Atomistics and the Divisibility of Space and Time

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§1. Introduction

In any attempt at a unified and consistent theory of elementary particles, its starting point was quantum field theory, although the direction of generalization or modification in one attempt may have been widely different from that in another. This was largely due to the great success of quantum electrodynamics, which came very close to perfection in its refined formulation by Tomonaga and also by Schwinger and Feynman. The remaining problem of consistency or convergence has been reduced, at this stage, almost to a subject of academic interest. Applications of quantum field theory to nuclear and subnuclear phenomena by including nucleon, pion, muon and neutrino fields in addition to electromagnetic and electron fields were successful in obtaining semi-quantitative, or, in some cases, even quantitative agreements with experiments, but the ever increasing number of particles and resonances defied the conventional scheme of ascribing specific field to each of so-called elementary particles. More recently, the models, in which only few of so-called elementary particles are regarded as more fundamental and all the others as composite systems of these fundamental particles, served considerably for systematization of a great variety of particles and resonances. In particular, the choice in Sakata model of three fundamental spin one-half fields seemed to be a clue to find interrelations between particles belonging to hadron group. As things went on in this direction further, however, it turned out that the fundamental particles associated with these three spin one-half fields are not likely to be identified with any set of familiar members of hadron group, but more likely to be identified with a very strange type of particles which were called quarks or aces in Gell-Mann-Zweig scheme or alternatively with less strange, but so far unobserved particles in modified schemes. Such a situation seems to reinforces our scepticism about straightforward applications of quantum field theory to subnuclear phenomena.

Long before the discoveries of great many kinds of particles and resonances, there were already doubts about the application of field concept to cases other than electromagnetic and gravitational fields, which were the two
outstanding examples in classical physics. The classical field was the totality of a certain physical quantity, such as the force acting on the material particle, which could be determined at each point by an appropriate experiment. In order that quantum mechanics is a consistent theory applicable to atomic phenomena accompanied by the emission and absorption of radiation, electromagnetic field was to be quantized just as dynamical variables for the electron as a point particle was subject to quantization. There was no doubt about it except unlimited validity of special relativity and/or quantum mechanics in the case of electromagnetic field which has infinite degrees of freedom. On the other hand, the equivalence of the quantum mechanical description of a system of point particles in configuration space to another description in terms of quantized field was proved rigorously both for bosons and fermions in the nonrelativistic limit. This was rather surprising because the quantized field for particles with non-zero rest mass had no correspondence to the classical field. Nevertheless, the generalization of the method of second quantization to relativistic region turned out to be both powerful and fruitful in dealing with all kinds of particles including photons.

Such was the situation until about fifteen years ago. Subsequent developments in high energy physics, however, did not guarantee the validity of quantum field theory as applied to high energy phenomena, in which each of the observed particles was represented by a quantized field separately. As already mentioned, one of the conspicuous trends in the current developments in the theory of elementary particles is to look for the next sub-level, on which all or most of the so-called elementary particles are reduced to systems consisting of few kinds of more fundamental entity. If one tries to go further in this direction, one meets with the fundamental question: Can this new entity be represented adequately by the quantized field in the usual sense? If one follows the line initiated by Chew which keeps to the so-called democratic view of particles, he would also be compelled to meet with a similar question, unless he gives up to go further than S-matrix formalism in order to describe subnuclear phenomena more in detail.

Under these circumstances, it seems to be worthwhile to remind again the wide gap between the concept of classical field and that of quantized field. In this connection, it is to be pointed out that there is another feature in classical field theory which has no counterpart in quantum field theory. Namely, classical field is associated, in general, with observable quantities at each point in space-time world and the gravitational field, in particular, is associated directly with the measurements of geometrical properties of the world, according to general theory of relativity. Furthermore, these properties, as represented by gravitational tensor field, and the dis-
tribution and motion of matter, as represented by energy-momentum tensor field, are connected with each other by simple equations. This unique feature of general theory of relativity has not been inherited to quantum field theory in the following sense. As a matter of fact, there have been attempts at the quantization of gravitational field, but their intention was to carry out the formal procedure in accordance with quantum field theory. One of the most essential points of general theory of relativity was the establishment of the interrelation between the geometry of large scale physical world and the distribution and motion of matter in it. If we suppose that there are similar interrelations also in the small scale world, those which are obtained as formal change of classical Einstein equations by quantizing gravitational field together with matter field must be relatively unimportant, because the characteristic length which is constructed from the gravitational constant, Planck's constant and velocity of light is very small compared with the length characterizing the scale of the world of elementary particles. Therefore, if we intend to resurrect the spirit of general theory of relativity in the world of elementary particles, we have to look for the possibility of finding interrelations between the geometrical properties, in a general and abstract sense, of the world of elementary particles and their more directly observable properties. There have been attempts at a unified theory of elementary particles based on the assumption that each particle has an extended structure in the space-time world. However, such an attempt is very likely to meet sooner or later with the problem of interrelation mentioned just above. Contrary to the case of general relativity dealing with classical field, the unlimited divisibility of space and time into smaller parts may well become the question in the theory of elementary particles which is to be taken seriously. In a sense, one can say that this very question was raised and solved already in atomic theory in its own way. Namely, according to nineteenth century view of atoms, matter consists of atoms each with a finite size. Hence, the measurement of length of a material body could not be more precise than the radius of an atom. The reason why such a situation did not contradict the assumption of unlimited divisibility of physical space came, of course, from the discovery of further divisibility on the part of the atom. An atom was divided into electrons and a nucleus, both of which turned out to be much smaller than the atom itself. Since the advent of quantum mechanics, most physicists have been inclined to think that the contradiction of the atomistic structure of matter and energy and the continuity of space and time disappeared forever by the unification of particle and wave aspects of matter and energy. Namely, in nonrelativistic quantum mechanics, only these systems which have finite degrees of freedom are treated, and furthermore, there is a correspondence in each case to a system consisting of a finite
number of point particles with or without spin. For instance, what is usually called rigid body in nonrelativistic quantum mechanics is nothing but an idealized case of many particle system, in which all the degrees of freedom other than the rotation as a whole can be ignored for some reason or other. However, if we go over to relativistic quantum theory, we have to deal also with systems which have infinite degrees of freedom according to the usual formalism. This gives rise to the year-old problem of divergence, which, in turn, brings about the following new question: In what way can the atomistic structure of matter and energy be reconciled with the assumption of unlimited divisibility of space and time on the subnuclear level? This is the question to which the author is going to try to give a tentative and crude answer in the subsequent sections. It will be different from the answer mentioned above on the atomic level.

§2. Elementary domains and elementary particles

The compatibility of the discreteness of matter and energy with the continuity of space and time on the atomic level is best shown by the second quantization formalism in nonrelativistic quantum mechanics. Namely, a quantum mechanical system consisting of any number of identical particles without spin and other extra degrees of freedom can be described by a scalar field operator \( \psi(x, t) \) and its hermitian conjugate \( \bar{\psi}(x, t) \) satisfying the commutation relations

\[
\begin{align*}
[\psi(x, t), \psi(x', t)] &= \delta(x-x'), \\
[\psi(x, t), \bar{\psi}(x', t)] &= 0, \\
[\bar{\psi}(x, t), \bar{\psi}(x', t)] &= 0,
\end{align*}
\]

where \( x \) stands for three space coordinates and the double sign \( \mp \) discriminates between bosons and fermions. \( \psi(x, t) \) is interpreted as the operator of annihilation at a point \( x \) and at an instant \( t \) of a particle, which can have only positive energy. Accordingly, \( \bar{\psi}(x, t) \) is interpreted as the creation operator of such a particle. The appropriateness of this interpretation is guaranteed by the positive definiteness of the Hamiltonian for the system.

By starting from this safe ground, one could have proceeded in various directions in order to approach a satisfactory theory of elementary particles. However, what has been done was, almost exclusively, the formal extension to relativistic cases retaining the infinitesimal or local character of commutation relations (1) for the local field. Roughly speaking, such an approach presupposes the decomposition of matter and energy into particles and quanta which can be localized at a point at any instant, while the continuity of space and time is taken for granted. Instead, one could have proceeded
in a somewhat different direction as shown in the following.

Let us imagine a small three dimensional space domain $D$ and introduce the operators

$$\varphi(D,t) = \frac{1}{\sqrt{V_D}} \int_D \varphi(x,t) \, dx,$$

$$\bar{\varphi}(D,t) = \frac{1}{\sqrt{V_D}} \int_D \bar{\varphi}(x,t) \, dx$$

which satisfy the commutation relation

$$[\varphi(D,t), \bar{\varphi}(D,t)] = 1,$$  \hspace{1cm} (3)

where $\int_D$ denotes the integration over the domain with the volume $V_D$. Further, if we consider another domain $D'$ and define the operators $\varphi(D',t)$ and $\bar{\varphi}(D',t)$ in the same way as (2), we have the commutation relations

$$[\varphi(D,t), \varphi(D',t)] = V_{D'D'} / \sqrt{V_D V_{D'}},$$

$$[\varphi(D,t), \bar{\varphi}(D',t)] = 0,$$

$$[\bar{\varphi}(D,t), \bar{\varphi}(D',t)] = 0,$$  \hspace{1cm} (4)

where $V_{D'D'}$ is the volume of the domain $D \cap D'$ common to $D$ and $D'$. Thus, if the domain $D'$ does not overlap with $D$, we have

$$[\varphi(D,t), \bar{\varphi}(D',t)] = 0.$$  \hspace{1cm} (4)'

$\varphi(D,t)$ and $\bar{\varphi}(D,t)$ can naturally be interpreted as the operators annihilating and creating all at once at an instant $t$ an extended particle which occupies uniformly the domain $D$. Accordingly the number operator for such particles is defined by

$$n(D,t) = \varphi(D,t) \bar{\varphi}(D,t)$$

which is commutative with $n(D',t)$ defined for any domain $D'$ other than $D$, only if there is no overlapping between $D$ and $D'$.

At this point, one can depart from the usual nonrelativistic quantum mechanics by forgetting the definition (2) of $\varphi$, $\bar{\varphi}$ in terms of local field operators and by starting right from the commutation relations (4). Of course, we have to impose some restriction on the size and shape of the domains. Otherwise, we shall come back again to usual theory in the limit, in which each of the domains shrinks to a point in space. One of the most simple ways of restriction is to consider only the spherical domains of radius $l_o$, which can be located arbitrarily in space. Let us call any one of such spherical domain an *elementary domain*. An elementary particle in this picture is an object which occupies an elementary domain all at
Atomistics and the Divisibility of Space and Time

once at any instant. If we consider a system consisting of any number of elementary particles in this sense, which are confined in a very large box, there are only finitely many number operators \( n(D_1, t), n(D_2, t), \ldots n(D_N, t) \), which are commutative with each other, where each of \( D_1, D_2, \ldots D_N \) is an elementary domain and none of them overlaps with another. If we consider further \( n(D', t) \), of an arbitrary elementary domain which does not belong to the original set \( \{D_1, D_2, \ldots D_N\} \), \( n(D', t) \) cannot be commutative with all of \( n(D_1, t), n(D_2, t), \ldots, n(D_N, t) \). Otherwise, \( n(D', t) \) had to be included in the set \( \{n(D_1, t), n(D_2, t), \ldots, n(D_N, t)\} \). In this sense, the set \( \{D_1, D_2, \ldots D_N\} \) can be called a complete set of elementary domains. Such a complete set is obtained by filling the whole space in the box with elementary spheres which do not overlap with each other, any further addition of elementary spheres being impossible without overlapping with those already in the box.

Obviously, there are great many possibilities in choosing a complete set. If we defined the center of any domain \( D \) by three coordinates

\[
X = \int_D x dx / \sqrt{V_D}
\]

(6)

the complete set \( \{D_1, D_2, \ldots D_N\} \) is characterized by \( 3N \) parameters \( X^{(\alpha)}, X^{(\beta)}, \ldots X^{(\gamma)} \) which are restricted by the conditions

\[
|X^{(\alpha)} - X^{(\beta)}| \geq 2l_0, \quad r, s = 1, 2, \ldots N
\]

(7)

where

\[
X^{(\alpha)} = \int_{D_r} x dx / \sqrt{V_r}, \quad V_r = 4\pi r_0^3 / 3
\]

(8)

and \( |X^{(\alpha)} - X^{(\beta)}| \) denotes the distance between two points \( X^{(\alpha)} \) and \( X^{(\beta)} \).

Now, in order that the description of our system by means of a complete set \( \{D_1, D_2, \ldots D_N\} \) is equivalent to the description by means of another set \( \{D'_1, D'_2, \ldots D'_{N'}\} \), we may assume linear relations

\[
\begin{align*}
\varphi'(D'_\rho, t) &= \sum_r c_{\rho r} \varphi(D_r, t), \\
\bar{\varphi}'(D'_\rho, t) &= \sum_r \bar{c}_{\rho r} \bar{\varphi}(D_r, t),
\end{align*}
\]

(9)

where \( \bar{c}_{\rho r} \) is complex conjugate of \( c_{\rho r} \). Since the transformation (9) must have its inverse, we have to restrict ourselves to the cases \( N' = N \). Obviously, the relations (9) cannot be derived from the original definition (2), but are the new assumptions, which keep us from going back to quantum mechanical system consisting of point particles. However, one can derive the commutation relations

\[
[\varphi(D_r, t), \bar{\varphi}(D_s, t)]_+ = \delta_{rs}, \quad \}
\]

\[
[\varphi(D_r, t), \bar{\varphi}(D_s, t)]_+ = \delta_{rs}, \quad \}
\]
from the relations (4) which we retain and the same commutation relations also hold for \( \varphi'(D_r, t) \) and \( \varphi'(D_s, t) \). Thus, it follows that the transformation (9) must be unitary:

\[
\sum_r c_{\rho r} \bar{c}_{\sigma r} = \delta_{\rho \sigma}, \quad \rho, \sigma = 1, 2, \ldots N.
\]

(11)

Of course, there is the transformation in our case that corresponds to the transformation of coordinate system in the usual sense. But the unitary transformation (9) from one set of operators to another is to be regarded as a new and more general kind of transformation peculiar to our case.

Now, we go over to dynamics of our system. The case of free particles in the usual theory can be easily modified so as to represent free extended particles in our case. Namely, the Hamiltonian operator \( H \) in the equations of motion

\[
\frac{i\hbar}{\Delta t} \frac{\partial \varphi_r}{\partial t} = [\varphi_r, H]
\]

(12)

is assumed to have the form

\[
H = \sum_r \bar{\varphi}_r H_r \varphi_r
\]

(13)

where

\[
\varphi_r = \varphi(D_r, t), \quad \bar{\varphi}_r = \bar{\varphi}(D_r, t)
\]

and \( H_r \) constitute a hermitian matrix with \( N \) rows and columns and \( \Delta t \) is a very small time interval of the order of \( \ell_0/c \). The correspondence to the usual Hamiltonian for free particles in the limit \( \ell_0 \to 0 \) is secured by choosing the matrix element \( H_r \) such that it does not vanish, only if \( D_r \) and \( D_s \) are neighboring domains to each other. More precisely, for two spheres \( D_r, D_s \) with

\[
2\ell_0 \leq |X^{(r)} - X^{(s)}| < 4\ell_0
\]

(14)

one can take \( H_r = -H_s \), which corresponds to a product of \( \bar{\varphi}(x, t) \) and the first derivative of \( \varphi(x, t) \) in the direction of the vector \( X^{(r)} - X^{(s)} \) in the limit \( \ell_0 \to 0 \). The situation will be only a little more complicated, if we try to reproduce the term which becomes a product of first derivatives both of \( \varphi(x, t) \) and \( \bar{\varphi}(x, t) \) in the limit \( \ell_0 \to 0 \).

In order that such a correspondence is not lost by changing the choice of the complete set of elementary domains, the coefficients \( c_{\rho r} \) appearing in the transformation (9) have to be severely restricted by conditions in ad-
Atomistics and the Divisibility of Space and Time

§3. Modifications of the model

The simple model, which was considered in §2, has a great deal of similarity to a classical model in dynamical theory of gases. In the latter, a great number of small rigid spheres with the same radius move in a large box and collide with each other and with the wall of the box according to laws of classical dynamics. Among two possible choices in the commutation relations (10), the fermion case corresponding to the anticommutativity resembles the classical model more than the boson case corresponding to the commutativity, because the impenetrability of matter is retained in the former case. However, there are also essential differences between our model and the classical model. For one thing, whereas the part of space in the box which is not occupied by the rigid sphere is simply an empty space in classical model, the whole space in the box is covered by elementary spherical domains irrespective of whether they are occupied or empty in our model. This stems from the fundamental difference between classical and quantum mechanics. Whereas the existence and non-existence of an object at a certain part of space at a certain instant are discriminated from each other without any ambiguity in classical dynamics, they are to be considered simultaneously with respective probability, in general, in quantum mechanics. Thus, even when a domain becomes empty, we may imagine that the domain itself remains from quantum theoretical point of view.

In spite of such a difference, the similarity above mentioned is advantageous for modifications of our simple model. In the classical model, the rotation of each of rigid spheres could be ignored by assuming that its surface is smooth. In our model as considered in §2, the rotational degrees of freedom could not come in, too, because the rotation of a spherical domain around its center does not mean anything. Hence, it is necessary to take into account the orientation of each elementary domain by modifying the simple model in some way or other, in order to deal with the rotation of an extended particle.

At this point, an abstraction of the intuitive model is needed. Namely, each elementary domain is supposed to be characterized by three Euler angles $\theta_1$, $\theta_2$, $\theta_3$ defining its orientation in addition to three coordinates $X_1$, $X_2$, $X_3$. The case of interacting particles in usual theory can also be transcribed in our language by adding fourth order terms to the right hand side of (13). However, it does not seem to be worthwhile to go into details, because our system is nothing more than a very crude and unrealistic model. Certainly, generalizations and abstractions of our model is necessary in order to approach a realistic theory of extended particles, as will be discussed in the next section.
\(X_2, X_3\) defining its center. Further, let us assume that the operators 
\(\varphi (X_1, X_2, X_3; \theta_1, \theta_2, \theta_3)\) \(, \varphi (X_1, X_2, X_3; \theta_1, \theta_2, \theta_3)\) ascribed to each oriented domain commute or anticommute with each other, if the centers of two domains coincide, but the orientations are different. Since it is not easy to derive such consequences from any simple modification or generalization of (1) and (2), from which we started, it is to be admitted that our modified model departs much further from the starting point than the simple model treated in §2.

In any case, the system now in question is described by the set of operators

\[
\varphi (X_1^{(r)}, X_2^{(r)}, X_3^{(r)}; \theta_1^{(r)}, \theta_2^{(r)}, \theta_3^{(r)}; t), \quad \varphi (X_1^{(s)}, X_2^{(s)}, X_3^{(s)}; \theta_1^{(s)}, \theta_2^{(s)}, \theta_3^{(s)}; t)
\]

with \(r=1, 2, \cdots, N\), where \(X^{(r)}, X^{(s)}, \cdots X^{(\infty)}\) are mutually restricted by the relations

\[
|X^{(r)} - X^{(s)}| \geq 2l_0, \quad \text{for } r \neq s
\]

and each set of \(\{\theta_1^{(r)}, \theta_2^{(r)}, \theta_3^{(r)}\}\) are continuous parameters identical with the Euler angles

\[
0 \leq \theta \leq \pi, \quad 0 \leq \psi < 2\pi, \quad 0 \leq \phi < 2\pi
\]

in the usual notation. As well-known, the wave function of a rigid body in quantum mechanics can be expanded into series of eigenfunctions

\[
u_{JKN}(\theta_1, \theta_2, \theta_3)
\]

belonging to eigenvalues of angular momentum, where \(J\) is either a positive integer including zero or a positive half integer and

\[
K = J, J-1, \cdots, -J+1, -J, \quad M = J, J-1, \cdots, -J+1, -J.
\]

The absolute square of angular momentum is \(\hbar^2 J(J+1)\) and \(K\) and \(M\) are its third components in the body frame and the laboratory frame respectively. The eigenfunction \(\nu_{JKN}(\theta_1, \theta_2, \theta_3)\) is either one valued or two valued according as \(J\) is integer or a half integer. We can expand \(\varphi\) in (15) into series of eigenfunctions \(\nu_{JKN}\):

\[
\varphi (X_1^{(r)}, X_2^{(r)}, X_3^{(r)}; \theta_1^{(r)}, \theta_2^{(r)}, \theta_3^{(r)}; t) = \sum_{JKN} \phi_{JKN}(X^{(r)}, t) u_{JKN}(\theta_1^{(r)}, \theta_2^{(r)}, \theta_3^{(r)})
\]

and expand \(\varphi\) similarly. If \(u_{JKN}\) is properly normalized, we obtain the commutation relations

\[
[\varphi_{JKN}(X^{(r)}, t), \varphi_{J'K'M'}(X^{(s)}, t)] \_ = \delta_{rr'} \delta_{JJ'} \delta_{KK'} \delta_{MM'},
\]

\[
[\varphi_{JKN}(X^{(r)}, t), \varphi_{J'K'M'}(X^{(s)}, t)] \_ = 0
\]
Atomistics and the Divisibility of Space and Time

\[ [\tilde{\varphi}_{JKM}(X^{(0)}, t), \tilde{\varphi}_{j'k'm'}(X^{(0)}, t)] = 0. \]

\( u_{JKM}(X^{(0)}, t), \bar{u}_{JKM}(X^{(0)}, t) \) can be interpreted as the annihilation and creation operators of a particle which occupies the domain \( D^{(0)} \) and is rotating with the spin angular momentum designated by \( J, K, M. \)

In this connection, it is to be remarked that Hara has shown recently the equivalence of \( \varphi_{JKM}(X^{(0)}, t) \) with Dirac wave functions in the case of \( J=1/2. \)

There is, however, another modification of our model which departs from the original model much less than the modification discussed just above. Namely, the operators \( \varphi(D), \bar{\varphi}(D) \) defined by (2) can be generalized to

\[
\begin{align*}
\varphi(f(x), t) &= \int_D f(x) \psi(x, t) \, dx, \\
\bar{\varphi}(f(x), t) &= \int_D \tilde{f}(x) \tilde{\psi}(x, t) \, dx,
\end{align*}
\]

where \( f(x) \) is an arbitrary function defined in the domain \( D. \) If we introduce the spherical coordinates \((r, \theta, \phi), \) with its origin at the center \( X \) of the domain \( D, f(x) \) can be expanded into series of spherical harmonics

\[ f(x) = \sum_{i,m} f_{in}(r) Y_{in}(\theta, \phi) \]

and the new set of operators

\[ \varphi_{in}(X^{(0)}, t) = \frac{1}{g_{in}} \int_{D_i} f_{in}(r) Y_{in}(\theta, \phi) \psi(x, t) r^2 \sin \theta \, dr \, d\theta \, d\phi \]

and their hermitian conjugates for the elementary spherical domains \( D_1, D_2, \cdots D_N \) satisfy the commutation relations

\[
\begin{align*}
[\varphi_{in}(X^{(0)}, t), \bar{\varphi}_{i'm'}(X^{(0)}, t)] &= \delta_{in}\delta_{im}\delta_{m'm'}, \\
[\varphi_{in}(X^{(0)}, t), \varphi_{i'm'}(X^{(0)}, t)] &= 0, \\
[\bar{\varphi}_{in}(X^{(0)}, t), \bar{\varphi}_{i'm'}(X^{(0)}, t)] &= 0,
\end{align*}
\]

where \( g_{in} \) in (24) is defined by

\[ g_{in} = \sqrt{\int_0^l f_{in}(r) f_{in}(r) r^2 \, dr}. \]

Thus \( \varphi_{in}(X^{(0)}, t), \varphi_{in}(X^{(0)}, t) \) can be interpreted as the annihilation and creation operators for a particle with the radius \( l_0, \) the center located at \( X^{(0)} \) and the spin angular momentum specified by \( l, m. \) An advantage of this modified model is that the vibrational motion can be taken into account, in addition to the rotational motion, by further decomposition of \( f_{in}(r) \) into vibrational eigenfunctions. However, there is a disadvantage that the spin
of a particle is restricted to zero or an integer.

One can think of various possibilities of further modification so as to include more degrees of freedom of internal motion which were considered already in nonlocal field theory, but this will be left for other papers which will come out subsequently. Instead, the essential difference between the present approach and the one based on the concept of nonlocal field will be discussed a little further in the next section.

§4. Concluding remarks

In local field theories, each point in space is the seat which a point particle can occupy or else is empty. Thus there is no need to discriminate between three dimensional space continuum and the whole set of seats for the particles. On the contrary, if we suppose that each of the elementary or more fundamental particles is an entity with spatially extended structure, each of the seats for it is to have also a finite volume. As already pointed out, there is no clear-cut distinction between the empty and occupied seats, but they are probabilistically connected with each other, if the entity is a quantum theoretical object. In such a case, the totality of seats remains irrespective of which seats are empty or occupied and is to be discriminated from the space continuum. The discreteness of matter and energy and the unlimited divisibility of space can be reconciled with each other in this way. Although the whole argument is very crude and incomplete, it may be a possible answer to, at least, a part of the question which was raised at the end of §1. The argument is very incomplete as for the question of unlimited divisibility of time, in particular. If we try to answer this part of the question, we have to consider the implication of relativity seriously as will be done in subsequent papers.

Now it is clear that nonlocal field theories hitherto developed differ from the present point of view in that there is no concept of empty seat in the former, although each particle is assumed to be extended in space. This gave rise to the new difficulty in the interpretation of the ground state for one particle case. Namely, the ground state can not be erased, because it is, in a sense, an empty seat itself. If we go over to many particle cases, we have to suppose such seats here and there. In order to deal with the case of arbitrary number of particles, we are obliged to imagine in advance the totality of seats which the particles can occupy. So the present point of view is not so strange as it looks at first sight. On the one hand, it can be regarded as an extension of atomistics to the part of space which is not occupied by particles. On the other hand, however, it can also be regarded as a new version of quantum theory of ether with globular structure.
References

1) See, for example, H. Yukawa, "Space-Time Description of Elementary Particles" in the Proceedings of the International Conference on Elementary Particles, Kyoto, 1965, p. 139, where the references to preceding papers are found.

2) An important modification of commutation relations between field operators for "Urmaterial" was made in Heisenberg's theory, but their infinitesimal character was retained.


4) This model corresponds to the case of bilocal field in the previous works as cited in reference 1).

5) This point of view is very close to the one which is based on the discrete space and/or time, but is still distinct from it. See, for instance, the article by D. Bohm, and also the articles by M. Schönberg and by I. E. Tamm in the Proceedings of the International Conference on Elementary Particles, Kyoto, 1965.