Current × Current Interaction and $|\Delta I|=1/2$ Rule

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The $|\Delta I|=3/2$ amplitudes of hyperon decays will be shown to be negligible in the limit of zero pion four momentum even if we assume that the non-leptonic weak interaction is a product of the weak charged current and its hermitian conjugate.

Suggested by leptonic and semi-leptonic decay interactions, many physicists have considered that the non-leptonic weak interaction is formed from the weak charged current times its hermitian conjugate. If we assume the Cabibbo model of semi-leptonic decays, the strangeness-changing part of this non-leptonic interaction is written as

$$H_w = \frac{G}{4\sqrt{2}} \cos \theta \sin \theta \int d^4 x [J^a_{(1)} J^{a(1)}_a + J^a_{(2)} J^{a(2)}_a + J^a_{(3)} J^{a(3)}_a + J^a_{(5)} J^{a(5)}_a], ~ (1)$$

where

$$J^a_{(i)}(x) = i\bar{q}(x)\gamma^a (1 + \gamma_5) \lambda_i q(x) = 2V^a_{(i)}(x) + 2A^a_{(i)}(x),$$

and

$$G = 1.02 \times 10^{-5} m_p^{-1}. $$

A difficulty with this interaction is that it consists of the sixth component of a unitary octet which obeys $|\Delta I|=1/2$ and a term that transforms like the representation $27$ which has both $|\Delta I|=1/2$ and $3/2$. To overcome this difficulty some have proposed that the octet part of the interaction is enhanced dynamically and some have introduced neutral currents to cancel out the $27$ or $|\Delta I|=3/2$ part of the interaction.

Recently it has been shown that the $s$-wave parts of hyperon decay amplitudes are proportional to the expectation values of the interaction (1) between

\[\begin{align*}
\text{*) Permanent address: Department of Physics, Tokyo University of Education, Tokyo.} \\
\text{**) If we introduce neutral currents, the interaction (1) becomes} \\
H_w = G \int d^4 x J^a_{(i)} J^{a(1)}_a. \quad (2)
\end{align*}\]
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the octet baryons in the limit of zero pion four momentum\(^5,8\) and that the $p$-wave parts of hyperon decay amplitudes are equal to the octet baryon pole contributions in the same limit\(^6,7\) if we make the following assumptions: (i) Currents $J^a_i(x)$ satisfy equal-time commutation relations based on the quark model, (ii) the partial conservation of the axial vector current (PCAC), and (iii) the decay vertices satisfy unsubtracted dispersion relations.\(^8\) From $s$-wave hyperon decays we find the following expectation value of the non-leptonic weak interaction,\(^9,**)\)

$$
\langle B_8^{(i)}|H_W|B_8^{(j)}\rangle = 2(DD_{6ij} + FF_{6ij} + T_{71}T_{43}),
$$

where

$$
D = -1.6 \times 10^{-5} \text{ MeV},
$$
$$
F = 3.6 \times 10^{-6} \text{ MeV}
$$

and

$$
T_{\pi} = 0,
$$

though these values and $g_A = 1.18 \pm 0.02$ give too small $p$-wave amplitudes.\(^9\) These results show that the $|\Delta I| = 3/2$ part or $T_{\pi}$ is negligible and the $D/F$ ratio is about $-1/2$.

If we use the identity

$$
\int d^3x \langle a|JJ|b\rangle = \sum_{n: \text{complete set}} \int d^3x \langle a|Jn\rangle \langle n|J|b\rangle,
$$

we can calculate the expectation value of the non-leptonic weak interaction making use of our knowledge of the matrix elements of vector and axial vector currents. In this article we neglect all the intermediate states in (4) except for the ground state octet baryons ($B_8$) and the ground state decuplet baryons ($B_{10}$). We know that matrix elements between these states and $B_8$ are large and we have rough information on their form factors. On $\langle B_8|V(x)|B_8\rangle$ we have precise information obtained from electron scattering experiments.\(^5,9,10\) For $\langle B_8|A(x)|B_8\rangle$ we apply Nambu's conjecture\(^5\) and assume that axial vector form factors are proportional to the electric form factors of nucleon, $G_E(q^2)$, suggested by the results of neutrino experiments.\(^9\) If we assume the $SU(3)$ static

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\(^8\) Some may doubt the validity of the method used in references 4)–7) if the current $\times$ current interaction is an effective one due to the existence of an intermediate boson. However, the method is justified even in this case if the boson is very massive. This problem will be discussed elsewhere.

\(**\) Explicitly,

$$
\langle B_8^{(i)}|H_W|B_8^{(j)}\rangle = D[\Sigma^*p + \Sigma^-n - (1/\sqrt{2})(\Sigma^0n + \Sigma^0p) - (1/\sqrt{6})(\bar{A}n + \bar{A}p)]
$$

$$
+ F[-\Sigma^*p + \Sigma^-n + (1/\sqrt{2})(\Sigma^0n - \Sigma^0p) - (3/2)^{1/2}(\bar{A}n - \bar{A}p)]
$$

$$
+ T[\Sigma^*p + \Sigma^-n + (1/\sqrt{2})(\Sigma^0n + \Sigma^0p) + (3/8)^{1/2}(\bar{A}n + \bar{A}p)] + \text{h.c.}
$$

\(***\) We use the empirical formula of form factors proposed by Chan et al.\(^5\)
model of $B_{\alpha}$, we can calculate $\langle B_{\alpha} | J | B_{\beta} \rangle$ from $\langle B_{\beta} | J | B_{\alpha} \rangle$ for small momentum transfer.\(^{10}\) We assume that form factors of transition elements are proportional to $G_{E\mu}$; this is also suggested by the recent neutrino experiments.\(^{10}\)

By making use of these assumptions, we have found the following contributions from $B_8$ intermediate state (first terms on the right-hand sides of (5)) and those from $B_{10}$ intermediate states (second terms),

$$D = -3.2 \times 10^{-8} \text{ MeV} = -(0.53 + 2.68) \times 10^{-8} \text{ MeV},$$

$$F = 3.8 \times 10^{-8} = (1.58 + 2.23) \times 10^{-8} \text{ MeV},$$

and

$$T_\pi = -0.1 \times 10^{-5} \text{ MeV} = (0.23 - 0.29) \times 10^{-5} \text{ MeV}.$$

In deriving the above result we have assumed\(^{10}\) $m(B_8) = m(B_{10}) = m = 1130 \text{ MeV}$. Details of the calculation will be given in the last part of this article. Thus, we have found that the $|A\ell|=3/2$ amplitudes are negligible even though we have assumed the large $|A\ell|=3/2$ interaction. Hence, in the zero four-momentum limit of pion, non-leptonic hyperon decays are found to satisfy the $|A\ell|=1/2$ rule. The $D/F$ ratio is about $-0.8$ which should be compared with the experimental value $-0.5$. As to the absolute values of $D$ and $F$ the agreement between (5) and (3) is fairly good.

The $s$-wave amplitudes of hyperon decays predicted by (5) are as follows (in an arbitrary unit):

$$A(\Lambda^+) = 4.8, \quad A(\Sigma^-) = -8.4, \quad A(\Sigma^0) = 0.2 \quad \text{and} \quad A(\Xi^-) = 9.8,$$

which should be compared with the experimental values $5.5, -7.3, 0.0$ and $7.0$, respectively.

From (5) we find that the contribution from the $B_{10}$ intermediate state is bigger than that from the $B_8$ intermediate state. This is because $B_{10}$ has a very big reduced width. For example, $\gamma(\Delta \rightarrow N + \pi) = 4.2 \text{ for } \Gamma(\Delta \rightarrow N + \pi) = 110 \text{ MeV}$, while $\gamma(N \rightarrow N + \pi) = 3.0$. Since the reduced widths of higher resonances into $N + \pi$ (or $B_8 + P_3$) are much smaller than $\gamma(\Delta \rightarrow N + \pi)$, [for example $\gamma(N^{**} \rightarrow N + \pi) \approx (1/5)\gamma(\Delta \rightarrow N + \pi)$], individual contributions from higher resonance intermediate states may be small. However, since there are an infinite number of intermediate states, our result (5) may have a considerable error.

For convenience we have tabulated individual contributions due to vector currents and axial vector currents in Table I. Because of the large isovector magnetic moment of the nucleon, the contribution due to the magnetic form factor of $B_8$ and that from $\langle B_8 | V | B_{10} \rangle$ (M 1 transition) are most important.

If there exist neutral currents\(^3\) (5) must be replaced by $D = 5.3 \times 10^{-5} \text{ MeV}$ and $F = 6.3 \times 10^{-5} \text{ MeV}$ and of course $T_\pi = 0$. We can neither support nor\(^3\) If we assume smaller $m$, we get bigger contributions.
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Table I. Expectation value of non-leptonic weak interaction.
The $B_8$ and $B_{10}$ stand for contributions from $B_8$ and $B_{10}$ intermediate states respectively. Unit is $10^{-5}$ MeV.

<table>
<thead>
<tr>
<th></th>
<th>$B_8$</th>
<th></th>
<th>$B_{10}$</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>VV</td>
<td>AA</td>
<td>VV</td>
<td>AA</td>
</tr>
<tr>
<td>charge</td>
<td>magnetic</td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$D$</td>
<td>-0.61</td>
<td>0.06</td>
<td>0.02</td>
<td>-1.87</td>
</tr>
<tr>
<td>$F$</td>
<td>0.00</td>
<td>1.06</td>
<td>0.52</td>
<td>1.56</td>
</tr>
<tr>
<td>$T$</td>
<td>0.27</td>
<td>-0.03</td>
<td>-0.01</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

reject the existence of neutral currents from our results.
In the following we shall derive numerical results (5).

(i) $B_8$ intermediate state
The vector currents of the octet baryons are

$$
\langle B^{(j)} | V_8^{(i)} \rangle \langle B^{(8)} \rangle = \left(1 + \frac{q^2}{4m^2} \right)^{-1} e^{-iqx}
$$

$$
\times \bar{u}(p') \{(p' + p, (2m)^{-1} F_{i\beta} G_{EP}(q^2)
$$

$$
+ \left( i \frac{\Gamma q^2}{4m^2} - \frac{i}{2m} \sigma_{\mu q'} \right) (d_{v} D_{i\beta} + f_{v} F_{i\beta}) G_{M}(q^2) \nu(p)
\right)
$$

where $q = p' - p$, $G_{EP}(0) = 1$, $G_{M}(0) = 1 = \mu_p = 4.71$, $d_v = 0.62$, $f_v = 0.38$, $m = m(B_8) = 1130$ MeV and

$$
G_{EP}(q^2) = G_{M}(q^2) = \left(1 + \frac{q^2}{0.71 \text{BeV}^2}\right)^{-2}
$$

$$
= \left(1 + \frac{q^2}{0.56m^2}\right)^{-2} \equiv F(q^2).
$$

For axial vector currents we assume the PCAC hypothesis,\(^{91,92}\)

$$
\langle B^{(j)} | A_8^{(i)} \rangle \langle B^{(8)} \rangle = \langle d_{A} D_{i\beta} + f_{A} F_{i\beta} \rangle e^{-iqx}
$$

$$
\times \bar{u}(p') \left[ \gamma_{\mu q'} + \frac{2mq_{\mu q'}}{q^2 + m(P_i)^2} \right] \nu(p) F(q^2),
$$

where $d_A = 0.74$, $f_A = 0.44$ and $d_A + f_A = g_A$.

Then, the contribution of the $B_8$ intermediate state to the expectation value (5) is

$$
2C \left[ D_{est} \left\{ - \frac{9}{10} I + \kappa^2 \left( - \frac{3}{10} d_v^2 + \frac{9}{10} f_v^2 \right) (J - I) \right. \right.
$$

$$
\left. \left. + \left( - \frac{3}{10} d_A^2 + \frac{9}{10} f_A^2 \right) (I + J + K) \right\} \right]
$$

\(^{91}\) Here, $m(P_i)^2 = \langle \pi_i \rangle^2$, $m(K)^2$ or $m(q)^2$. 

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\[ + F_{ik} \left[ \kappa^2 d_v f_v (J - I) + d_A f_A (I + J + K) \right] \]
\[ + T_{ik} \left[ \frac{2}{5} I + \kappa^2 \left( \frac{2}{15} d_v^2 - \frac{2}{5} f_v^2 \right) (J - I) \right. \]
\[ + \left. \left( \frac{2}{15} d_A^2 - \frac{2}{5} f_A^2 \right) (I + J + K) \right] \}

where

\[ C = \frac{G}{\sqrt{2}} \cos \theta \sin \theta \times \frac{4\pi m^3}{(2\pi)} = 1.56 \times 10^{-4} \text{ MeV}, \]
\[ I = \int_0^\infty dE (E^2 - 1)^{1/2} \left( 1 + \frac{2(E - 1)}{0.56} \right)^{-4} = 0.043, \]
\[ J = \int_0^\infty dE (E^2 - 1)^{1/2} E \left( 1 + \frac{2(E - 1)}{0.56} \right)^{-4} = 0.056, \]

and

\[ K = \int_0^\infty dE (E^2 - 1)^{1/2} m_{\sigma}^2 m_{K}^2 \left[ m_{\sigma}^2 + 2m^2(E - 1) \right]^{-1} \left[ m_K^2 + 2m^2(E - 1) \right]^{-1} \]
\[ \times \left( 1 + \frac{2(E - 1)}{0.56} \right)^{-4} = 0.002. \]

(ii) \( B_{13} \) intermediate state

We assume the \( SU(3) \) static model to \( B_{13} \), i.e. we assume the decuplet baryons are bound states of the octet baryons and the octet pseudoscalar mesons. Then, matrix elements are\(^{(11,12)}\).

\[ \langle \bar{J}^{++} | V_{\sigma}^{(1)} (x) + i V_{\sigma}^{(2)} (x) | p \rangle = 6.8 e^{-iqx} \]
\[ \times \bar{A}_{++}^+ (p') \left\{ \delta_{\sigma\gamma} + \frac{i q_{\sigma}}{2m} \gamma_{\mu} \right\} i\not{\partial} (p) F(q^2), \]
\[ (9) \]

and\(^{(13,14)}\).

\[ \langle \bar{J}^{++} | A_{\mu}^{(1)} (x) + i A_{\mu}^{(2)} (x) | p \rangle = 1.7 e^{-iqx} \]
\[ \times \bar{A}_{++}^+ (p') \left\{ \delta_{\mu\nu} - \frac{q_{\mu} q_{\nu}}{q^2 + m(P')^2} \right\} i\not{\partial} (p) F(q^2), \]
\[ (10) \]

\(^{(11)}\) If we use the estimate of reference 12), our factor 6.8 should be replaced by 7.8.

\(^{(12)}\) If we assume the Goldberger-Treiman relation for the \( J \to N^{++} l^+ \bar{\nu} \) decay, our factor 1.7 should be replaced by 1.9.

\(^{(13)}\) Furlan et al. have obtained a different expression for this current.\(^{(13)}\) However, there is little difference for the numerical result.
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where $\Gamma(J\rightarrow N+\pi)$ is assumed to be $110$ MeV and $m(B_{\bar{s}}) = m(B_s) = m$.

The contribution of the $B_{\bar{s}}$ intermediate state to the expectation value (5) is

$$2C \left\{ \frac{3}{20} D_{stk} - \frac{1}{8} F_{stk} - \frac{1}{60} T_{stk} \right\}$$

$$\times \left\{ (6.8 \sqrt{2})^2 \frac{2}{3} (J-I) + (1.7 \sqrt{2})^2 \frac{1}{6} (5I+4J-L) \right\},$$

(11)

where we have replaced a factor, $F^a(E-1) (4-E+E^2)$, by $4F^a(E-1)$ in the integrand to avoid a divergence,

$$L = \frac{\alpha}{\gamma} dE (E^2-1)^{1/2} E^2 \left( 1 + 2(E-1) \right)^{-4} = 0.081$$

and $m_r^2$ and $m_s^2$ are neglected.

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Note added. The original version of this article is the report IC/66/44 of the International Centre for Theoretical Physics. Since the report was written on April 1966, similar works have been done by Chiu, Schechter and Ueda\textsuperscript{14,15} and by Biswas, Kumar and Saxena\textsuperscript{16} by making use of slightly different assumptions.

References