam. Note that the hardness of the inverse power law found in this work is comparable to the hardness found by Dieckmann (2005). The logarithmic binning in Dieckmann (2005), without a normalization to the momentum width of the bins, has overemphasized the momentum density at large \( p_{x} \) and a power law \( N_{e}(p_{x}) \sim p_{x}^{-1.3} \) has been found. The break energy has been the same here and in Dieckmann (2005). Above this spectral break, the electron momentum distribution may be exponentially decreasing.

The protons of the ambient plasma, for the simulations with \( R = 1 \) and 2, loose energy to the electrons during the late simulation times, when the power law builds up. In the simulation with \( R = 10 \), that did not show a loss of proton beam energy, the spikes could not be fully developed during the simulation run time and the interaction between the protons of the ambient plasma and the electrons remained weak. The high-energy tail has remained exponential like.
We may thus conclude that the interaction between all electrons and the protons of the ambient plasma is the origin of the inverse power-law distribution and that this, probably stochastic, interaction together with the spike formation also accelerates electrons to peak momenta of up to \( p_e/m_e c \approx 10 \Gamma(v_b) \) in the jet frame of reference. This value is in line with the peak momentum found by Dieckmann (2005). The peak momentum the electrons have reached has remained unchanged, despite an increase of the electron numbers by an order of magnitude relative to that in Dieckmann (2005). This is evidence for a physically, rather than numerically, motivated cut-off of the electron distribution at this peak momentum.

The simulations in this work have further shown that the electron momentum distributions at the end times of the simulations decrease exponentially for \( p_e < 0 \). The distributions have been identical for all \( R \), implying that only the jet density is important in this context. Finally, we have found that the peak kinetic energy which the jet electrons reach, in the frame of reference of the ambient plasma, is about \( 3 \times 10^7 m_e c^2 \) or 15 GeV. The reference frame of the ambient plasma is, for GRBs, moving at Lorentz factors of only a few relative to the reference frame of the Earth. The two-stream instability would thus yield a significant number of ultraenergetic electrons, which may contribute to the energetic radiation emitted by GRBs, provided that the electrostatic modes can develop. Future work has to examine the electron acceleration efficiency of mixed and filamentation modes.

An interesting aspect of the findings in this work has been that spikes form in response to energetic electrostatic fields that interact with a cool plasma. This scenario could develop even if the GRB ejecta are so hot that no streaming instability can develop. Provided that the GRB plasma and the plasma of the ambient medium are well-separated in momentum space, the large electrostatic fields due to thermal noise (Dieckmann et al. 2004b) could trigger cavitational processes in the presumably much cooler ambient plasma. If, for example, the ambient medium is the stellar wind of the progenitor star, its temperature may be even less than the 400 eV used here, e.g., a few eV in the case of the solar wind. Cavitation is facilitated by low plasma thermal energies.

The stochastic interaction between electrons and the charge density fluctuations of a relativistically moving proton beam may also be important for more generally valid plasma conditions than those modelled here. An interesting aspect is that the large-scale 3D simulations by Hededal et al. (2004) have found a power law with the same hardness as the one discussed here, albeit for a beam flow speed with \( \Gamma(v_b) = 15 \) and a beam density ratio of 3. The simulation in Frederiksen et al. (2004) with a similar setup but with a lower beam speed has shown significant proton heating which results in large amplitudes of the electrostatic noise. Stochastic interactions between the beam protons and the electrons could thus be an important mechanism for the development of a power-law momentum distribution, also in higher dimensions.

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