is of importance in analyzing the data in Fig. 5 and makes the conclusion that there were no particle size effects doubtful. All of the other data presented were obtained using 55-micron particles at relatively high air velocities. For these, the difference between U_a and the calculated value of U_p is too small to affect the results significantly.

One cannot calculate the ratio D_{D_p}/C_{D_a} as indicated because the dust-concentration values given in Fig. 5 would be incorrect if U_a/U_p is much greater than unity. The dust concentration was determined by dividing the measured value for the dust flow in gms/sec by the quantity of air pushed through the tube per sec. To be correct, each value should be multiplied by U_a/U_p. Taking this factor into account, the values of D_{D_p}/D_{D_a} calculated by assuming there are no particle interaction effects would be as follows:

d (microns)	C_{D_p}/C_{D_a}
470	3.8
155	1.26
55	1.03

U_p for the 470 micron particles must be about one quarter of the theoretical value for dust-free air or there is some particle size effect or some experimental error. Repeat experiments in which U_p is measured are needed.

The Reynolds number for flow in the tube leading to the sensing unit was always greater than 4000. This would indicate that it would not be necessary to take into account the velocity profile.

A Photoelastic Approach to Transient Stress Problems Employing Low-Modulus Materials[1]

E. VOLTERRA.[2] The authors are to be congratulated upon their excellent paper which represents a very important contribution in the field of dynamic photoelasticity. Since the early 1940's much work has been done on the dynamic characteristics of low-modulus materials. Perhaps it is correct to state that the study of this subject started in Cambridge, England, during the Second World War when, under the supervision of Sir Geoffrey Taylor, research on the dynamic properties of some plastics and rubberlike materials under impact loading was carried out for the British Admiralty [1, 2, 3].[3]

The main object of the early experiments was to derive, from a rheological point of view, the dynamic stress-strain relationships of these materials under impact loading. This was the object also of experiments which were carried out in later years at the Illinois Institute of Technology and at Rensselaer Polytechnic Institute in this country under the sponsorship first of the Office of Naval Research and later of the Air Force Office of Scientific Research [4–8].

In all the earlier experiments mentioned, the apparatus used to produce impact loading on the specimens was the same as that used by the authors of the present paper. On the other hand, in all the experiments, methods of recording stress and strain were continually being improved. In fact, whereas in the early Cambridge experiments optical methods were used to record the distance between the pendulums during impact (which involved a double differentiation to be performed in order to derive the stress-strain relationship), in more recent experiments, performed at Rensselaer Polytechnic Institute, a more exact method based on the use of an accelerometer fixed to one of the pendulums was used to record stresses and strains. In this paper the authors, in addition to the accelerometer, use a linear differential transformer mounted between the pendulums and a Fastex motion-picture camera. It is interesting to learn that the three methods give the same results as to stress-strain curves. However, there are three questions in which the writer requests comment:

1 What is the influence of the end conditions of the specimen? How were the ends of the specimens lubricated?

2 During impact did barreling of the specimens occur and, if so, how did it affect the assumption of uniform stress distribution in the specimen?

3 Is the assumption justifiable that lateral inertia of the specimen during impact can be neglected?

References

1 G. I. Taylor, "The Testing of Material at High Rates of Loading," *Journal of the London Institution of Civil Engineers*, vol. 26, 1946, pp. 486–519.

2 E. Volterra, "Sulla Relazione Dinamica tra Sforzi Specifici e Deformazioni Specifiche per i Materiali Elastici e Plastici," *Giornale del Genio Civile*, Fasc. 3, May and June, 1946.

3 E. Volterra, "Alcuni Risultati di Prove Dinamiche Sui Materiali," *La Rivista del Nuovo Cimento*, vol. 4, 1948, pp. 1–28.

4 E. Volterra, "A Mathematical Interpretation of Some Experiments on Plastics and Rubber-Like Materials," Proceedings of the Second International Congress on Rheology, Oxford, England, July, 1953, pp. 73–78.

5 E. Volterra, R. A. Eubanks, and D. Muster, "Some Experiments at High Rates of Loading," Proceedings of the Eighth International Congress of Applied Mechanics, Istanbul, Turkey, August, 1952.

6 E. Volterra, R. A. Eubanks, and D. Muster, "An Investigation of the Dynamic Properties of Plastics and Rubber-Like Materials," *Proceedings of the Society for Experimental Stress Analysis*, vol. 8, 1950, pp. 85–96.

7 D. Muster and E. Volterra, "A Rotating Drum Camera for Recording Impact Loading Deformations," *Proceedings*, American Society of Motion Picture and Television Engineers, vol. 59, 1952, pp. 44–48.

8 E. Volterra and C. S. Barton, "An Impact Testing Machine for Plastics and Rubber-Like Material," *Proceedings of the Society for Experimental Stress Analysis*, vol. 16, 1958, pp. 157–166.

Authors' Closure

The authors are very grateful to Prof. Volterra for his kind comments and constructive criticisms.

The validity of the dynamic stress strain determination from the double pendulum method depends upon the assumption that the stress distribution is uniform over the entire specimen. Three factors may influence the validity of this assumption:

1 Frictional constraint at the ends of the specimen.
2 Lateral inertia of the specimen.
3 Viscoelastic characteristics of the specimen material.

The first factor is strongly dependent on the lubrication at the interface between the specimen and the pendulums. In this experiment a liberal quantity of a silicone grease was applied to the right-hand pendulum and a thin layer of the same grease to the left-hand pendulum. The photographs shown in Fig. 4 indicated almost no frictional constraint on the right-hand side, yet they do indicate some effects of friction on the left-hand side. Barreling will be dependent on frictional effects at the ends, and the magnitude of the strain applied to the specimen. The photoelastic fringe patterns were obtained at low values of strain and

[1] By J. W. Dally, W. F. Riley, and A. J. Durelli, published in the December, 1959, issue of the JOURNAL OF APPLIED MECHANICS, vol. 26, TRANS. ASME, vol. 81, series E, pp. 613–620.

[2] Professor of Engineering Mechanics, The University of Texas, Austin, Tex. Mem. ASME.

[3] Numbers in brackets designate References at end of discussion.

the barreling of the specimen were not large enough to be noticeable.

The authors believe the effects of lateral inertia are small and much less serious than the end conditions. Referring again to Fig. 4, the 1.5, 4.5, 6.5, and 7.5 fringes are nearly uniformly distributed over the width of the specimen. The other fringes are not as well distributed, but it is believed that the conditions at the ends of the specimen produced the irregularities rather than lateral inertia effects.

If the material were perfectly elastic and if one neglected lateral inertia effects, the fringes would build up as a consequence of wave propagation and reflection. Thus a difference in the fringe order over the length of the specimen would always exist. Inspection of Fig. 4 shows this wave-propagation effect for the first 6 frames as the one-half fringe propagates down the specimen and reflects from the right-hand pendulum. However, from frame 7 on there is little evidence of wave propagation and the fringe order at both ends of the specimen is equal. Any local differences in the fringe order at the ends of the specimen may be attributed to frictional effects.

It is the authors belief that lateral inertia and wave propagation effects will not influence the distribution of the stress in a specimen of a low-modulus material to any appreciable extent. Frictional constraint is the greatest factor which introduces experimental error and it should be the first concern of the investigator.

Response of a Nonlinear String to Random Loading[1]

R. H. LYON.[2] In his paper, Caughey has presented calculations for the mean square displacement of a string governed by the equation of motion (his notation)

$$\rho\left(\frac{\partial^2 u}{\partial t^2}+\beta\frac{\partial u}{\partial t}\right)=\left[T_0+\frac{AE}{2L}\int_0^L\left(\frac{\partial u}{\partial x}\right)^2dx\right]\frac{\partial^2 u}{dx^2}+f(x,t)\quad(1)$$

where $f(x, t)$ is governed by the correlation function

$$\overline{f(x,t_1)f(\eta,t_2)}=4D\delta(x-\eta)(\sin\omega_c\tau)/\tau.\quad(2)$$

It is the purpose of this discussion to show that his results may be obtained by elementary considerations and to suggest that recent work of the present author[3] has discounted the validity of using equation (1) in this problem.

The mean square displacement of the i^{th} linear mode may be easily found to be [see Caughey's equation (21)]

$$\sigma_{i,0}^2=2DL/i^2\pi^2\beta T_0.\quad(3)$$

The effect of the total motion is merely to increase the average tension by an amount

$$\overline{\Delta T}=\frac{EA\pi^2}{4L^2}\sum_{i=1}^{N}i^2\sigma_{i,0}^2=\frac{EAD}{2\beta T_0L}N.\quad(4)$$

[1] By T. K. Caughey, published in the September, 1959, issue of the Journal of Applied Mechanics, vol. 26, Trans. ASME, series E, vol. 81, pp. 341–344.

[2] Associate Professor of Electrical Engineering, University of Minnesota, Minneapolis, Minn. Currently on leave as National Science Foundation Postdoctoral Fellow at the Department of Mathematics, The University, Manchester, England.

[3] R. H. Lyon, "The Random Vibration of Elastic Strings—Theoretical," WADC Technical Report 58-570, September, 1958.

Since

$$\overline{U_0^2}=\sum_{i=1}^{\infty}\sigma_{i,0}^2=\frac{DL}{3\beta T_0}$$

and

$$\overline{U^2}=\sum_{i=1}^{\infty}\sigma_i^2=\frac{DL}{3\beta(T_0+\overline{\Delta T})},$$

one has

$$\overline{U^2}/\overline{U_0^2}=(1+\overline{\Delta T}/T_0)^{-1}=(1+\alpha\sigma_{1,0}^2N)^{-1}\quad(5)$$

which is Caughey's result for $\alpha\sigma_{1,0}^2N$ small, which it must be for the analysis to be consistent. It is obvious that the new linear system with tension $T_0+\overline{\Delta T}$ will satisfy equipartition.

The work just referred to does not use equation (1) but rather more general equations similar to those developed by Carrier.[4] Although the assumption is made that $EA \gg T_0$, the result does not reduce to (5) but rather

$$\sigma_i^2/\sigma_{i,0}^2=(1+2Q_i\overline{\Delta T}/T_0)^{-1}\quad(6)$$

where Q_i is the quality factor for the i^{th} mode. Since for a large number of systems $Q_i \gg 1$, the reduction in displacement is stronger than (5) would indicate. This result, when traced back, occurs by an interaction between longitudinal and transverse motions of the string.

Author's Closure

The author wishes to thank Dr. Lyon for his interesting comments on the paper. The author would like to refute the claim that equation (1) of the above paper is invalid. To this end, consider Carrier's equations for the string,[4] modified to include damping and external forces.

$$f(x,t)=\rho u_{tt}+ru_t-\partial_x[T\sin\theta]\quad(1)$$

$$O=\rho V_{tt}+\eta V_t-\partial_x[T\cos\theta]\quad(2)$$

where

$$\sin\theta=\frac{u_x}{1+e}\quad(3)$$

and the local strain e is given by

$$e=((1+V_x)^2+u_x^2)^{1/2}-1\quad(4)$$

$$T=T_0+EAe\quad(5)$$

If the following assumptions are made:

(a) $V_x=O(u_x^2)\ll 1$

(b) $T_0\ll EA$

(c) $w_c<\Omega_1$

where w_c is the highest frequency excited in transverse vibration and Ω_1 is the lowest eigen value for longitudinal vibrations.

If assumption (c) is satisfied, the time derivative terms in (2) may be neglected. Hence,

$$\partial_x[T\cos\theta]=0\quad(6)$$

$$\therefore T\cos\theta=g(t)\quad(7)$$

where $g(t)$ is independent of x.

Using assumption (a) and equations (3) and (4),

$$T\simeq T_0+EA\left(V_x+\frac{1}{2}u_x^2\right)\quad(8)$$

[4] G. F. Carrier, "On the Nonlinear Vibration Problem of the Elastic String," *Quarterly of Applied Mathematics*, vol. 3, 1945, pp. 157–165.