

extrapolated for smaller deformations. The behaviors of the whole cell and ghost were seemingly so similar that it is hard to imagine any structural difference.

The current trend in red cell mechanics is to consider the cell as simply a membranous structure. This research supports that trend. It indicates that further attention would be more usefully directed toward understanding the many complex membrane forces, rather than relying on an internal network to explain the shape of the cell.

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DISCUSSION

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The authors have introduced a very interesting experimental technique that appears to be a potentially useful means for the controlled testing of red blood cells to obtain quantitative data on deformability. The purpose of this discussion is simply to provide an idealized model for the force-deformation characteristics of the folded cell wedged in the tapered glass tube as shown in Fig. 1(c).

The primary forces acting on the cell and responsible for the deformed shape are assumed to be a set of normal resultant forces F_n at the walls, a set of friction forces μF_n , and a net axial load due to the fluid pressure drop across the cell. These forces will be assumed to be coplanar to simplify the discussion. For a total tube taper angle of α , force equilibrium in the axial direction requires that

$$\Delta F_p = 2F_n \sin \frac{\alpha}{2} + 2\mu F_n \cos \frac{\alpha}{2} \quad (1)$$

where ΔF_p is the net axial load due to the fluid pressure drop. The value of ΔF_p may be estimated by assuming an inviscid, incompressible flow between the reservoir and the exit to the tube and by application of the Bernoulli theorem to the top of the reservoir, just upstream and just downstream of the folded cell and at the tube exit. In addition, the equation of continuity is required throughout the system. Subject to these assumptions, the resulting pressure drop across the cell is

$$\Delta p = \rho g z \left[\frac{1}{\left(1 + \frac{2l}{d_e} \tan \frac{\alpha}{2}\right)^4} - \frac{1}{\left(1 + \frac{2(l+l_c)}{d_e} \tan \frac{\alpha}{2}\right)^4} \right] \quad (2)$$

where l is the distance from the trailing edge of the cell to the exit of the tube, d_e is the exit diameter, l_c is the deformed length of the cell in the axial direction, and $\rho g z$ is the hydrostatic pressure in the reservoir. If f is the fraction of tube area occupied by the folded cell of diameter d_f , then

$$\Delta F_p = f \Delta p \frac{\pi}{4} d_f^2. \quad (3)$$

The geometrical parameters in equation (2) may be estimated from Fig. 4 which leads to the result

$$\Delta p \cong \frac{8l_c}{d_e} \rho g z \tan \frac{\alpha}{2} \quad (4)$$

$$\cong 0.4 \rho g z.$$

If the tube becomes totally plugged by the cell, the coefficient of equation (4) is simply 1.0. Substitution of equation (4) into equation (3) yields

$$\Delta F_p = \frac{2\pi d_f^2 l_c}{d_e} f \rho g z \tan \frac{\alpha}{2} \quad (5)$$

and substitution of equation (5) into (1) gives

$$\frac{2\pi d_f^2 l_c}{d_e} f \rho g z \tan \frac{\alpha}{2} = 2F_n \left(\sin \frac{\alpha}{2} + \mu \cos \frac{\alpha}{2} \right) \quad (6)$$

which completes the static analysis of equilibrium.

To investigate the deformation of the cell, the force-deformation relation is required for large deformations in the folded shape. We assume a linear relation given by

$$\Delta d = CF_n \quad (7)$$

where $\Delta d = d_i - d_f$ is the reduction in diameter, d_i is the initial cell diameter, C is an elastic compliance, and F_n is the normal force component. Substituting equation (7) into (6) yields

$$\frac{2\pi d_f^2 l_c}{d_e} f \rho g z \tan \frac{\alpha}{2} = \frac{2(d_i - d_f)}{C} \left(\sin \frac{\alpha}{2} + \mu \cos \frac{\alpha}{2} \right) \quad (8)$$

or

$$\frac{2\pi l_c}{d_e} f \rho g z \left[\tan \frac{\alpha}{2} \right] d_f^2 + \frac{2}{C} \left[\sin \frac{\alpha}{2} + \mu \cos \frac{\alpha}{2} \right] d_f - \frac{2}{C} \left[\sin \frac{\alpha}{2} + \mu \cos \frac{\alpha}{2} \right] d_i = 0 \quad (9)$$

which is a quadratic expression for d_f . For small taper angles, equation (9) may be simplified and solved for d_f . The positive root of the simplified expression is

$$d_f = \frac{(\alpha + 2\mu) d_e}{2\pi \alpha C f \rho g z l_c} \left[\sqrt{1 + \frac{4\pi \alpha C f \rho g z l_c d_i}{(\alpha + 2\mu) d_e}} - 1 \right] \quad (10)$$

For a given tube the quantities α , d_e , and $\rho g z$ are fixed by geometry or are held constant. Therefore, as equation (10)

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indicates, any variation in measured values of d_f between runs for a given cell in a given tube is presumably caused by variations in μ , the coefficient of friction between the cell and the tube. This is true if f , C , and l_c are assumed constant for a given cell. In addition, when two different cells are compared using a fixed tube configuration, the measured data would also contain differences due to the initial diameters, d_i , as well as any potential differences in C which is a more direct measure of the deformability of a cell than measurement of d_f alone.

We conclude, therefore, that to the extent that this highly simplified model is appropriate to the experiment presented by the authors, that an independent control of initial cell diameter, preferably by direct measurement, is desirable for any quantitative evaluation of cell deformability.

Authors' Closure

Professor Adams has presented an interesting idealized model for the force-deformation characteristics of the red cell as tested in our experiment. We feel that his model would be potentially useful if the many independent variables such as tube diameter, initial cell diameter and shape, folded cell diameter and shape, coefficient of friction between cell and wall, and the many uncertainties in cellular material properties; could be reasonably measured or estimated. Unfortunately, the inherent limitations of microscopic experimentation place all of these parameters well beyond the realm of even reasonable estimation. For this reason we feel that the idealized model is only that and lends little to today's understanding of the structure of the red cell.