It is shown in the most general case of electron-phonon system that the variation principle in the quantum theory of transport phenomena can be reduced to the variation principle of the Umeda-Kohler-Sondheimer type. The reduced variation principle corresponds to that which is obtained from the investigation by Sondheimer in proving the Kelvin relations in the electron-phonon system and has been investigated more generally by Ziman and Kasuya. Owing to this result the Kelvin relations in thermoelectricity hold in the most general case. It is in fact a natural consequence from the present point of view for the reason that those relations hold in the quantum theory from which the variation principle in the classical theory has been derived.

§ 1. Introduction

In the thermoelectric phenomena in the electron-phonon system, the phonons deviate more or less from the thermal equilibrium states, which has been considered as a main reason for certain anomalies in the thermoelectric behaviour of semiconductors. From this point of view Sondheimer, who noticed some calculations on thermoelectricity of which results conflict with the Kelvin relations, has proved that the Kelvin relations hold even if the phonon distributions depart from the thermal equilibrium in the electron-phonon system involving the interactions between electrons and phonons.

In connection with such an investigation some variation principles can be formulated, which are a generalization of the variation principles proposed by Umeda and Kohler. By means of this the thermoelectric coefficients can be calculated efficiently. Ziman and also Kasuya have formulated such a variation principle in a more general system which involves the interactions more than the electron-phonon interaction.

In contrast with these investigations based on the Boltzmann-Bloch equation of the electron-phonon transport we have recently formulated a variation principle in the quantum statistical theory of transport phenomena and have shown that it is reduced to the variation principle of the Umeda-Kohler-Sondheimer type in the classical theory by means of a perturbational treatment in the case of the system of the electrons scattered by impurities.
It will be shown in this paper (§ 2) that the quantum variation principle can be reduced as well to the variation principle of the classical type obtained by Ziman and Kasuya in the electron-phonon system involving interactions between electrons and phonons, between electrons and impurities, between phonons and impurities and among phonons. In connection with such a reduced variation principle the Kelvin relations in thermoelectricity are shown to hold; this is a natural consequence from the present point of view as explained in § 3. Some conjectures are given as well in § 3 in relation to removing some restrictions used in the reduction in § 2.

§ 2. Reduction from the quantum to the classical variation principles

The variation principle in the quantum theory of transport phenomena has been presented in respect of the operators \( \Phi^{(+)} \) and \( \Phi^{(-)} \) which have relations with the time-dependent density matrix \( \rho(t) \) expressed as

\[
\rho(t) = \rho_0 \pm \rho_0 \int_0^t d\lambda e^{\lambda H} \Phi^{(\pm)} e^{-\lambda H} e^{st},
\]

where \( H \) represents the Hamiltonian of the system acted by no external disturbances, and \( \rho_0 = \rho_0(H) \) denotes the density matrix of the system with this Hamiltonian at thermal equilibrium. We have supposed two cases that the system is at thermal equilibrium at \( t = -\infty \) reflecting the influence of an external disturbance which develops like \( \exp(st) \) with a small positive parameter \( s \) in one case to which \( \Phi^{(+)} \) in (1) corresponds, and in the other case which corresponds to \( \Phi^{(-)} \) the system is at thermal equilibrium at \( t = \infty \) reflecting an external disturbance varying with time proportionally to \( \exp(-st) \).

The variation principle has been suggested in the following form. The operators \( \Phi^{(+)} \) and \( \Phi^{(-)} \) should be determined so as to make the expression stationary:

\[
W(\Phi^{(+)}, \Phi^{(-)}) = \frac{1}{i\hbar} \left[ H, \Phi^{(+)} \right] s \Phi^{(+)} + (\Phi^{(+)} + \Phi^{(-)}, \gamma),
\]

where the inner product \( (\Phi, \Psi) \) of two operators \( \Phi \) and \( \Psi \) has been defined by

\[
(\Phi, \Psi) = \int_0^\beta d\lambda \, \text{Tr} \left( \rho_0 e^{\lambda H} \Psi e^{-\lambda H} \right) = (\Psi, \Phi)
\]

and \( \gamma \) represents an external disturbing agent expressed in the form

\[
\gamma = X_1 \cdot J + X_2 \cdot Q
\]

*) The definition of \( \Phi^{(-)} \) in (1) differs from that in a previous paper in its sign.
in terms of the operators $J$ and $Q$ for the electric and thermal currents in the system and the disturbing forces

$$X_i = E + \frac{1}{e} F \zeta, \quad X_i = -\frac{1}{T} F T. \quad (5)$$

In (5) $E$, $\zeta$ and $T$ denote the external electric field, chemical potential and temperature respectively, and $e$ denotes the magnitude of electron charge. In the special case that the forces in (5) vanish except for a component $E_{\mu}(\mu = x, y \text{ or } z)$ of $E$ in a Cartesian coordinate and it equals the unity, the stationary value of (2) gives the electrical conductivity of the system.

We shall investigate the electron-phonon system whose Hamiltonian is expressed as

$$H = H_e + H_p + H_{e-i} + H_{e-p} + H_{p-p} + H_{p-i}, \quad (6)$$

where the respective components represent the energies of individual electrons and phonons and the interaction energies between electrons and impurities, between electrons and phonons, among phonons and between phonons and impurities. They can be written as

$$H_e = \sum_k \varepsilon_k a_k^* a_k, \quad H_p = \sum_q \hbar \omega_q b_q^* b_q,$$

$$H_{e-i} = \sum_k \sum_{k-k'} \langle k|v|k'\rangle a_k^* a_{k'},$$

$$H_{e-p} = \sum_k \sum_q C_q a_k^* a_k (b_q + b_q^*), \quad (7)$$

$$H_{p-p} = (3!)^{-1} \sum_{q,q',q''} F(q, q', q'') \xi_{q} \xi_{q'} \xi_{q''},$$

$$H_{p-i} = (2!)^{-1} \sum_{q,q'} F(q, q') \xi_{q} \xi_{q'},$$

where $a_k$ and $a_k^*$ denote the annihilation and creation operators for an electron with a wave number $k$, and $b_q^*$ and $b_q$ those for a phonon of a certain mode with a wave number $q$, and $\varepsilon_k$ and $\hbar \omega_q$ represent the energies of these electron and phonon. $\xi_q$ means a Fourier component of the displacement of lattice ions which relates with $b_q$ and $b_q^*$:

$$\xi_q = \sqrt{\frac{\hbar}{2M\omega_q}} (b_q + b_q^*), \quad (8)$$

where $M$ denotes the mass of a lattice ion. $\langle k|v|k'\rangle$ denotes a matrix element of the perturbing potential due to an impurity atom occupying the origin of the referred coordinate system and $\rho_{k-k'}$ a Fourier component of the distribution of impurity atoms scattered randomly on the lattice. $C_q$ denotes a coupling parameter of the electron-phonon interaction. In $H_{p-p}$ only three-phonon processes are taken into account and $F(q, q', q'')$ denotes a strength of the interaction causing such a process.

In relation to the form of the interaction Hamiltonians in (7) we can as-
sume the undetermined operators $\Phi^{(+)}$ and $\Phi^{(-)}$ as
\[
\Phi^{(+)} = \sum_{\kappa} \sum_{\ell} \langle \mathbf{k} | \phi_{\ell}^{(+)\dagger} | \mathbf{k} \rangle a_{\ell}^{\dagger} a_{\kappa} + \sum_{\kappa} \sum_{\ell} \langle \mathbf{q} | \phi_{\ell}^{(+)\dagger} | \mathbf{q} \rangle b_{\kappa}^{\dagger} b_{\ell}
\]
\[
+ \sum_{\kappa} \sum_{\ell} \left( \langle \mathbf{k} + \mathbf{q} | \phi_{\ell}^{(+)\dagger} | \mathbf{k} \rangle a_{\kappa} a_{\ell} b_{\ell} + h.c. \right)
\]
\[
+ \sum_{\kappa} \sum_{\ell} \sum_{\ell'} \left( \langle \mathbf{q} | \phi_{\ell}^{(+)\dagger} | \mathbf{q} \rangle b_{\ell}^{\dagger} b_{\ell'} + h.c. \right),
\]  

where h.c. means the hermitian conjugate. Using (9) as well as (7) in which $\xi_q$ is substituted by (8) and making a similar calculation as in the previous paper, we can obtain
\[
W = \left[ \frac{1}{2} \sum_{ \kappa, \ell, \kappa', \ell'} f_{\kappa, \ell} f_{\kappa', \ell'} \left( \{ s + i (\varepsilon - \varepsilon') / \hbar \} \langle \mathbf{k}' | \phi_{\ell}^{(-)} | \mathbf{k} \rangle \langle \mathbf{k} | \phi_{\ell'}^{(+)\dagger} | \mathbf{k}' \rangle \right)
\]
\[
+ \frac{i}{\hbar} \rho_{\kappa, \ell} \langle \mathbf{k} | \varepsilon | \mathbf{k}' \rangle \langle \mathbf{k}' | \phi_{\ell}^{(-)} | \mathbf{k} \rangle \langle \phi_{\ell}^{(-)} | \mathbf{k} - \phi_{\ell}^{(+)} | \mathbf{k} \rangle \right)
\]
\[
- \sum_{\kappa, \ell} \frac{f (1 - f')}{\hbar} \frac{N - (1 - f') f' (N + 1)}{\varepsilon' - \varepsilon - i \hbar \omega} \langle \mathbf{k} + \mathbf{q} | \phi_{\ell}^{(-)} | \mathbf{k}, \mathbf{q} \rangle
\]
\[
\langle \mathbf{k}, \mathbf{q} | \phi_{\ell}^{(+)\dagger} | \mathbf{k} + \mathbf{q} \rangle + \frac{i}{\hbar} G_q q_q' \left( \langle \mathbf{q} | \phi_{\ell}^{(-)} | \mathbf{k} + \mathbf{q} \rangle \langle \phi_{\ell}^{(+)\dagger} | \mathbf{k} + \mathbf{q} \rangle \langle \phi_{\ell}^{(-)} | \mathbf{k} + \mathbf{q} \rangle - (\pm) \right)
\]
\[
+ \frac{1}{2} \sum_{\kappa, \ell} \sum_{\kappa', \ell'} \frac{N - N'}{\hbar} \langle \mathbf{q} | \phi_{\ell}^{(-)} | \mathbf{q} \rangle \langle \phi_{\ell}^{(-)} | \mathbf{q} \rangle \langle \phi_{\ell}^{(-)} | \mathbf{q} \rangle
\]
\[
+ \frac{i}{\hbar} G_{q_q', q_q''} \left( \langle \mathbf{q} | \phi_{\ell}^{(-)} | \mathbf{q} \rangle \langle \phi_{\ell}^{(+)} \dagger | \mathbf{q} \rangle \langle \phi_{\ell}^{(-)} | \mathbf{q} \rangle + (\pm) \right)
\]
\[
+ \frac{1}{2} \sum_{\kappa, \ell} \left( \frac{\partial f}{\partial \varepsilon_k} \left( \phi_{\ell}^{(+\dagger)} + \phi_{\ell}^{(-)} \right) \eta_{\kappa, \ell} + \sum_{\ell} \frac{\partial N}{\partial h \omega} \left( \phi_{\ell}^{(+\dagger)} + \phi_{\ell}^{(-\dagger)} \right) \eta_{\kappa, \ell} \right),
\]  

where h.c. means the hermitian conjugate of the whole terms in the preceding square bracket. Use has been made of the abbreviations
\[
G_{q_q', q_q''} = \left( \frac{\hbar}{2 M} \right)^{3/2} (\omega_q \omega_{q'} \omega_{q''})^{-1/2} F(q, q', q''),
\]
\[
F(q, q', q'') = \frac{\hbar}{2 M} (\omega_q \omega_{q'} \omega_{q''})^{-1/2} F(q, q', q'')
\]
\[
\varepsilon = \varepsilon_k, \varepsilon' = \varepsilon_k', \omega = \omega_q, \omega' = \omega_{q'}, \omega'' = \omega_{q''}, f_f = f_0 (\varepsilon_k),
\]
\[
f' \equiv f_0 (\varepsilon_k') \text{ or } f_0 (\varepsilon_{k+q}), N = N_q(\omega), N' = N_q'(\omega'), N'' = N_q''(\omega''),
\]
\[
N = N_{q+q'}(\omega), N' = N_{q+q'}(\omega'), \text{ and } N'' = N_{q+q'}(\omega''),
\]
where $f_0 (\varepsilon)$ and $N_\varepsilon (\omega)$ denote the Fermi and Planck distribution functions of
electron and phonon respectively with energies $\varepsilon$ and $\hbar\omega$. The notation $(\pm)$ means the interchange between the superscripts $(\pm)$ and $(-\pm)$ in each curly bracket in (10). One more remark is needed on the notation in (10), that is, $\phi_{\gamma k}$ and $\phi_{\alpha q}$ denote the diagonal elements $\langle \mathbf{k}|\phi_{\gamma k}|\mathbf{k}\rangle$ and $\langle \mathbf{q}|\phi_{\alpha q}|\mathbf{q}\rangle$ and such terms as expressed as $\langle \mathbf{k}|\phi_{\gamma k}^{(+)}|\mathbf{k}\rangle$ and $\langle \mathbf{q}|\phi_{\alpha q}^{(+)}|\mathbf{q}\rangle$ should be regarded to have only off-diagonal elements. The operator $\eta$ given by (4) representing the effect of the external disturbance can be written in the form

$$\eta = \sum_k \eta_{\alpha,k} a_k^* a_k + \sum_q \eta_{\beta,q} b_q^* b_q. \quad (11)$$

We have thus the last two terms in (10).

By equating the derivatives of $W$ in (10) in respect of

$$\langle \mathbf{k}'|\phi_{\gamma k}^-|\mathbf{k}\rangle, \langle \mathbf{q}'|\phi_{\alpha q}^-|\mathbf{q}\rangle, \langle \mathbf{k} + \mathbf{q}|\phi_{\gamma q}^(-)|\mathbf{k}, \mathbf{q}\rangle \text{ and } \langle \mathbf{q}'|\phi_{\alpha q}^-|\mathbf{q}\rangle$$

to zero owing to the stationary condition for $W$, we get the relations

$$\langle \mathbf{k}'|\phi_{\gamma k}^+|\mathbf{k}\rangle = (\varepsilon' - \varepsilon + i\hbar s)^{-1} \rho_{\gamma k-k}\langle \mathbf{k}|\phi_{\gamma k}^+|\mathbf{k}\rangle, \langle \mathbf{q}'|\phi_{\alpha q}^+|\mathbf{q}\rangle = h^{-1}(\omega' - \omega + i\hbar s)^{-1} \rho_{\alpha q-q}\langle \mathbf{q}|\phi_{\alpha q}^+|\mathbf{q}\rangle, \quad (12)$$

and similarly the relations among the matrix elements of $(-\pm)$ species.

By substituting (12) and the similar relations among the matrix elements of $(-\pm)$ species into (10) and by making use of the transformation which is useful to discuss the dissipative phenomena

$$\phi = (\phi^{(+)} + \phi^{(-)})/2, \quad \psi = (\phi^{(+) - \phi^{(-)})}/2, \quad (13)$$

we can obtain in the limit $s \to +0$

$$W = 2\langle \phi, \eta \rangle - \langle \phi, L\phi \rangle + \langle \psi, L\psi \rangle, \quad (14)$$

by the use of the classical inner product

$$\langle \phi, \psi \rangle = -\sum_k \left[ \frac{\partial}{\partial \varepsilon_k} f_0(\varepsilon_k) \phi_{\gamma k} \psi_{\gamma k} - \sum_q \frac{\partial}{\partial \hbar\omega_q} N_0(\omega_q) \phi_{\alpha q} \psi_{\alpha q} \right] \quad (15)$$

of the operation $L$ applied to $\phi$ expressed by the inner product with a functional vector $\chi$

$$\langle \chi, L\phi \rangle = \sum_k \sum_q L_{\chi q} \langle \mathbf{k}', \mathbf{k} \rangle \chi_{\gamma k} \phi_{\gamma k} - \chi_{\alpha q} \phi_{\alpha q} \quad (16)$$

$$+ \sum_k \sum_q L_{\chi q} \langle \mathbf{k}', \mathbf{q} \rangle \chi_{\gamma k} \phi_{\gamma k} + \chi_{\alpha q} \phi_{\alpha q}$$

$$+ \sum_q \sum_{\chi q'} L_{\chi q'} \langle \mathbf{q}', \mathbf{q} \rangle \chi_{\gamma q} \phi_{\gamma q} - \chi_{\alpha q} \phi_{\alpha q}$$

$$+ \sum_{\chi q'} \sum_{\chi q''} L_{\chi q''} \langle \mathbf{q}', \mathbf{q}'', \mathbf{q} \rangle \chi_{\gamma q} \phi_{\gamma q} - \chi_{\alpha q} \phi_{\alpha q}$$

$$+ \sum_{\chi q''} \sum_{\chi q''' q'''} L_{\chi q'''} \langle \mathbf{q}', \mathbf{q}'', \mathbf{q} \rangle \phi_{\gamma q} - \chi_{\alpha q} \phi_{\alpha q}$$

$$+ \sum_{\chi q''} \sum_{\chi q''' q'''} L_{\chi q'''} \langle \mathbf{q}', \mathbf{q}'', \mathbf{q} \rangle \phi_{\gamma q} - \chi_{\alpha q} \phi_{\alpha q}$$
where the coefficients are given by

\[
L_{e-i}(k, k') = -\frac{\pi}{\hbar k T} n_i \langle k | v | k' \rangle^2 f_0(\varepsilon_k) [1 - f_0(\varepsilon_{k'})] \delta (\varepsilon_k - \varepsilon_{k'}),
\]

\[
L_{e-s}(k, q) = \frac{2\pi}{\hbar k T} |C_q|^2 f_0(\varepsilon_k) [1 - f_0(\varepsilon_{k + q})] N_0(\omega_q) \delta (\varepsilon_k - \varepsilon_{k + q} - \hbar \omega_q),
\]

\[
L_{p-i}(q, q') = \frac{\pi}{\hbar k T} |G_{q, q'}|^2 N_0(\omega_q) \left[ N_0(\omega_{q'}) + 1 \right] \delta (\omega_q - \omega_{q'}),
\]

\[
L_{p-s}(q, q', q'') = \frac{\pi}{\hbar k T} |G_{q, q', q''}|^2 \left[ N_0(\omega_q) + 1 \right] N_0(\omega_{q'}) N_0(\omega_{q''}) \delta (\omega_q - \omega_{q'} - \omega_{q''}).
\]

In deriving \( L_{e-i}(k, k') \) in (16), an averaging has been taken on random distributions of the impurities whose concentration is denoted by \( n_i \).

The expression (14) should be made stationary in respect of any variations of \( \phi \) and \( \psi \). That condition on \( \phi \) is quite trivial and is satisfied by taking \( \phi = 0 \), where (14) is minimum with regard to \( \phi \). Then (14) reduces to

\[
W(\phi) = 2\langle \phi, \eta \rangle - \langle \phi, L\phi \rangle,
\]

which should now be made maximum as a functional of \( \phi \). This is just a classical variation principle of the Umeda-Kohler type in the electron-phonon system.

In fact, on the basis of the Boltzmann-Bloch equation for the electron-phonon system, Sondheimer\(^3\) has obtained the second summand for the electron-phonon collision operation and has shown that the Kelvin relations in thermoelectricity hold even if the phonon distribution is allowed to deviate from thermal equilibrium. Ziman and Kasuya have generalized more so as to take account of interactions other than the electron-phonon interaction and have discussed the variation principle.\(^9\) The variation principle can be established on the basis of the positive-definite (strictly speaking non-negative) and self-adjoint properties of \( L \)

\[
\langle \phi, L\phi \rangle \geq 0, \quad \langle \phi, L\phi \rangle = \langle \phi, L\phi \rangle.
\]

Ziman has shown\(^6\) that the rates of temporal changes in the entropy of the electron-phonon system due to the collision processes and to the drift of a phase space point can be expressed respectively as

\[
\left\langle \frac{\partial S}{\partial t} \right\rangle_c = \langle \phi, L\phi \rangle / T, \quad \left\langle \frac{\partial S}{\partial t} \right\rangle_d = -\langle \phi, \eta \rangle / T.
\]

Then the variation principle can be expressed in the thermodynamical language that the distribution \( \phi \) should be determined in a steady state so as to make

\[-2(\partial S/\partial t)_c - (\partial S/\partial t)_d \]

maximum. It can be stated alternatively that \( (\partial S/\partial t)_c \) should be made maximum under the condition \( (\partial S/\partial t)_c + (\partial S/\partial t)_d = 0 \), or that \( (\partial S/\partial t)_c^2/(\partial S/\partial t)_c \) should be maximum. The maximum value is equal to the entropy production \( (\partial S/\partial t)_c \) due to the collision process in the steady state,
which is equal to the electrical conductivity in the case that $\eta$ equal to a component $E_\mu=1\{\mu=x,y,\text{or} z\}$ of the electric field in a Cartesian coordinate.

The reciprocal relations between the transport coefficients or the Kelvin relations in thermoelectricity can be shown to hold on the basis of the self-adjoint property of $L$. If we write $\phi=X_1\cdot \phi_1+X_2\cdot \phi_2$ corresponding to $\eta$ written in such a form as in (4), the averaged values of $\overline{J} = \langle \phi, J \rangle$ and $\overline{Q} = \langle \phi, Q \rangle$ can be expressed as

$$\overline{J} = \langle \phi_1, L\phi_1 \rangle \cdot X_1 + \langle \phi_2, L\phi_2 \rangle \cdot X_2,$$

$$\overline{Q} = \langle \phi_2, L\phi_2 \rangle \cdot X_1 + \langle \phi_1, L\phi_1 \rangle \cdot X_2,$$

(20)

where the reciprocal relations $\langle \phi_i, L\phi_j \rangle = \langle \phi_j, L\phi_i \rangle$ for $i, j=1 \text{ or } 2$ or the Kelvin relations are satisfied on account of the self-adjoint property of $L$.

The parallelism between the quantum and the classical variation principles is appearing not only in the fact that there is a similarity between (2) and (14) which should be made stationary and (2) can be reduced to (14) by means of a perturbational treatment, but also in the fact that the definition of the inner product (3) in the quantum case quite analogous to the definition (15) in the classical case can be regarded as a natural extension of (15) and is reduced to (15) by means of a perturbational approximation. In fact if we use the expression (9) for $\Phi^{(\pm)}$ in the inner product $(\Phi^{(\pm)}, \Phi^{(\pm)})$ defined by (3) and retain only diagonal terms such as $\langle k | \phi^{(\pm)}_\sigma | k \rangle = \phi_{\sigma,k}$ and $\langle q | \phi^{(\pm)}_\nu | q \rangle = \phi_{\nu,q}$ neglecting all the others which are off-diagonal and of higher order in the perturbation, we get

$$(\Phi^{(-)}, \Phi^{(+)} = -\sum_{\mathcal{K}} \left[ \frac{\partial}{\partial \varepsilon_k} f_k(\varepsilon_k) \right] \phi_{\sigma,k}^{(+) - \sum_{\mathcal{Q}} \frac{\partial}{\partial \omega_q} N_q(\omega_q) \phi_{\nu,q}^{(-)} = \langle \phi, \phi \rangle,$$

(21)

where use has been made of the fact that $\sum_k f_k(\varepsilon_k) \phi_{\sigma,k}^{(+) = \sum_q N_q(\omega_q) \phi_{\nu,q}^{(-)} = 0$ for the reason that $\phi_{\sigma,k}$ and $\phi_{\nu,q}$ which are driven by the forces $\tau_{\sigma,k}$ and $\tau_{\nu,q}$ should be vector-like as well as their agents $\eta_{\sigma,k}$ and $\eta_{\nu,q}$. We can also obtain more directly, in the lowest order of perturbation, from (1) of plus sign in which (9) is substituted

$$f_k = \text{Tr} (\rho (t) a_k^\dagger a_k) = f_0(\varepsilon_k) - \frac{\partial f_0(\varepsilon_k)}{\partial \varepsilon_k} \phi_{\sigma,k},$$

$$N_q = \text{Tr} (\rho (t) b_q^\dagger b_q) = N_0(\omega_q) - \frac{\partial N_0(\omega_q)}{\partial \omega_q} \phi_{\nu,q},$$

(22)

which represents distribution functions respectively for electrons and phonons. Thus in many respects it can be said that our quantum variation principle is a natural extension of the classical variation principle of the Umeda-Kohler type.
§ 3. Concluding remarks

The quantum variation principle on the transport phenomena in the electron-phonon system has been reduced to the classical variation principle of the Umeda-Kohler type by means of a perturbational treatment. The effect of Umklapp processes as well as the multiband effect has been neglected at all and only a single mode of phonon polarization has been taken into account, but it is straightforward to remove such restrictions in our reduction as well as to introduce any more modification in the form of the interaction Hamiltonian without changing the essential feature of the reduction.

It is also remarked that the quantum linear response theory of thermal as well as dynamical disturbances on which the quantum variation principle is based shows that the reciprocal relations hold between the transport coefficients; therefore, from the present point of view the existence of these relations or the Kelvin relation in thermoelectricity is quite evident in the classical variation principle which has been derived from the quantum one.

References

   D. ter Haar and A. Neaves, Phil. Mag. Suppl. 5 (1956), 391.
   M. Kohler, Z. Physik 124 (1948), 772.