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Fitting depth–discharge relationships in rivers with floodplains

Asgeir Petersen-Øverleir

ABSTRACT

A statistical method based on a simple uniform flow depth–discharge model for a two-stage main channel-floodplain river section has been developed and tested for data from four hydrometric gauging stations. The depth–discharge rating curve fitting procedure is formulated as a piecewise regression problem. A simulated annealing algorithm is used to obtain the least-squares rating curve parameters, including the main channel-floodplain change-point. Uncertainty of all parameter and discharge estimates is approximated by bootstrap techniques.

The application of the methodology to field data showed an accurate agreement between the measured and the estimated depth–discharge relationships. A good agreement between the observed and estimated main channel-floodplain change-points was also achieved. Some difficulties, caused by factors such as unequal measurement error variance, infinite parameter estimates and the presence of more than two depth–discharge change points, were apparent; these are discussed.

Key words | change point, compound channel, depth–discharge relationships, piecewise regression, simulated annealing, steady uniform flow

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NOMENCLATURE

A	Cross-sectional area
B	Channel surface width
b_1	Main channel surface width
b_2	Floodplain channel surface width
C	General symbol for a function of statistical parameters
$D-1$	Number of segments in a piecewise regression
d	Index
e	Regression residual
F	Nominal friction coefficient in the generalized uniform flow equation
f	General symbol for a regression function
g	Residual sum of squares function, or function to be optimized in simulated annealing
H	The Heaviside step function
h	Flow depth
i	Index
j	Index
k	Coefficient of proportion between the left and right floodplain widths
l	Index
L	Number of bootstrap simulations
$m_p(\cdot)$	Linear least-squares solution for the parameter p given the parameters in $(\cdot)$.
N	Exponent of the hydraulic radius in the generalized uniform flow equation
n	Number of available stage-discharge measurements
P	General symbol for parameters in f
P_0	Vector of the regression parameters $(N_1, N_2, \theta_1, \theta_2, \lambda, \omega)$.
Q	Discharge
q	Log-discharge
R	Hydraulic radius

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S_f	Friction slope
T	Temperature in the simulating annealing algorithm
V	Mean velocity over cross-section
$W_{1:2}$	Regression basis functions
x	General symbol for an independent regression variable
y	General symbol for a dependent regression variable
β	$\log(\lambda)$
ε	Stochastic measurement error
η	Exponent of the friction slope in the generalized uniform flow equation
θ	$\log(FS_f^{\eta}b)$
λ	b_2/b_1
σ	Standard deviation of ε
ω	Flow depth at the main channel-floodplain intersection
$\hat{}$	Placed above parameters that are estimated using the method of least squares

INTRODUCTION

A central problem in hydrological data production is the transformation of recorded stage time-series to discharge time-series, where stage is the water surface height in relation to a local datum. This is usually achieved by means of an estimated stage-discharge rating curve. The rating curve can be developed using indirect methods which are intrinsically deterministic and based on open-channel hydraulics. In its simplest form, one applies a uniform flow model (typically the Manning or the Chézy equation) to the physical characteristics of the river channel. More elaborate approaches are numerically-driven and apply the governing equations for gradually varied flow, or even the full Saint Venant equations.

Most hydrometric offices, however, prefer direct methods for setting up rating curves, i.e. measuring the discharge at different stages using current meter, dilution or hydro-acoustic methods. The pairs of stage-discharge measurements are used to build up a scatter of points describing the true rating curve for the channel section containing the gauging station. The standard approach is to assume a one-to-one relationship between stage and discharge and fit a parameterized and smooth curve to the measurements, something which provides an easy and efficient way of transforming archived stage data to discharge in a computerized hydrometric database. Typically, one uses a power-law rating curve formula and objective statistical regression methods for curve fitting (Venetis 1970; ISO 1998; Moyeed & Clarke 2005; Reitan & Petersen-Øverleir 2007). Unfortunately, this procedure is not always straightforward.

Gauging stations with a long reach acting as the hydraulic control are often encountered in natural rivers, a setting addressed in this study. This type of channel control often consists of a main channel flanked by floodplain on one or both sides. As described later in this paper, this situation involves a compound stage-discharge relationship, such that the data need to be divided into two parts to be used separately in rating curve fitting. The change point is traditionally decided manually, often without detailed information about the physical and geometrical properties of the channel control.

For example, Norwegian hydrometric practice for gauging stations, as in many other countries, does not include routine sampling and analysis of channel and floodplain topography and bed roughness characteristics. Even if some information is available from field reconnaissance, it is difficult to pinpoint the exact location of the change point for the following two reasons. Locally, the flow-regime transition from the main channel to the floodplains might be gradual due to a gradual change in channel geometry and hydraulic resistance. Secondly, a natural channel reach is seldom prismatic and so the flow tops the main channel banks by degrees along the effective channel control, causing the stage-discharge relationship at the gauging station to gradually merge into one of the two states even if the local physical changes are abrupt. The determination of the change point is therefore subject to uncertainty, and setting it manually will yield a subjective result. In addition, assigning uncertainty to the estimated rating curve parameters and calculated discharges will not be possible, at least in a sense that is statistically consistent.

To the knowledge of the author, there exist only two studies on how to statistically fit compound stage-discharge relationships to data: Zarzer (1987) and Petersen-Øverleir & Reitan (2005). Both studies proposed power-law rating curve models and segmented least-squares methods for handling the problem. However, their approaches involve several weaknesses. Firstly, the residual sum of squares functions used take values in a three- or higher dimensional space and numerical minimization methods had to be invoked to obtain the parameter estimates.

In that respect, both studies used Newton-based local minimization schemes. This, as clearly stated in the most recent of the two studies, is an unreliable technique due to the presence of compatible local minima and non-differentiable areas in the corresponding leas-squares surfaces. Furthermore, due to the complexity of the least-squares surfaces, standard asymptotic maximum likelihood theory, which was used for inference in Petersen-Øverleir & Reitan (2005), might produce grossly misleading uncertainty measures when the likelihood surface exhibits multimodality (Mäkeläinen *et al.* 1981).

An additional weakness is found in Zarzer's paper: an additive discharge measurement error was assumed. Clearly this assumption does not hold (Sorooshian & Dracup 1980; Petersen-Øverleir 2004) and will lead to severe unequal measurement error variance (heteroscedasticity) in the regression. This is in general an unwanted situation in regression since significant heteroscedasticity has a tendency to alter inference. Also, particularly for piecewise regression, unequal variance might make the least-squares estimator give change point estimates that account for heteroscedasticity rather than for shifts in the hydraulics.

This study provides a simpler and easier method than the two above-mentioned studies. The approach is hydraulics motivated, in the sense that the depth–discharge model is an extended version of the Manning's equation. Although it is based on several simplifying assumptions, it will be shown to perform adequately in natural two-stage rivers. The depth–discharge model is fitted to measurements using the method of least squares. Rating curve parameter estimates, including the main channel-floodplain change point, can be readily obtained by a grid search, though this study applies a simulated annealing scheme for this purpose. Uncertainty in a discharge estimated by the fitted depth–discharge relationship is approximated using the bootstrap method. The main goal of this paper is to provide an automated method that requires no data beyond the available depth–discharge measurements for setting up rating curves in two-stage river sections, and to assess its applicability using several real-life datasets. A subsidiary purpose is to illustrate that the combination of hydraulics, hydrometry and computational statistics can provide a tool for objective estimation and consistent uncertainty analysis of an open-channel flow problem.

Note that this study assumes that the cease-to-flow stage is identifiable and known, i.e. the stage values are the same as the flow depths. The term stage will therefore be synonymous with flow depth.

BACKGROUND

Hydraulics of two-stage channels

Deterministic discharge assessment in compound channels, where the calculation of the total hydraulic resistance is the chief problem, have traditionally been done in two ways: the single channel and separate channel methods. The former regards the entire wetted perimeter as one channel of one roughness value; the latter divides the cross-section into main channel and floodplain zones, which in turn are treated separately.

Sellin (1964) identified the vortex structures at the interface of the main channel and the floodplain channels. These interactions are present because the water velocity of the main channel is, in general, greater than on the floodplains. The differences give rise to a zone of mixing between the two channel parts. The transfer of momentum occurring in this zone produces turbulent shear stress that results in an additional flow resistance. It is well known that the traditional approaches, which do not take into account this phenomenon, will either over- or underestimate the discharge capacity significantly. Since the work of Sellin, a large number of one-dimensional (1D) methods that account for the main channel-floodplain interference have been proposed for calculating the discharge capacity of a river channel (e.g. Myers 1978; Wormleaton *et al.* 1982; Knight & Demetriu 1983; Prinos & Townsend 1984; Ackers 1992, 1993; Bousmar & Zech 1999). Two-dimensional models have been explored too (e.g. Shiono & Knight 1991; Ervine *et al.* 2000; Bousmar & Zech 2004). Studies of discharge prediction in meandering compound channels are also encountered in the literature (e.g. Toebes & Sooky 1967; Ervine *et al.* 1993; Shiono *et al.* 1999; Liu & James 2004).

In some river channels, the transition from the main channel to floodplain is geometrically smooth. The floodplain is often more vegetated than the main channel, and the channel section will therefore have roughness varying along the wetted perimeter since it is partially vegetated. Some studies (e.g. Naot *et al.* 1996; Helimö 2004) have shown that a shear layer also exists in channels of varying stream bank vegetation. Hence, compound stage-discharge relationships might also be encountered in single-shaped channels.

Generalized uniform flow equation

The methods referred to in the previous sections are based on mechanistic models, and are intended to be applied in rivers without objective statistical calibration. As several stage-discharge measurements are typically available at a hydrometric gauging station, objective model calibration ought to be a high priority. However, using a compound channel flow model based on rigorous and accurate physics for this purpose would generally be a very difficult task, mainly because information beyond the stage-discharge measurements and recorded stage is not commonly available at hydrometric gauging stations. This fact perhaps explains why most of the information on compound channel hydraulics is based on laboratory or hypothetical measurements, rather than observations in natural river channels. Even if sufficient geometrical and flow information should be available, the calibration procedure would most likely be mathematically intractable due to high model complexity. Alternatively, one can consider a simpler model, so long as it produces an adequate fit to the observed data. A simple model is provided by the generalized friction law equation for uniform flow:

$$V = FS_f^{\eta}R^N \tag{1}$$

where V is the average cross-sectional flow velocity, F is a coefficient including all sources of hydraulic resistance in the channel section, S_f is the friction slope, R is the hydraulic radius and N and η are parameters. This model is encountered in many open-channel flow textbooks, for example Chow (1959), p. 91, and Dingman (1984), p. 114. Its theoretical explanation can be derived from the works of Chen (1991) and Venutelli (2005). In most applications, N and η are fixed at 2/3 and 1/2, respectively, according to the Manning's equation. However, applications of Equation (1) to data from natural rivers have shown that the two parameters do vary from site to site (e.g. Bray 1979; Jarret 1984; Dingman & Sharma 1997), which implies that they should best be considered free parameters in order to facilitate model flexibility.

Piecewise regression

A reasonable way to formulate a compound channel rating curve fitting procedure is as a piecewise regression problem. This well-known statistical approach is applied in order to model shifting response-covariate relationships of unknown change point levels, or segmentation limits, and is mathematically formulated as

$$y_i = f(x_i; P) = \begin{cases} f_1(x_i; P_1) + \varepsilon_i, & x_i \le \omega_1 \\ f_2(x_i; P_2) + \varepsilon_i, & \omega_1 < x_i \le \omega_2 \\ \vdots & \vdots \\ f_D(x_i; P_D) + \varepsilon_i, & \omega_{D-1} < x_i \end{cases} \tag{2}$$

where y is the dependent variable, x the independent variable or covariate, ε is the independent and identically distributed noise of zero expectancy and finite variance σ^2, i is an index running from 1 to n measurements, P_d is a vector of parameters, ω_d are the change points and P is a vector of the parameters$(P_1, \ldots, P_D, \omega_1, \ldots, \omega_{D-1})$. In many physical applications, it is necessary to impose continuity so the regression function does not jump from one segment to the next, i.e.

$$f_d(\omega_d; P_d) = f_{d+1}(\omega_d; P_{d+1}) \tag{3}$$

A continuous stage-discharge relationship is to be expected in the majority of cases, although discontinuities have been found in channels with a narrow, deep main channel and wide rough floodplains (Smart 1992). Discontinuities have also been observed for unsteady flow over sand-bed channels (Simons *et al.* 1962).

The statistical literature on continuous piecewise regression is large. Numerous studies have been published after the early works on this topic such as Robison (1964), Hudson (1966) and Hinkley (1969). It is beyond the scope of this paper to give a detailed bibliography on the topic, but the interested reader can consult Shaban (1980) and Seber & Wild (1989) for a comprehensive overview of segmented regression.

Piecewise regression estimation is known to be intractable due to two factors. Firstly, the estimates cannot be obtained in closed form. Secondly, and perhaps more important, the RSS (residual sum of squares) surface is, in general, very complex due to local minima and discontinuous derivatives. The presence of these characteristics has been demonstrated in many practical applications (e.g. Lerman 1980; Solow 1987; Julious 2001; Piepho & Ogutu 2003; Petersen-Øverleir & Reitan 2005). To overcome these difficulties, one has to invoke global and derivative-free optimization methods. In addition, the inferential methods one must use must work in such a complex setting, since traditional likelihood-methods require a unimodal likelihood surface with continuous derivatives.

The literature on piecewise regression in hydraulics, hydrology and related sciences is, however, scant. Esterby & El-Shaarawi (1981) applied two-phase linear regression to discharge time-series of the Nile River to detect shifts due to climatic changes. Solow (1987) used a two-phase linear regression model to test for changes in a time-series of Southern Hemisphere temperatures. Bates (1990) applied log-piecewise linear models to fit at-a-station hydraulic geometry to data from Australian rivers. Ryan *et al.* (2002) applied models with several linear segments in order to model discharge–bedload transport rate relationships.

Global optimization

Local minima which give spurious least-squares estimates must be avoided for a successful application of piecewise regression. Techniques exist to deal with this problem by perturbing the RSS function (e.g. Pronzato & Walter 2001; Wood 2001), but they are not considered in this study.

Another way to achieve safe estimation is to use an appropriate global optimization (minimization or maximization) algorithm. Basically, there are two sorts of global optimization. The first option is to apply some kind of grid search. This brute-force approach is trustable, in the sense that it is guaranteed to give a good approximation to the global optimum within the selected grid, given that the range and density of points is high enough. The method provides much information about the characteristics of the RSS surface, which can be used for post-modelling assessment purposes. Grid search has the major drawback of becoming extremely costly when the number of parameters increases, and especially if it is used in an intensive simulation scheme. For three-parameter problems and beyond, this method might be deemed unfeasible.

The second way to global optimization is to use a type of probabilistic iterative algorithm, e.g. multi-start local optimization, tunnelling, genetic algorithm or simulated annealing. The latter method, simulated annealing (SA), is an easy-to-implement algorithm and works at an affordable cost. It has become a popular global optimization technique in several application fields, for example in the calibration of surface and sub-surface hydrological models and the optimization of the operation of water systems. SA appears to have found little application in the calibration of open-channel flow models. This is somewhat surprising, as most river hydraulic models have free parameters that need tuning against observed geometric and/or flow data.

The theory of SA is well known and covered in numerous books and papers (e.g. Corana *et al.* 1987; Van Laarhoven & Aarts 1987; Otten & van Ginneken 1989; Bertsimas & Tsitsiklis 1993). The interested reader can consult the original articles by Metropolis *et al.* (1953) and Kirkpatrick *et al.* (1983).

Bootstrap inference

The bootstrap, introduced to regression problems by Efron (1979), is a computer-driven method for estimating the accuracy of statistical estimates. In many respects, it surpasses conventional statistical techniques, of which the method of maximum likelihood is the most important. Above all, the bootstrap is often applicable in situations far too complicated for the traditional techniques. Also, it is not based on asymptotic approximations which can be grossly inaccurate in finite-sample situations. A piecewise regression problem exhibits, as mentioned, several complicating factors that necessitate going beyond the traditional inferential techniques in order to obtain plausible uncertainty measures.

Basically, the bootstrap idea is to emulate how the given sample was generated, by estimating the original model from the observed data. This model estimate is then used in a Monte Carlo simulation to obtain samples that are used to quantify bias, variance and confidence limits. There is an extensive literature on bootstrap methods. Efron & Tibshirani (1993) and Davison & Hinkley (1997) cover much of the methodology and relevant references. Bootstrapping has been performed in piecewise regression, e.g. Ryan *et al.* (2002) used pair bootstrap in linear piecewise regression modelling, and Toms & Lesperance (2002) bootstrapped non-parametrically linear regression models of both sharp and smooth change point transitions.

For regression problems, there are basically two options. The first is to apply the bootstrap parametrically by fitting a distribution, typically a normal of zero expectancy and constant and finite variance, to the post-regression residuals and use this to sample replicate data. A second option is to make no assumptions concerning the distribution of the measurement error besides constant variance and use draws with replacement from the original and centred residuals. The latter method is thus more robust against effects caused by misclassification of the underlying error distribution, and is therefore applied in this study. The procedure can be described as follows. First, one calculates the 1 to n centred residuals:

$$e_i = y_i - f(x_i;\hat{P}) - \frac{1}{n}\sum_{i=1}^{n}[y_j - f(x_j;\hat{P})] \quad (4)$$

where $\hat{P}$ symbolizes the vector of regression parameter estimates, here assumed to have been found by the method of least squares. The following algorithm is then invoked:

Algorithm 1

For $l = 1, \ldots, L$,

1. for $j = 1, \ldots, n$
   a) draw e_j^* from $e_1, \ldots, e_n$with replacement;
   b) set $x_j^* = x_j$ and then set $y_j^* = f(x_j^*;\hat{P}) + e_j^*$
2. use $(y_1^*, x_1^*)_l, \ldots, (y_n^*, x_n^*)_l$ to obtain a least-squares estimate $\hat{P}_l^*$, then
3. calculate the desired statistic, or parametric function, $C_l^*(P_l^*)$.

The resulting sample $C_1^*, \cdots, C_L^*$ is then used to derive inferential measures such as standard deviation and confidence intervals for the statistic. The chosen number of simulations L must be sufficiently large so that Monte Carlo variability is negligible compared to the measures approximated by the simulation.

Bootstrapping does not always work well. One problem is correlated residuals, where successive residuals in time sequences have a tendency to be more alike than otherwise. If this happens, the bootstrap estimate of variance could be badly wrong. Heteroscedastic residuals are another potential problem when constant measurement error variance is assumed. If the variance is not properly stabilized, re-sampling residuals will give misleading results. The way bootstrapping is applied in this study introduces some unwanted effects of discreteness.

If the sample is very small, this effect can be severe in the sense that the re-sampling distribution lacks the all-over density required to give reliable quantiles. Another difficulty for small datasets is the chance of encountering a bootstrap sample that only contains replicates of a very few, at worst one, of the original data elements. This can lead to ill-conditioning and identifiability problems (Seber & Wild 1989, chapter 3). Finally, there is the problem of outliers in the data. The inclusion of stage-discharge measurements that have been erroneously performed or miscomputed could seriously affect the simulation output.

METHOD

Simplifying assumptions

It is necessary to introduce some initial simplifying assumptions for the part of the river channel affecting the hydraulics at the gauging station. (1) Both the main river channel and the floodplains are prismatic and have rectangular shapes (Figure 1). (2) The channel is wide such that the hydraulic radius can be well approximated by the hydraulic depth. (3) The stage-discharge relationship is practically unaffected by

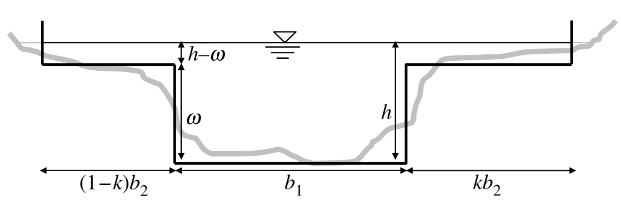

Figure 1 | A compound rectangular cross-sectional approximation to a hypothetical natural main channel-floodplain cross-section.

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hysteretic effects caused by unsteady flow and/or variable backwater. (4) The mean cross-sectional flow velocity can be described by the generalized uniform flow equation shown in Equation (1). (5) The hydraulic characteristics of the channel section are not subject to significant temporary changes during the time considered.

Model development

According to Figure 1, the flow area A and surface width B are given by

$$A(h;(b_1,\lambda,\omega)) = \begin{cases} b_1 h & \text{if } h \leq \omega \\ b_1(h + \lambda h - \lambda\omega) & \text{if } h > \omega \end{cases} \tag{5a}$$

$$B(h;(b_1,\lambda,\omega)) = \begin{cases} b_1 & \text{if } h \leq \omega \\ b_1(1+\lambda) & \text{if } h > \omega \end{cases} \tag{5b}$$

where $\lambda = b_2/b_1$. By applying Equation (1), and acknowledging the fact that the discharge Q is given by $Q = VA$ and also that R can be accurately approximated by the hydraulic depth A/B since the channel is wide, one obtains

$$Q(h;P_0) = \begin{cases} \exp(\theta_1) h^{N_1+1} & \text{if } h \leq \omega \\ \exp(\theta_2)(h + \lambda h - \lambda\omega)^{N_2+1} & \text{if } h > \omega \end{cases} \tag{6}$$

where $\exp(\theta_1) = F_1 S_f^{\eta} b_1$, $\exp(\theta_2) = F_2 S_f^{\eta} b_1/(1+\lambda)^{N_2}$ and $P_0 = (N_1, N_2, \theta_1, \theta_2, \lambda, \omega)$. With $h > 0$, Equation (6) is equivalent to

$$q(h;P_0) = \begin{cases} \theta_1 + (N_1+1)\log(h) & \text{if } h \leq \omega \\ \theta_2 + (N_2+1)\log(h + \lambda h - \lambda\omega) & \text{if } h > \omega \end{cases} \tag{7}$$

where $q = \log(Q)$. Equation (7) admits a possibility for the stage-discharge relationship to be discontinuous for $h = \omega$, which is not wanted. Imposing continuity on Equation (7) yields

$$q(h;P_0) = \begin{cases} \theta_1 + (N_1+1)\log(h) & \text{if } h \leq \omega \\ \theta_1 + (N_1 - N_2)\log\omega + (N_2+1)\log(h + \lambda h - \lambda\omega) & \text{if } h > \omega \end{cases} \tag{8}$$

Note that the upper segment rating curve can be written in similar form to the lower segment rating curve, but with a zero plane displacement, i.e. $\tilde{\theta}_2 + (N_2+1)\log(h+\delta)$ where $\delta = -\omega\lambda/(1+\lambda)$ and $\tilde{\theta}_2 = \theta_1 + (N_1 - N_2)\log\omega + (N_2+1)\log(1+\lambda)$. Keeping the original parameterization, it is convenient to write Equation (8) as a single expression. This can be achieved by introducing the Heaviside function:

$$H(h-\omega) = \begin{cases} 0 & \text{if } h - \omega \leq 0 \\ 1 & \text{if } h - \omega > 0 \end{cases} \tag{9}$$

such that Equation (8) can be expressed

$$q(h;P_0) = \theta_1 + (N_1+1)W_1(h;(\lambda,\omega)) + (N_2+1)W_2(h;(\lambda,\omega)) \tag{10}$$

where the two basis functions are defined as

$$W_1(h;(\lambda,\omega)) = \log h + H(h-\omega)(\log\omega - \log h) \tag{11a}$$

$$W_2(h;(\lambda,\omega)) = H(h-\omega)[\log(h + \lambda h - \lambda\omega) - \log\omega] \tag{11b}$$

The discharge measurement error is typically assumed to be multiplicative, proportional to and substantially smaller than the expected discharge. Hence, having n stage-discharge measurements available, one obtains the regression model

$$q_i = f(h_i;P_0) = \theta_1 + (N_1+1)W_1((\lambda,\omega);h_i) + (N_2+1)W_2((\lambda,\omega);h_i) + \varepsilon_i \tag{12}$$

where the noise terms are assumed to be identically and independently distributed with zero expectancy and a finite variance σ^2. Equation (12) is a piecewise regression problem and identical to Equation (2) for $D = 2$. In order to avoid physically impossible model parameters, the following constraints are introduced: $h_{\min} \leq \omega < h_{\max}$ where $h_{\min}$ and $h_{\max}$ are the lowest and highest vales of $(h_1, h_2, \cdots, h_n)$, respectively, and $\lambda > 0$. Note that the second restraint allows a change point that is exclusively caused by change in friction occurring at ω, i.e. $b_1 \gg b_2$. It is also worth noting that the first restraint allows the regression model to include one-segmented solutions, i.e. when $\omega = h_{\min}$. The two constraints are assumed to apply throughout the study. Constraining λ to be larger than zero can be done by introducing the re-parameterization $\lambda = \exp(\beta)$. The RSS valid for Equation (12) then becomes

$$g(P_0) = \sum_{i=1}^{n} [q_i - \theta_1 - (N_1+1)W_1((\beta,\omega);h_i) - (N_2+1)W_2((\beta,\omega);h_i)]^2 \quad (13)$$

Equation (13) is separable such that the parameters (θ_1, N_1, N_2) can be eliminated by using standard linear regression methods, i.e. $\theta_1 = m_{\theta_1}(\beta,\omega)$, $N_1 + 1 = m_{N_1}(\beta,\omega)$ and $N_2 + 1 = m_{N_2}(\beta,\omega)$ (for explicit expressions for the m functions, see for example Draper & Smith 1981, p. 196). Applying this principle is appropriate since Equation (13) has a unique solution given the parameters β and ω. Hence, Equation (13) takes the form

$$g(\beta,\omega) = \sum_{i=1}^{n} [q_i - m_{\theta_1}(\beta,\omega) - m_{N_1}(\beta,\omega)W_1((\beta,\omega);h_i)] - m_{N_2}(\beta,\omega)W_2((\beta,\omega);h_i)]^2 \quad (14)$$

and is a highly nonlinear function taking values in the 2D space defined by $\beta \in \langle -\infty, +\infty \rangle$ and $\omega \in [h_{\min}, h_{\max}]$. The standard deviation of the measurement error σ can be estimated by

$$\hat{\sigma} = \sqrt{\frac{1}{n-5}\sum_{i=1}^{n} e_i} \quad (15)$$

where

$e_i = q_i - \hat{\theta}_1 - (\hat{N}_1 + 1)W_1((\hat{\lambda},\hat{\omega});h_i) - (\hat{N}_2 + 1)W_2((\hat{\lambda},\hat{\omega});h_i)$ is typically referred to as the residuals. The 'hat' symbol denote that the associated parameter value is estimated using the least-squares method. Note that the residuals should be centred if they are applied in non-parametrically bootstrap schemes, as indicated in Equation (4).

Estimation and inference

The grid search method described in the section on global optimization can be used for obtaining the parameters that minimize Equation (14), given that some computer power is available. One then calculates the RSS for all the values of an appropriate grid for ω and β. This discretized picture of the RSS surface can not only be used to derive the parameter estimates, but also for a graphical analysis. An example taken from one of the Norwegian datasets used in the preliminary SA test studies is shown in Figure 2.

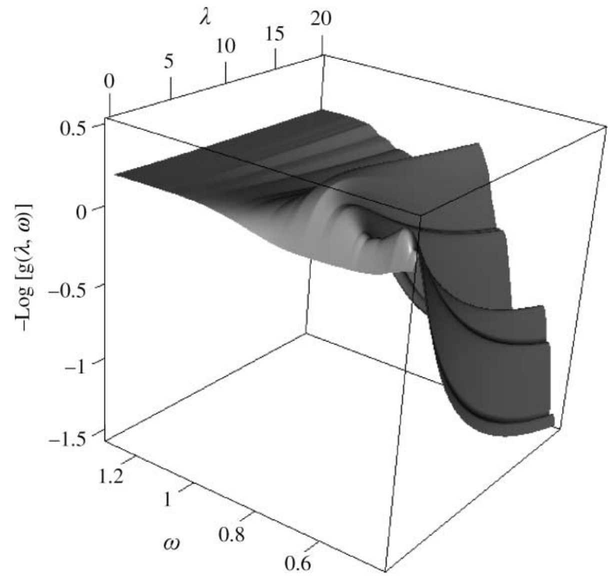

Figure 2 | Example of the negative log-residual sum of squares surface for the piecewise regression model used in the analysis as a function of the two nonlinear parameters. Data used are from the River Sameti at Sametibru, Norway.

The extreme complexity of the RSS surface is striking. The high degree of multimodality, the presence of non-smooth areas and extensive flatness in parts, clearly make local minimization by numerical methods quite unreliable. This difficulty is easily overcome by the grid search method, where one can readily pick out the global minimum and obtain the correct parameter estimates.

The presence of several local minima in the RSS surface seen in Figure 2, of which at least two are compatible, illustrates that standard asymptotic likelihood inference will not, in general, apply to the problem. Invoking bootstrap methods disposes of this difficulty. This implies furthermore that grid search becomes costly, since Equation (14) must be minimized for each of a large number of bootstrap simulations. Two methods were tested for this purpose. One was based on multi-start local minimization. Here a simplex, or Nelder-Meade, procedure was launched for several widely spread starting values, and then selected the parameters of the iteration(s) yielding the smallest minima. The starting values were taken from a constructed rectangular block of points covering the region where the estimates of ω and β were assumed to be located. A scheme where the starting points were randomly generated from the rectangular region was also tested. The reliability of the multi-start local minimization method was, not surprisingly, proportional to both the span and density of starting values considered. In terms of efficiency and reliability, it was not considered better than the SA algorithm, which was the second method tested. However, this conclusion must be qualified by the fact that the testing was limited in many ways. A comprehensive comparison would involve: (1) the consideration of different configurations of the annealing and the stochastic generating mechanisms in the SA algorithm and different ways for generating starting values in the multi-start local minimization algorithm; (2) various computation techniques such as parallel programming; and (3) more datasets with a varying degrees of complexity. The SA method was thus applied for the minimization of Equation (14).

A symmetric (bivariate normal) distribution was used as the proposal distribution in the SA algorithm. This gives the acceptance probability $\exp[g(\beta_{\text{new}}, \omega_{\text{new}}) - g(\beta_{i-1}, \omega_{i-1})]/T_i$. Many temperature decrease algorithms were tried in preliminary experiments. Using test data from five Norwegian rivers with two-segmented rating curves, it was found that $T_i = 1/i$ was appropriate with start and end temperatures of 0.5 and 0.001, respectively. That is, in each of the five preliminary case studies, the SA algorithm successfully found the global minimum a thousand consecutive times, with an SA variance of $<0.02\%$ of the mean of the parameter estimates. A single processor workstation with a CPU of about 500 MHz and a BogoMips value (a much-used indication of the processor speed) of approximately 1000 was used for computation. The average iteration times were between 2.07 and 2.99 seconds. This speed is acceptable for scientific purposes. However, for practical applications it might be deemed too slow. The effectiveness of the SA method can be significantly enhanced by using parallel computing (Azencott 1992).

The bootstrap scheme outlined in Algorithm 1 was used for inference. Preliminary testing showed that it was acceptable to use $L = 1{,}000$.

APPLICATIONS TO FIELD DATA

The rating curve assessment used in this study is based on a very simple model and also on several simplifying assumptions which, at best, are only partially met at a gauging station in a natural river channel. Hence, the capability of the proposed method must be evaluated by application to stage-discharge data from natural river channels of various sizes where the main channel-floodplain change point is approximately known from geometrical survey. The paper will now outline results from four rivers draining a wide range of (decreasing) catchment areas.

Data, rating curve fit and change point estimation

The river stretch containing the gauging station on the River Paraná at Corrientes, Argentina, drains a catchment of 2,051,720 km^2 (García & Vargas 1998) and has a slope of about 8.5×10^{-5}. It exhibits a multi-channelled pattern with many bars and islands (Orefo & Stevaux 2002). The reach just downstream of the station consists of two main channels separated by a low flat island. The right channel is 500 m wide with a flow depth between 15 and 18 m, whereas the left is 2 km wide with a depth of 20 m. The only floodplain is on the right side of the river and is on average 8.1 km wide and forested. The river tops the main channel banks when the stage above the local gauge datum is approximately 6 m (Clarke *et al.* 2000). As the cease-to-flow stage has been measured as 11.67 m (personal communication, Franzetti *et al.* 2005), this means that the main channel-floodplain change point is located at a flow depth around 18 m.

The 368 stage-discharge measurements used in this study were taken in the period of 1980–2003, using either current meter or ADCP (Acoustic Doppler Current Profiler). Looking at Figure 3, the measurements appear noisier than one would expect. An analysis of the data suggests this phenomenon is caused not by unsteady flow or variable backwater, but by complex channel changes in the period during which the measurements were collected. This accords with Amsler *et al.* (2005) who identified a link between climate and morphological changes in the River Paraná channel.

Clearly, the hydraulic situation on the River Paraná at Corrientes violates assumptions (1) and (5) of Section 3.1. Figure 3 illustrates that the estimated rating curve nevertheless gives a good overall description of the measurements, except some evidence that the high discharges are underestimated. Also, the change point is estimated with

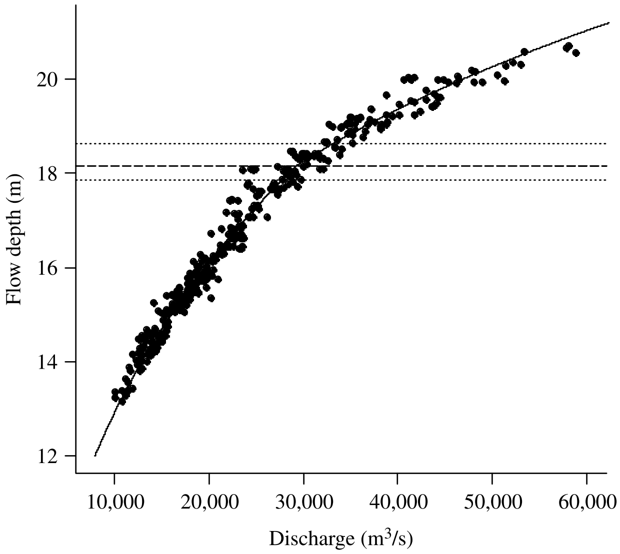

Figure 3 | Depth–discharge measurements and the fitted compound rating curve (solid line) for the River Paraná at Corrientes, Argentina, the estimated main channel-floodplain intersection point (dashed line) and the corresponding 95% bootstrap confidence limits (dotted lines).

relatively high accuracy, with a point estimate of 18.17 m and upper and lower 95% bootstrap confidence limits yielding 18.63 m and 17.87 m, respectively. This interval is in good agreement with the observed main channel-floodplain intersection point. A systematic deviance can be discerned between the estimated rating curve and the highest measurement. The reason for this might be the river reaching a higher floodplain (García & Vargas 1998) at very high flows.

The River Severn at Montford Bridge, above Shrewsbury, England, has been subject to several hydraulic studies (e.g. Darby & Thorne 1996; Karamisheva *et al.* 2006). The catchment area at the gauging station is 2,025 km^2. The corresponding reach, which has a slope of approximately 2.0×10^{-4}, comprises two vegetated floodplains flanking the gravel bed main channel at a flow depth of approximately 5.75 m (see Figure 4(b)). Stage-discharge measurements (873 in total) collected between 18 November 1976 (the date of the last datum change) and the end of 2006 were initially applied to the least-squares estimator, which then exhibited two notable minima. The global minimum gave a change point estimate of 0.85 m, whereas the compatible local minimum corresponded to a point in the vicinity of the main channel-floodplain intersection. The global estimate was clearly not the one sought, but most likely the flow depth where a low-flow control, not describable by the main channel geometry shown in Figure 4(b), is being drowned. The low change point estimate might also be an artefact caused by the mild degree of heteroscedasticity present in the River Severn data.

In order to obtain an objective main channel-floodplain change point, all measurements with a flow depth lower than 2 m were censored. The remaining (239) measurements were supplied to the least-squares estimator which then gave an estimated change point of 5.24 m. Even if the estimate clearly reflects the main channel-floodplain intersection when the 95% confidence limits are taken into account, it is rather low compared to the observed change point. Nevertheless, the estimated rating curve shown in Figure 4(a) gives a good overall description of the

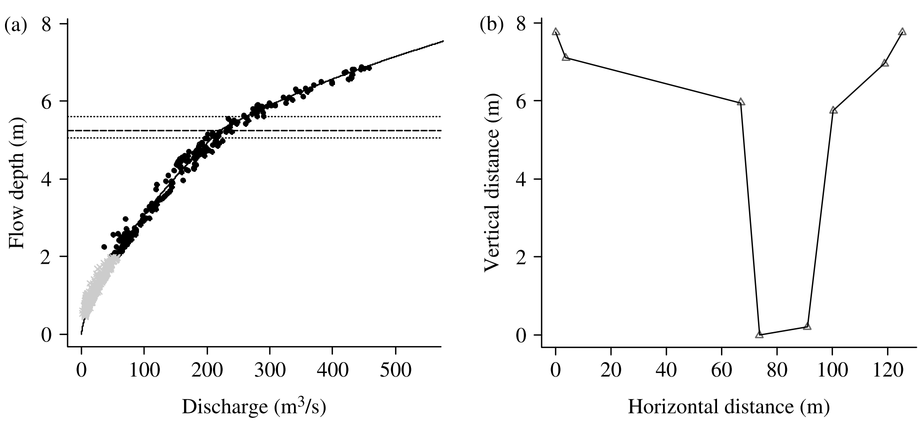

Figure 4 | Depth–discharge measurements (black and grey dots represent included and omitted measurements, respectively) and the fitted compound rating curve (solid line) for the River Severn at Montford, England: (a) the estimated main channel-floodplain intersection point (dashed line) and the corresponding 95% bootstrap confidence limits (dotted lines) and (b) cross-sectional profile for the River Severn at Montford, UK.

available measurements, including the censored low-flow measurements.

A reach of the River Main in Ulster, Northern Ireland, has been used for studies on the hydraulic behaviour of two-stage river channels. The channel cross-section at the gauging site consists of a deep central section flanked by two floodplains (see Figure 5(b)). The bed material in the main channel consists of coarse gravel, while the floodplains are covered with grass and weeds. The experimental reach, with a slope of 3×10^{-3}, has an upstream catchment area of approximately 250 km^2. More details concerning this river reach can be found in Myers & Lyness (1994). The 24 stage-discharge measurements available from this site were made by current meter. The bankfull depth at this station is observed to be at a flow depth of 1 m (Myers & Lyness 1994), which is a little lower than suggested by the cross-section shown in Figure 5(b). As seen in Figure 5(a), the change point at 1 m is well estimated by the rating curve model, although the bootstrap confidence limits enclosing the estimated change point are relatively wide. The noticeable uncertainty is primarily ascribed to the measurement being noisier around the main channel-floodplain interaction. Additionally, there are few measurements available just above the change point. Figure 5(a) demonstrates that the estimated rating curve fits the measurements accurately.

The River Blackwater at Farnham in Hampshire, England, is a small, managed two-stage river. Sellin & van Beesten (2004) give a thorough description of this river section, which is highly vegetated. The instrumented reach has a thalweg slope of 8.5×10^{-4} and drains a catchment of 35 km^2. The channel form is complex, with irregularities in plan and cross-sections being specific to their location. A representative cross-section for the reach has therefore been constructed. According to information on the homepage of the NCRFS (network on conveyance in river floodplain systems) database (Babaeyan-Koopaei *et al.* 2001), from which the data used in this study have been taken, the representative cross-section has a compound trapezoidal shape with a cease-to-flow stage at 0.4 m flow depth and a bankfull depth at 0.75 m. The hydraulic conditions in the reach change drastically with the growing season, and consequently any particular rating curve is valid for only a short period.

Measurements collected during 18–23 January 1995 were chosen for this study. This period was selected because the measurements range well above and below the floodplain. Discharge is measured by means of an electromagnetic flow gauge installed just upstream of the study reach. Figure 6 shows that the estimated rating curve provides a very good description of the available measurements. However, the change point estimate is somewhat low according to the bankfull depth given by the nominal cross-section (0.75 m). In fact, the 95% confidence limits do not include 0.75 m. The reason for this, besides estimation imprecision and model inadequacy, could be that that the cease-to-flow stage for the representative cross-section is set too low. This is supported by Sellin & van Beesten (2004) who applied a cease-to-flow point of 0.5 m.

This section shows that the presented method is capable of reproducing depth–discharge relationships for a broad

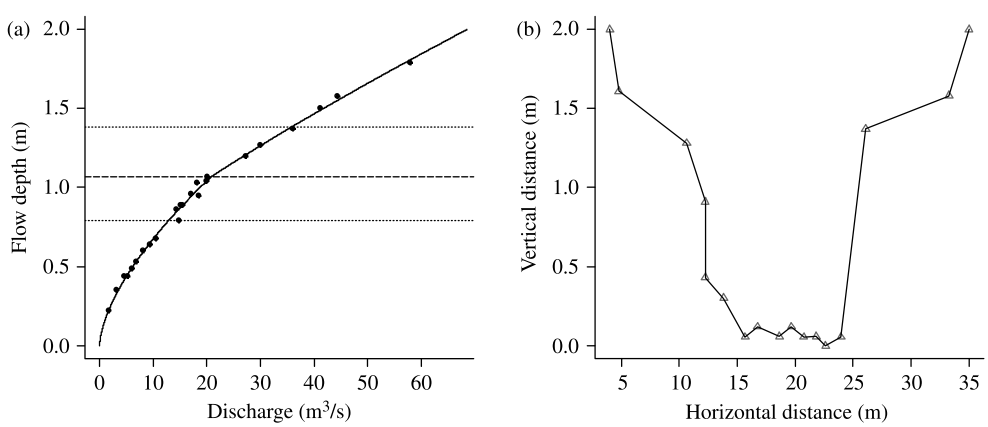

Figure 5 | Depth–discharge measurements (black dots) and the fitted compound rating curve (solid line) for the River Maine at Ulster, Northern Ireland (a) the estimated main channel-floodplain intersection point (dashed line) and the corresponding 95% bootstrap confidence limits (dotted lines) and (b) cross-sectional profile for the River Maine at Ulster, Northern Ireland.

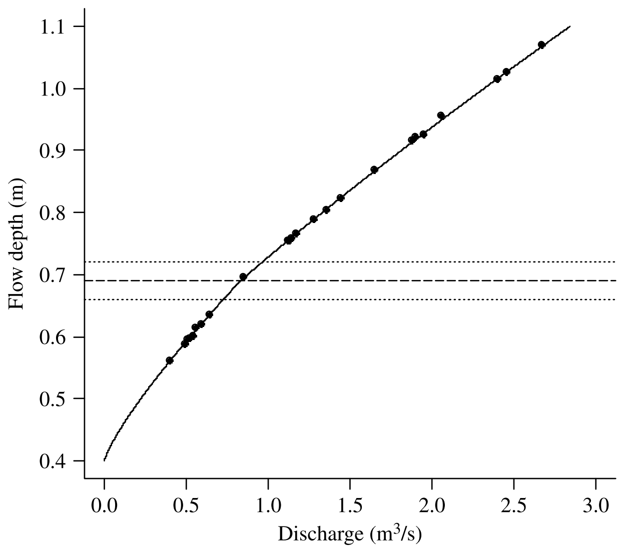

Figure 6 | Depth–discharge measurements and the fitted compound rating curve (solid line) for the River Blackwater, UK: the estimated main channel-floodplain intersection point (dashed line) and the corresponding 95% bootstrap confidence limits (dotted lines).

range of two-stage channel sections in natural rivers. It should, however, be recognized that the proposed methodology is based on several simplifying assumptions, and cannot be expected to fully reflect the true physics of the flow in the channel section containing the gauging station. Hence, even if the estimated rating curve accurately estimates the observed depth–discharge characteristics in the measured area, it should not be applied in the extrapolated part where it might give completely misleading discharge estimates.

Parameter and discharge estimations

Table 1 reports the least-squares rating curve parameter estimates and their 95% bootstrap confidence intervals. Using the fitted rating curve parameters for the minimum, median and maximum stage values in the measurements, the corresponding three discharges were calculated. The discharge estimates and the associated 95% bootstrap confidence limits are shown in Table 2.

The change point parameter ω, which describes the flow regime change, shares little dependence with the rest of the parameters which account for the stage-discharge growth characteristics. As the effect of ω is clearly identifiable, it is sensible to assess the change point estimation in conjunction with the true river channel characteristics, as was done in the previous section. On the other hand, it is doubtful whether the other rating curve parameter estimates will reflect the true hydraulic characteristics of the river stretch containing the gauging station because the parameter estimates will be highly correlated, and because an idealized channel shape is used rather than the true one. However, taking the 95% confidence intervals into account, most of the least-squares parameter estimates in Table 1 do not seem unreasonable.

Further perusal of the results in Table 1 reveals that some of the parameters were estimated with poor precision. In particular, $\lambda = \exp(\beta)$ was inaccurately determined. Even in the case of the Paraná River at Corrientes where no less than 368 data points were available, a 95% confidence interval for this parameter ranges from 2.0×10^{-2} to 8.4. In fact, for several of the datasets, the left hand confidence limit for the estimate of β appeared to be $-\infty$. Alternatively, the left hand confidence limit of $\hat{\lambda}$ is zero.

This phenomenon was further explored using some of the model runs in the bootstrap simulation. A local minimization algorithm, in the form of the simplex method, was launched at the point where the SA algorithm converged. The simplex algorithm took the β further towards $-\infty$ while the other parameter values and the

Table 1 | Depth–discharge rating curve estimates (95% bootstrap confidence intervals in parentheses)

Station	$\hat{\omega}$	$\hat{\beta} = \log(\hat{\lambda})$	$\hat{\theta}_1$	$\hat{N}_1$	$\hat{N}_2$
The Paraná River at Corrientes	18.17 (17.87, 18.63)	−3.08 (−3.92, 2.13)	1.15 (0.92, 1.41)	2.15 (2.06, 2.23)	3.70 (−0.73, 3.91)
The River Severn at Montford	5.24 (5.06, 5.60)	−3.95 (−4.52, 2.20)	2.94 (2.90, 2.99)	0.48 (0.45, 0.52)	1.61 (−0.52, 1.75)
The River Maine at Ulster	1.07 (0.79, 1.38)	−0.01 (−7.23, 3.11)	2.93 (2.87, 2.96)	0.60 (0.52, 0.65)	0.18 (−0.72, 1.09)
The River Blackwater at Farnham	0.69 (0.66, 0.72)	−1.33 (−2.26, −0.59)	1.35 (1.27, 1.42)	0.24 (0.19, 0.28)	0.20 (0.04, 0.30)

Table 2 | Estimated discharges for the maximum, median and minimum flow depths of the measurements and various associated bootstrap statistics

					Bootstrap quantiles		
Station	**Statistic**	**Rating curve estimate**	**Bootstrap mean**	**Bootstrap S.D.**	**0.025**	**0.975**	**0.500**
The Paraná River at Corrientes	$\hat{Q}(h_{max})$	55,415	54,970	951	53,172	56,805	54,976
	$\hat{Q}(h_{median})$	21,338	21,331	85.3	21,164	21,491	21,330
	$\hat{Q}(h_{min})$	10,515	10,521	91.4	10,347	10,697	10,522
The River Severn at Montford	$\hat{Q}(h_{max})$	454	451	10.6	427	470	451
	$\hat{Q}(h_{median})$	173	173	1.51	170	176	173
	$\hat{Q}(h_{min})$	53	53.0	0.60	51.9	54.2	53.1
The River Maine at Ulster	$\hat{Q}(h_{max})$	56.8	56.6	2.40	52.2	62.2	56.5
	$\hat{Q}(h_{median})$	15.5	15.5	0.24	15.0	16.0	15.5
	$\hat{Q}(h_{min})$	1.65	1.67	0.06	1.56	1.81	1.66
The River Blackwater at Farnham	$\hat{Q}(h_{max})$	2.67	2.67	0.01	2.64	2.69	2.67
	$\hat{Q}(h_{median})$	1.16	1.16	<0.01	1.15	1.17	1.16
	$\hat{Q}(h_{min})$	0.41	0.41	<0.01	0.40	0.41	0.41

RSS barely differed from the initial SA estimates. The fact that the least-squares estimator yields an infinite value to β should not cause significant concern, as all the corresponding estimated discharges are finite and calculable for $\lambda = 0$. It can, however, waste optimization time, as a large proportion of the available iterations will be used to move β towards minus infinity. Clearly, this unfortunate property can be readily mitigated by applying an appropriate stopping criterion in the optimization routine. Changes to the upper-segment friction law parameter N_2 seemed to have little effect on the model. However, the degree of insensitivity for N_2 was far less than was the case for β, in the sense that no infinite estimates were observed for N_2. In conclusion, these findings suggest that the model could be less parameterized, e.g. by further idealizing the channel shape by assuming that the floodplain–main channel surface widths are equal and thus fixing λ to 1. Other simplifications might also be considered.

Examination of the bootstrap statistics listed in Table 2 reveals that the presented rating curve model is capable of estimating discharges with high accuracy. Thus, the imprecision in the rating curve parameter estimates is not manifested in the discharge estimation accuracy. This is clearly a result of highly dependent parameter estimates, and implies that an increase/decrease of some parameter values will be balanced by an increase/decrease of the remaining ones when the estimated discharge is calculated.

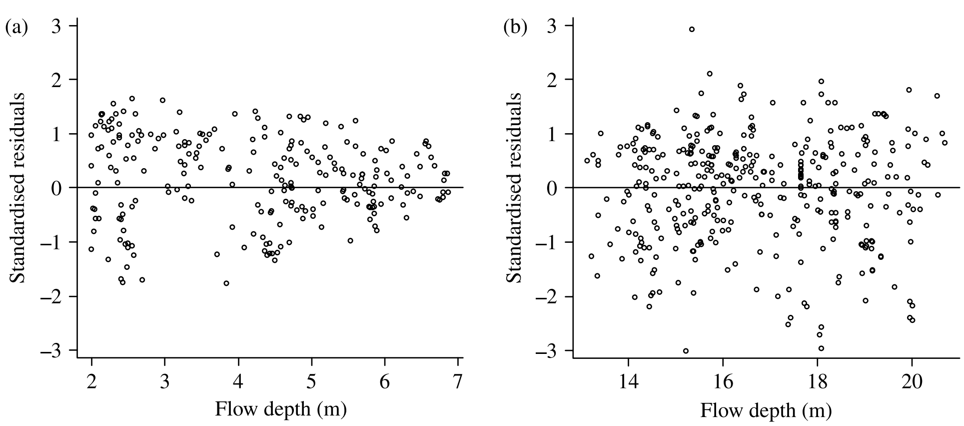

Figure 7 | Depth-residual plot for (a) the River Severn at Montford, UK and (b) the River Paraná at Corrientes, Argentina.

Some of the depth-residual plots displayed a slight trumpet shape, exemplified by the River Severn data in Figure 7(a) (4 outlying residuals were omitted from the plot). This phenomenon is typically caused by some degree of heteroscedasticity. Studies have highlighted that rating curve estimation can be plagued by this unwanted feature (Petersen-Øverleir 2004; Whitfield & Hendrata 2006). A trumpet-like depth-residual shape could also be caused by curvature effects (Seber & Wild 1989). However, simulation studies performed during this study strongly suggest that the proposed model is not plagued by curvature effects. Simulated datasets with homoscedastic measurement error produced no signs of unequal variance in the depth-residual plots, but did show characteristics similar to the depth-residual plot for the Corrientes data in Figure 7(b). In any case, it is recognized that the mild heteroscedasticity found in some of the case studies may mean that the range of the confidence intervals shown in Table 2 might be slightly incorrect. That is, the uncertainty corresponding to the lower and higher discharge estimates could be a little under- and overestimated, respectively.

An important feature to be noted in Table 2 is that the bootstrap densities seem, in general, to be symmetric around their respective mean values which differ little from the original estimates. These facts have two important implications: (1) the estimation bias is negligible; and (2) the bootstrap standard deviation, which requires fewer bootstrap replications for an accurate calculation than bootstrap percentiles, could be used to form the approximate confidence intervals.

CONCLUSIONS

This paper presents a continuous generalized steady uniform flow model of the depth–discharge relationship for main channel-floodplain river channels, a piecewise regression technique based on least squares for fitting the model to observed depth–discharge data, a simulated annealing routine for obtaining the least-squares model parameter estimates numerically and a bootstrap methodology to evaluate the accuracy of the fitted model. The results of the application of this framework to data from four gauging stations are presented and evaluated.

The applications show that the proposed method is capable of estimating this main channel-floodplain intersection point adequately, although the corresponding statistical uncertainty is sometimes large. The other estimated rating curve parameters showed noticeable uncertainty. In contrast, the rating curve discharge estimates were obtained with high precision. However, the results must be qualified by the fact that they are only for four hydrometric gauging stations.

The methodology presented here is statistics- and data-driven and works without hydraulic information beyond the available depth–discharge measurements. It should therefore be useful for many applications since detailed information of hydraulic characteristics such as channel topography and bed roughness are not commonly available at hydrometric gauging stations. In contrast to conventional compound rating curve assessment, the methodology is automated and amenable to objective statistical inference. This paves the way for the analysis of compound rating curve uncertainty propagation in any subsequent modelling that requires discharge data, e.g. the planning of hydraulic structures, rainfall–runoff model calibration, flood frequency analysis, geomorphologic studies and the assessment of water-borne pollutants.

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