**K→2π Decays and the ΔI=1/2 Rule**

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A study is given of the problem whether or not, and how, the deviations from the exact ΔI=1/2 rule observed in the K→2π decays can quantitatively be explained as a consequence of electromagnetic corrections to a purely ΔI=1/2 weak interaction. A sort of the current X current type weak interaction is considered. It is shown that the experimental values of a pair of the amplitude ratios, \( A(K^+→π^+π^0)/A(K^0→π^+π^-) \) and \( A(K^0→π^+π^0)/A(K^0→π^+π^-) \), can consistently be explained from such a point of view if one considers the \( λ_τ \)-like interaction, besides the usually adopted \( λ_5 \)-like one, and takes the effect of the \( τ−π^0 \) mixing into account.

The effect of the possible vector meson intermediate states is found to be in a direction favorable for explaining these amplitude ratios if \( m_{K^+}−m_{K^0}<0 \). The implication of the fact that the \( λ_τ \)-like term plays a more important role is discussed.

§ 1. Introduction

The so-called ΔI=1/2 rule is a useful hypothesis in the study of the nonleptonic decays of strange particles, and is known to be satisfied in various phenomena. However, for the K→2π decays, one encounters a serious discrepancy between the predictions and the experimental results. Let \( R_1 \) and \( R_2 \) be a pair of the amplitude ratios defined by

\[
R_1 = \frac{A(K^+→π^+π^0)}{A(K^0→π^+π^-)},
\]
\[
R_2 = \frac{\sqrt{2} A(K^0→π^+π^0)}{A(K^0→π^+π^-)}. \tag{1.1}
\]

The ΔI=1/2 rule predicts \( R_1 = 0 \) and \( R_2 = 1 \), while their experimental values\(^1\) are

\[
|R_1^{\text{exp}}| = 0.0460 \pm 0.0009,
\]
\[
|R_2^{\text{exp}}| = 0.934 \pm 0.024. \tag{1.2}
\]

We would like to emphasize that not only the value of \( R_1^{\text{exp}} \) but also the value of \( R_2^{\text{exp}} \) shows a significant deviation from their predicted values, and that perhaps these are the most serious evidence against the validity of the exact ΔI=1/2 rule. In this connection, it is worth noting that, according to the recent experimental analysis,\(^2\) the \( Σ→Nπ \) decay parameters are found to satisfy the Gell-Mann-Rosenfeld triangle relation within errors.

It is often suggested that the aforementioned discrepancy should be considered as a consequence of electromagnetic violation of a purely ΔI=1/2 weak interaction. It has also been emphasized\(^3\) that, since the exact \( SU(3) \) symmetry together with \( CP \) invariance forbids the \( K^2→2π \) decays, the value of \( R_1^{\text{exp}} \) can...
qualitatively be understood without introduction of the $\Delta I=3/2$ interaction of non-electromagnetic origin. However, as far as we know, there has been no satisfactory quantitative explanation of these points.

On the other hand, Das and Mahanthappa, and also Sudarshan, have recently derived an interesting sum rule among three decay amplitudes of $K\to 2\pi$:

$$2A(K^+\to \pi^+\pi^0) = A(K^0\to \pi^+\pi^-) - \sqrt{2} A(K^0\to \pi^+\pi^0), \quad (1.3)$$

or equivalently

$$2R_1 + R_2 = 1, \quad (1.3')$$

which is in agreement with experiment within errors. However, since their derivations are essentially based upon the isospin selection rule only, namely the absence of a $\Delta I=5/2$ term, it is not clear whether the $\Delta I=3/2$ term involved has something to do with the electromagnetic violation of the $\Delta I=1/2$ rule.

In this paper, we want to search for a quantitative explanation of these decay modes of $K$ mesons. The questions are: whether or not, and how, can one understand the values of $R_{np}$ and $R_{np}$ without introduction of the $\Delta I\geq 3/2$ weak interaction? Whether or not, and how, can the sum rule, Eq. (1.3), result from electromagnetic corrections to a purely $\Delta I=1/2$ weak interaction?

We shall adopt the pole model that the non-leptonic decay proceeds mainly through the $SU(3)$-invariant strong interaction and the $CP$-invariant two-body weak interaction which transforms as the $\Delta Q=0, |\Delta S|=1$ components of the $SU(3)$ octet. For $K\to 2\pi$, to which our discussion is restricted, the intermediate states are further assumed to be dominated by the vector meson nonet. We adopt this model because it has been shown to give a reasonably quantitative picture of all the $\Delta I=1/2$ non-leptonic decays of hyperons and $K$ mesons, and also because the effective interaction for $K\to 2\pi$ in this model can be regarded as a sort of the current $\times$ current type weak interaction (see § 6 for the details). As electromagnetic corrections, we shall take into account the mass splittings between the members of the same isomultiplets and also the effect of the possible $\eta - \pi^0$ transition.

We give, in § 2, the relevant graphs and interactions, and, in § 3, the discussion of the sum rule. Using the values of the parameters estimated in § 4, we discuss the problem whether or not, and how, the values of $R_{np}$ and $R_{np}$ can be reproduced consistently in § 5. Section 6 is devoted to the discussion of the relation between the model adopted and the current $\times$ current picture of the weak interaction. The results obtained are summarized in § 7.

The observed small $CP$ violation is neglected throughout this paper.

§ 2. Graphs and interactions

At the outset, we present in Fig. 1 the Feynman graphs which contribute to $K\to 2\pi$ in the model we adopt.
Fig. 1. Feynman graphs contributing to $K \rightarrow 2\pi$ in the model adopted.

The interactions are written as follows:

$$H = H_1 + H_2 + H_3$$

with

$$H_1 = ig_1 \text{Tr}(V_{\rho} [M \bar{\rho} M]) ,$$  

$$H_2 = g_2 \text{Tr}(\{V_{\rho} \rho M\} + \lambda_4) + ig_3 \text{Tr}([V_{\rho} \bar{\rho} M] - \lambda_7) ,$$  

$$H_3 = g_{\rho \gamma} \tilde{\eta}^0 ,$$

where $M$ and $V_{\rho}$, the usual $3 \times 3$ matrices representing the pseudoscalar meson octet and vector meson nonet respectively, are defined such that, under hermitian conjugate and $CP$ transformation,

$$M^+ = M , \quad V_{\rho}^+ = V_{\rho} ,$$

$$(CP) M (CP)^{-1} = -M^T , \quad (CP) V_{\rho} (CP)^{-1} = V_{\rho}^T .$$

The relevant part of Eq. (2.1b) is, if written explicitly,

$$H_2 = (g_2 - g_2') \rho \bar{\rho} K^+ + (g_2 + g_2') (K^{\ast} + \partial_{\mu} \rho - \frac{1}{\sqrt{2}} K_{\mu} \bar{\rho} \tilde{\eta}^0 )$$

$$- \frac{1}{\sqrt{6}} (g_2 - 3g_2') K_{\mu} \rho \bar{\rho} \tilde{\eta} + \text{h.c.} \quad (2.1b')$$

The essential point is that we consider the $\lambda_7$-like weak interaction, the second term in Eq. (2.1b), together with the $\lambda_7$-like one. It should be noted that there is no reason to prefer the latter to the former from the argument of "octet enhancement" alone, which we regard as a reasonable extension of the notion of the $AI=1/2$ rule. More detailed discussion of the type of weak interactions will be given in § 6.

The symbols, $\tilde{\eta}^0$ and $\tilde{\eta}$, appearing in the above interaction Hamiltonians denote the "bare" fields of the $\pi^0$ and $\eta$ mesons which are different from their "physical" fields, $\pi^0$ and $\eta$, due to the presence of the $\eta - \pi^0$ coupling, Eq. (2.1c); the latter can be expressed in terms of the former:

\(^{8)}\) The sign of the fourth component is reversed. The superscript $T$ denotes the transpose of matrices.
\[ \begin{align*}
\{ \pi^0 &= \beta \bar{\pi}^0 - \gamma \bar{\eta}, \\
\eta &= \gamma \bar{\pi}^0 + \beta \bar{\eta},
\end{align*} \quad (2.3) \]

where \( \beta \) and \( \gamma \) are the mixing parameters satisfying
\[ \beta^2 + \gamma^2 = 1 \quad (2.4) \]

The value of \( \beta \) or \( \gamma \) is to be estimated from the “bare” and “physical” masses of \( \pi^0 \) and \( \eta \) (see the Appendix).

\section{3. Matrix elements and sum rule}

With the aid of Fig. 1 and Eqs. (2.1a), (2.1b') and (2.3), the matrix elements for each process can be readily written down as (apart from a common numerical factor):
\[ A(K_1^0 \rightarrow \pi^+ \pi^-) = \sqrt{2} g_1 g_2 (1 + \alpha) (m_{K^0}^2 - m_{\pi^+}^2) / m_{K^0}^2, \]
\[ A(K_1^0 \rightarrow \pi^0 \pi^0) = g_1 g_2 \left[ (1 + \alpha) - \frac{1}{3} \right] (m_{K^0}^2 - m_{\pi^0}^2) / m_{K^0}^2, \]
\[ A(K^+ \rightarrow \pi^+ \pi^0) = \frac{1}{\sqrt{2}} g_1 g_2 \left[ (\beta - \sqrt{3} \gamma) (1 + \alpha) (m_{K^+}^2 - m_{\pi^0}^2) / m_{K^0}^2, \right. \]
\[ \left. - \left( \beta (1 + \alpha) - \frac{1}{3} \right) (1 - 3\alpha) \right] (m_{K^0}^2 - m_{\pi^0}^2) / m_{K^0}^2 + 2\beta (1 - \alpha) (m_{\pi^+}^2 - m_{\pi^0}^2) / m_{\pi^+}^2, \quad (3.1)^* \]

where
\[ \alpha = g_1' / g_2. \quad (3.2) \]

We now define \( \Delta \)'s and \( m \)'s by
\[ \begin{align*}
m_{\Delta^+} &\; = m_A \left( 1 + \frac{\Delta_A}{2} \right), \\
&\; = m_A \left( 1 + \frac{\Delta_A}{2} \right), \quad A = (\pi, K, \rho, K^*). \quad (3.3) \]

All of the \( \Delta \)'s and \( \gamma \) are quantities of order \( \lesssim 10^{-2} \). Equations (3.1) become, to the first order of the \( \Delta \)'s and \( \gamma \),
\[ A(K_1^0 \rightarrow \pi^+ \pi^-) = \sqrt{2} g_1 g_2 (1 + \alpha) (1 - \Delta_{K^0}) (m_{K^0}^2 - m_{\pi^+}^2) / m_{K^0}^2, \]
\[ A(K_1^0 \rightarrow \pi^0 \pi^0) = g_1 g_2 \left[ (1 + \alpha) (1 + \Delta_{K^0}) + \frac{2}{3} \right] (1 + 3\alpha) \gamma] (m_{K^0}^2 - m_{\pi^0}^2) / m_{K^0}^2, \]

\( ^* \) All of these amplitudes vanish in the symmetry limit (i.e. \( m_K = m_\pi \)) as they should.
\[ A(K^+ \rightarrow \pi^+\pi^-) = \sqrt{2} g_{\pi^+} \left[ (1 + \alpha) - 2(1 - \alpha) \frac{m_{K^0}^2/m_{\pi^+}^2}{m_{K^+}^2 - m_{\pi^+}^2} \right] \]
\[ - (1 + \alpha) \frac{g_{\pi^+}}{\sqrt{3}} (1 + 3\alpha) \gamma \left( m_{K^+}^2 - m_{\pi^+}^2 \right)/m_{K^0}^2. \]

(3.1')

It is easily verified from Eqs. (3.1) and (3.1') that
i) if \( \alpha = 0 \) and \( m^* = m_{K^*} \), the sum rule (1.3) holds approximately;
ii) if \( \Delta_{K^0} = 0 \) and \( \gamma = 0 \) in addition to \( \alpha = 0 \) and \( m^* = m_{K^*} \), the sum rule holds exactly.

Thus we may say that the sum rule (1.3) can be derived from self-energy type electromagnetic corrections to the purely \( \Delta I = 1/2 \) weak interaction, and that this derivation is intimately connected with the pure \( \lambda_6 \)-like behavior of the interaction.

Though the above conditions lead exactly or approximately to the sum rule consistent with experiment, we should go further to inquire whether the values of \( R_1^{exp} \) and \( R_2^{exp} \) themselves can be reproduced or not.

§ 4. Estimates of the values of \( \Delta_{K^*} \) and \( \gamma \)

It is seen from Eq. (3.1') that the value of \( R_1 \) and \( R_2 \) are functions of the parameters \( \Delta_{K^*} \), \( \gamma \), and \( \alpha \), if the experimental values of \( m_{\pi^+}, m_{K^0}, m_{K^+}, m_{K^*}, m_{K^0} \) and \( m^* \) are inserted. In this section we shall estimate possible limiting values of the first two, \( \Delta_{K^*} \) and \( \gamma \).

A. The value of \( \Delta_{K^*} \)

The experimental value of \( \Delta_{K^*} \) is not well established yet. There have been several theoretical estimates of it. From the formula,\(^7\) \( m_{K^*}^+ - m_{K^0}^+ = m_{K^*} - m_{K^0} \), one gets \( \Delta_{K^*} \approx -0.0045 \); while, from his quark model approach, Konno\(^6\) has inferred that \( m_{K^*}^+ - m_{K^0}^+ = -8.3 \text{ MeV} \), and hence \( \Delta_{K^*} \approx -0.009 \), which seems to be the largest value that has been obtained. In the numerical calculations, we shall consider the case in which \( m^* = m_{K^*} \), \( \Delta_{K^0} = 0 \) and the case in which \( m_{K^0}^2/m_{\pi^+}^2 = 1.4 \), \( \Delta_{K^*} = -0.009 \).

B. The value of \( \gamma \)

From the \( SU(3) \) mass formula\(^8\)
\[ g_{\pi^+} = -\frac{1}{\sqrt{3}} \left( m_{K^0}^2 - m_{K^0}^+ + m_{\pi^+}^2 - m_{\pi^0}^2 \right), \]
one gets \( g_{\pi^+} \approx -0.003 \text{ BeV} \) and hence (using Eq. (A.5))
\[ \gamma \approx -0.011. \]

From the assumption that the observed mass difference between \( \pi^+ \) and \( \pi^0 \) is

\[^7\] This value is consistent with the recently reported experimental value:
\[ m_{K^0}^+ - m_{K^0}^+ = (6.5 \pm 3.8) \text{ MeV}. \]
due solely to the effect of $\eta - \pi^0$ coupling, one finds (Eq. (A.7) of the Appendix)

$$|\gamma| \approx 0.067.$$  

(4·2)

We regard this value, though unreasonably large, as the possible maximum value for $|\gamma|$.

§ 5. Whether and how the values of $R_1^{\exp}$ and $R_2^{\exp}$ can be reproduced

In this section, we discuss the problem whether or not, and how, the values of $R_1^{\exp}$ and $R_2^{\exp}$ can consistently be reproduced by a suitable choice of the value of the parameter $\alpha$.

From Eqs. (3·1'), one obtains, to the first order of the $\Delta$'s and $\gamma$,

$$R_1 = \Delta_s \left[ 1 - 2(1-\alpha) \frac{m_K}{m_\pi} \right] \frac{m_\pi^2}{(m_K^2 - m_\pi^2)}$$

$$- \Delta_{K*} \left[ - \frac{1}{\sqrt{3}} \gamma (1 + 3\alpha) / (1 + \alpha) \right],$$

(5·1a)

$$R_2 = 1 + 2 \Delta_s m_\pi^2 / (m_K^2 - m_\pi^2) + 2 \Delta_{K*} + \frac{2}{\sqrt{3}} \gamma (1 + 3\alpha) / (1 + \alpha).$$

(5·1b)

Let us first consider the following two special cases:

Case I. $\alpha = 0$ (or $g' = 0$)

The values of $R_1$ and $R_2$ can be calculated by inserting the values of $\Delta_{K*}$ and $\gamma$ estimated in the preceding section, the result being presented in Table I.

<table>
<thead>
<tr>
<th>$m_{K*} = m_\rho$, $\Delta_{K*} = 0$</th>
<th>$\gamma = 0.067$</th>
<th>$\gamma = 0$</th>
<th>$\gamma = -0.067$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>$R_2$</td>
<td>$R_1$</td>
<td>$R_2$</td>
</tr>
<tr>
<td>$-0.041$</td>
<td>$1.082$</td>
<td>$-0.003$</td>
<td>$1.006$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$m_{K*}^2/m_\rho^2 = 1.4$, $\Delta_{K*} = -0.009$</th>
<th>$\gamma = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.035$</td>
<td>$1.065$</td>
</tr>
</tbody>
</table>

As seen from Table I, though it is not impossible to reproduce the value of $R_2^{\exp}$, the value of $R_1^{\exp}$ can never be satisfactorily explained even if the $\eta - \pi^0$ mixing parameter $\gamma$ is as large as $|\gamma| \approx 0.067$.*)

Case II. $\gamma = 0$

In contrast to the previous case, it is not impossible in this case to reproduce the value of $R_1^{\exp}$ by choosing the value of $\alpha$ to be $\sim -1.5$ or $\sim -0.8$. On the other hand, the value of $R_2$ is independent of $\alpha$ and deviates appreciably from the value of $R_2^{\exp}$ (see the fifth row of Table I).

*) The conclusion remains unaltered even when the experimental value has been used for $g_{\rho\pi\pi}/g_{K^*K\pi}$. 

\[ K \to 2\pi \text{ Decays and the } \Delta I=1/2 \text{ Rule} \]
From the above results we see that either Case I or Case II fails to explain the values of $R_1^{\exp}$ and $R_2^{\exp}$ consistently.

We should proceed to consider the general case, that is, to regard both of $\alpha$ and $\gamma$ as free parameters and to determine them from the values of $R_1^{\exp}$ and $R_2^{\exp}$ by making use of Eqs. (5.1a) and (5.1b). Taking $R_1=0.046$ and $R_2=0.934$, one can easily solve these equations for $\alpha$ and $\gamma$ to get

$$\begin{cases}
\alpha \approx -7.2 & \text{for } m_K = m_\rho, \quad \Delta_{K^+} = 0, \\
\gamma \approx -0.018
\end{cases} \quad (5.2a)$$

or

$$\begin{cases}
\alpha \approx 27 & \text{for } m_K^2/m_\rho^2 = 1.4, \quad \Delta_{K^+} = -0.009, \\
\gamma \approx -0.016
\end{cases} \quad (5.2b)$$

In view of the large value obtained for $\alpha$, we should further consider another special case:

Case III. $\alpha = \infty$ (or $g_2 = 0$)

$R_1$ and $R_2$ reduce in this case to

$$
R_1 = \Delta_+ \left(1 + 2 m_{K^0}/m_\rho^2 \right) m_\rho^2 \left(\frac{m_K^2 - m_\rho^2}{m_K^2 - m_\rho^2} \right) - \Delta_{K^+} - \sqrt{3} \gamma,
R_2 = 1 + 2 \left[\Delta_+ m_\rho^2 \left(\frac{m_K^2 - m_\rho^2}{m_K^2 - m_\rho^2} \right) + \Delta_{K^+} + \sqrt{3} \gamma \right].
$$

(5.3)

It can be easily verified that both of the values $R_1^{\exp}$ and $R_2^{\exp}$ can consistently be reproduced by suitably choosing the value of $\gamma$. In fact, from Eq. (5.3) one finds that

$$\begin{cases}
R_1 = 0.0465 & \text{if } m_K = m_\rho, \quad \Delta_{K^+} = 0 \text{ and } \gamma = -0.022, \\
R_2 = 0.929
\end{cases} \quad (5.4a)$$

or

$$\begin{cases}
R_1 = 0.0456 & \text{if } \frac{m_K^2}{m_\rho^2} = 1.4, \quad \Delta_{K^+} = -0.009 \text{ and } \gamma = -0.015, \\
R_2 = 0.936
\end{cases} \quad (5.4b)$$

Note that $R_1$ and $R_2$ of Eq. (5.3) satisfy the relation

$$2R_1 + R_2 - 1 = 4 \left(1 + \frac{m_{K^0}^2}{m_\rho^2} \right) \Delta_+ m_\rho^2 \left(\frac{m_K^2 - m_\rho^2}{m_K^2 - m_\rho^2} \right),
$$

(5.5)

which is in better agreement with experiment than the sum rule, (1.3').

It should be also noted that the values obtained for $\gamma$ in Eqs. (5.2a), (5.2b), (5.4a) and (5.4b) are all reasonable in magnitude and agree in sign with the $SU(3)$ prediction, Eq. (4.1).

§ 6. Discussion of the type of effective weak interactions

We have so far worked with a sort of the vector meson dominant model.
However, as explicitly seen in Eq. (3·1), if the mass splittings among the vector mesons are neglected, the interactions $H_1 + H_2$ of Eq. (2·1) reduce effectively to the following one for over-all processes:

$$H_W = G \text{Tr} \{ [J^V_\mu, J^A_\mu] + \lambda_0 \} + iG' \text{Tr} \{ [J^V_\mu, J^A_\mu] - \lambda_0 \}$$

(6·1)

with

$$J^V_\mu = i[M \partial_\mu M]_-, \quad J^A_\mu = m \partial_\mu M, \quad G = g_1 g_2 / m^3, \quad G' = g g'/m^3,$$

where $m$ denotes the degenerate vector meson mass. Conversely speaking, the model adopted is no more than the model in which the vector octet current $J^V_\mu$ and the axial vector octet current $J^A_\mu$ are supposed to couple predominantly through the vector meson intermediate states. Therefore, what have been obtained for the case $m_P = m_K$, $A_{K^0} = 0$ in the previous sections should be naturally taken as the results which follow from the direct $J^V_\mu \times J^A_\mu$ type interaction, Eq. (6·1).

We have found that, in explaining consistently the values of $R_{\pi^\pm\pi^\mp}$ as a consequence of electromagnetic violations of the exact $\Delta I = 1/2$ rule, the $\lambda_\tau$-like term in the interaction (2·1b) or (6·1) plays an essential role. What does this fact imply?

In the ordinary theory of the current $\times$ current type weak interaction, the current $J_\mu$ is considered as the sum of the vector current $J^V_\mu$ and the axial vector current $J^A_\mu$, namely

$$J_\mu = J^V_\mu + J^A_\mu.$$  

(6·2)

Then, the interaction type for the strangeness violating processes is uniquely determined, under the assumptions of octet enhancement and $CP$ invariance, to be

$$\text{Tr} \{ J_\mu J_\mu \lambda_0 \} = \text{Tr} \{ (J^V_\mu J^V_\mu + J^A_\mu J^A_\mu) \lambda_0 \} + \text{Tr} \{ (J^V_\mu J^A_\mu) \lambda_0 \}.$$  

(6·3)

In other words, there has no room to consider the $\lambda_\tau$-like interaction for the parity violating processes like $K \to 2\pi$ as well as for the parity conserving processes. This is the reason why the interaction type is customarily confined to the $\lambda_\tau$-like one. In this sense, the results we have obtained may be taken as the evidence that the theory based upon the ordinary current $\times$ current type weak interaction is not an adequate one for the explanation of the $K \to 2\pi$ decays.

On the other hand, Kunimasa(13) has pointed out that the $\lambda_\tau$-like term is admissible in his modified current $\times$ current interaction. Following him, let us define the current $J_\mu$, instead of Eq. (6·2), to be

$$J_\mu = c_V J^V_\mu + c_A J^A_\mu.$$  

(6·4)

with arbitrary complex numbers $c_V$ and $c_A$. Then, from octet enhancement and $CP$ invariance, the $|\Delta S| = 1$ part of weak interaction can be written as
\[
\frac{1}{2} f \text{Tr}(\{J_\mu, J_\mu^\dagger\} \lambda_8) + \frac{1}{2} f' \text{Tr}([J_\mu, J_\mu^\dagger] \lambda_8) \\
= f |c_v|^2 \text{Tr}(J_\mu^\dagger J_\mu' \lambda_8) + |c_d|^2 \text{Tr}(J_\mu^\dagger J_\mu^\dagger \lambda_8) \\
+ f \text{Re}(c_v c_\ast_d) \text{Tr}(\{J_\mu, J_\mu^\dagger\} \lambda_8) + f' \text{Im}(c_v c_\ast_d) \text{Tr}([J_\mu, J_\mu^\dagger] \lambda_8),
\]

where \( f \) and \( f' \) are real parameters. It is seen that the parity violating part of Eq. (6.5) agrees exactly with Eq. (6.1) if \( f \text{Re}(c_v c_\ast_d) \) and \( f' \text{Im}(c_v c_\ast_d) \) are identified respectively with \( G \) and \( G' \).

The above discussion and the results obtained in the preceding section suggest that the ordinary current \( \times \) current weak interaction with octet enhancement, Eq. (6.3), should be abandoned, or at least modified, in order to explain the values of \( R_{1\pi^0} \) and \( R_{2\pi^0} \) consistently as a consequence of electromagnetic corrections to the purely \( \Delta I = 1/2 \) weak interaction.

\section*{§ 7. Summary}

We have concentrated ourselves on the study of the problem of whether or not, and how, the deviations from the exact \( \Delta I = 1/2 \) rule observed in the \( K \to 2\pi \) decays can be quantitatively explained without introduction of \( \Delta I \geq 3/2 \) interactions of non-electromagnetic origin. The results of our analysis can be summarized as follows:

i) If one starts with the ordinary current \( \times \) current weak interaction, Eq. (6.3) or the first term of Eq. (6.1), which transforms as \( \lambda_8 \)-component of octet, the sum rule (1.3) results from self-energy type electromagnetic corrections to this purely \( \Delta I = 1/2 \) weak interaction. However, the value of \( R_{1\pi^0} \) itself cannot satisfactorily be reproduced even when the effect of the possible large \( \eta - \pi^0 \) mixing is taken into account.

ii) The effect of the vector meson intermediate states is such that, if \( m_{\eta^*} - m_\pi < 0 \), it increases the value of \( R_1 \) and decreases that of \( R_2 \), in a direction favorable for explaining their experimental values. However, the discrepancy between \( R_{1\pi^0} \) and \( R_{2\pi^0} \) still cannot be removed by taking this effect into account.

iii) If one abandons, or modifies, the ordinary current \( \times \) current weak interaction and admits the \( \lambda_7 \)-like term in addition to the \( \lambda_8 \)-like term as in Eq. (6.1) or (2.1b), it becomes possible to reproduce the values of \( R_{1\pi^0} \) and \( R_{2\pi^0} \) consistently by a suitable choice of the ratio \( \alpha \) of the strength of these two terms. The \( \lambda_7 \)-like interaction is found to play a more important role than the \( \lambda_8 \)-like one.

iv) In the special case in which the \( \lambda_7 \)-like interaction is absent, \( R_1 \) and \( R_2 \) satisfy the relation (5.5), which is in better agreement with experiment than the sum rule (1.3), and the values of \( R_{1\pi^0} \) and \( R_{2\pi^0} \) themselves can also be satisfactorily explained by choosing the \( \eta - \pi^0 \) mixing parameter \( \gamma \) to be \(-0.015\sim\)
-0.022, a value reasonable in magnitude and in agreement with the SU(3) prediction in sign.

Thus we conclude that both of the $\lambda_r$-like interaction and the $\eta - \pi^0$ mixing play an important role for the consistent explanation of the values of $R^{\text{exp}}_1$ and $R^{\text{exp}}_3$ as a consequence of electromagnetic violations of the exact $\Delta I=1/2$ rule.

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Appendix

In this Appendix, we give relations among the “bare” and “physical” masses, the transition mass and the mixing parameters of the $\pi^0 - \eta$ system.

The mass terms of the Lagrangian of the system can be written in terms of the “bare” quantities:

$$L = \tilde{m}^{\pi^0}_{\ast} \tilde{m}_{\eta} + \tilde{m}_2 \tilde{\eta} \tilde{\eta} + g_{\ast \ast} (\tilde{m}^{\pi^0}_{\ast} \tilde{\eta} + \tilde{m}_{\eta} \tilde{\eta})$$

and in terms of the “physical” quantities:

$$L = m^{\pi^0}_{\ast} m^\eta + m^2_\eta \tilde{\eta}$$

The “physical” fields $\pi^0$ and $\eta$ are the mixtures of the “bare” fields $\tilde{\pi}^0$ and $\tilde{\eta}$:

$$\begin{align*}
\pi^0 &= \beta \tilde{\pi}^0 - \gamma \tilde{\eta}, \\
\eta &= \gamma \tilde{\pi}^0 + \beta \tilde{\eta},
\end{align*}$$

with

$$\beta^2 + \gamma^2 = 1.$$  

Substituting Eq. (A·3) into Eq. (A·2) and comparing with Eq. (A·1), one finds

$$\begin{align*}
\tilde{m}^2_{\ast \ast} &= m^2_{\ast \ast} \beta^2 + m^2_\eta \gamma^2, \\
\tilde{m}^2_\eta &= m^2_{\ast \ast} \gamma^2 + m^2_\eta \beta^2, \\
g_{\ast \ast} &= (m^2_\eta - m^2_{\ast \ast}) \beta \gamma.
\end{align*}$$

From Eqs. (A·4) and (A·5), it follows that

$$\begin{align*}
m^2_{\ast \ast} + m^2_\eta &= \tilde{m}^2_{\ast \ast} + \tilde{m}^2_\eta, \\
g_{\ast \ast} &= (m^2_\eta - m^2_{\ast \ast}) (\tilde{m}^2_{\ast \ast} - m^2_{\ast \ast}), \\
\gamma &= (\tilde{m}^2_{\ast \ast} - m^2_{\ast \ast}) / (m^2_\eta - m^2_{\ast \ast}).
\end{align*}$$

If $\tilde{m}_{\ast \ast}$ is taken to be $m_{\ast \ast}$, one finds
|γ| \approx 0.067 \quad \text{for } \bar{m}_e = m_e^*.

We may regard this as the possible maximum value for |γ|. It is therefore a good approximation to take β \approx 1.

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