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Sample Functions as Indicators of Aquifer Heterogeneities

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An adaptive slope matching method is used to evalute the aquifer heterogeneity from constant rate aquifer test data. The basic aquifer parameters such as the storativity and transmissivity are considered to vary spatially and the effects of such changes appear as temporal variations during a pumping test period. It is stressed that this is due to the depression cone expansion with time around the pumping well. The plots of individual parameters versus time are referred to as the sample functions. These functions have erratic variations indicating that the aquifers considered in this study have local heterogeneities. Furthermore, a moving average technique is applied to each parameter sample function for the purpose of finding regional trends in the parameter variations. It is interesting to notice that the arithmetic average values of sample functions yield aquifer parameter values as obtained by the classical type curve matching techniques which assume that the aquifer concerned is homogeneous. Hence the use of the sample function concept is recommended for the detailed interpretation of the aquifer heterogeneity.

Introduction

Usually there is an apparent lack of consistency between the parameters such as the hydraulic conductivity, transmissivity and storage coefficient determind by means of the aquifer tests and those obtained in the laboratory. This is due to the fact that the scale of observations is different and an aquifer may be considered as heterogeneous if sampled at different points by small sample volumes. However, in any aquifer test the same aquifer can represent homogeneity at least in the statistical sense. In other words, regional properties differ from local heterogeneous subunit properties. Hence, a natural aquifer can be idealized either as an equivalent homogeneuos aquifer at a macro-scale or as a random field at a micro-scale for any parameter considered. The latter case can be viewed under the regionalized variable theory as suggested by Matheron (1965).

Likewise, expansion of the depression cone with time during an aquifer test leads to the idea that initially this cone will have a small volume and it will attain its maximum expansion after a long time when the quasi-steady state flow is reached, (Şen 1987). Therefore, during such an expansion the depression cone moves across different heterogeneous subunits. As a result, the heterogeneity affects the drawdown and consequently, the aquifer heterogeneities appear as hidden components within the recorded time-drawdown data in the field. It is very essential to develop a technique in order to clearly detect and define aquifer heterogeneities and this is one of the most attractive aspects of modern aquifer testing interpretations. Testing can also indicate hidden anomalies which might not have been detected by other aquifer modeling techniques. Numerous kinds of heterogeneities influence the time-drawdown response during aquifer testing. Lack of interpretation models hindered our ability to detect most heterogeneities. However, it is now possible to employ modeling and observation matching techniques to extract extremely useful information from time-drawdown data as will be explained in this paper. In fact, the main purpose of this paper is to present a technique by which the heterogeneities can be depicted and presented in a scientific manner in the forms of sample functions.

Almost all of the aquifer test data evaluation techniques are based on the homogeneity assumption which is simplification in the analytical solution of groundwater flow problems. None of these techniques is capable of detecting the hidden components but they rather yield a single average value for the parameter estimate. There are few techniques that are known to the authors that deal with the heterogeneous aquifer parameters but again all of them consider systematic heterogeneities in the forms of either horizontal layering or vertical geological facies change or hydrologic barriers. Features such as faults, channels, leakages, vertical fractures, double porosity medium, layered and patchy aquifers and pinch outs all give rise to systematic heterogeneities. Ferris *et la.* (1962) and Nind (1965) have analyzed the influence of absolute and partial hydrologic barriers on aquifer test results. Hantush (1962) has presented analytical solutions for the drawdown in a non-leaky aquifer of exponentially increasing thickness. Recently, Şen (1987b) presented type curve solutions for systematic vertical facies changes around a pumping well. Field experiences, since the original work of Theis (1935), have indicated that always there are local discrepancies between the type curves and the time-drawdown data plots. At times, if the time drawdown plot deviates systematically from the type curve, that is expected theoretically, then the interpretation may be that either an impermeable or constant-head boundary may exist; or, if the deviations are rather in the form of a random pattern, then the results of the analysis may be labeled as unreliable with the observation that the deviations may well be due to either heterogeneous aquifer properties or variations of these parameters sample function in this paper.

Hence it is among the objectives of this paper to estimate the spatial variations of aquifer parameters around the pumping well from observed time-drawdown measurents by means of the sample functions. The local heterogeneities are represented as individual deviations from the regional trend hidden within the sample function. Then by using different orders of moving averages, the regional trends of these functions are depicted. It is observed that the overall arithmetic mean value of any sample function corresponds to the parameter estimates obtained from classical type curve matching procedure.

Simple Model for Heterogeneity Effect on Aquifer Parameters

Hydraulic parameters such as the transmissivity, storativity, hydraulic conductivity, leakage factor *etc.* are related to the material and geometric properties of the aquifer domain. Almost in any analytical treatment of groundwater movement the material is assumed to be isotropic and homogeneous. Consequently, a single value for each parameter emerges after the application of the classical type curve matching methods that are available. It is a known fact that even though the aquifer might be isotropic and homogeneous the values of the hydraulic conductivity and storage coefficient effect the depression cone around the pumping well; the smaller the hydraulic condutivity the narrower is the depression cone as shown in Fig 1. However, a low storativity gives rise to an extensive depression cone as shown in the same figure. It is obvious that K and S effect the drawdown in the same direction *i.e.* increase in K and/or S causes decrease in drawdown. It is then physically plausible to ask whether some erratic changes during an aquifer test in the drawdown measurements are due to the expansion of depression cone with time? The answer to this question is affirmative because, for instance, any change in the geological facies such as the existence of an impervious boundary gives rise to an increase of drawdown more than usual after a certain time. It is by now accepted classically that such an increase of drawdown or effect of the barrier starts to occur at a time when the extent of the depression cone reaches this barrier. This is to say that the aquifer parameters obtained from aquifer test data, in fact, reflect the properties of material changes (heterogeneities) within the effective domain of depression cone. The question is then if erratic changes appear in S or K (*i.e.* heterogeneous aquifer material), will there be reflections of such a situation in the drawdown measurements? No doubt, there will be significant effects but none of the existing analytical solution methods of groundwater flow toward wells

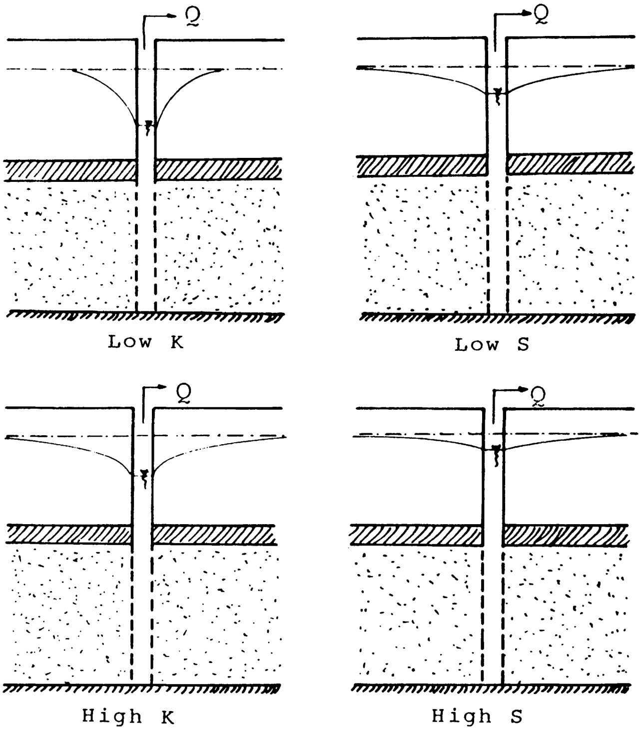

Fig. 1.
Parameters effect on the depression cone.

accounts for such local variations. Logically, the increment *ds*, in the drawdown during a time interval *dt*, means there is an increment in the depression cone volume *dV*, and also some additional aquifer domain is sampled, *dr*, within the depression cone dominance (see Fig 2). If there exists any heterogeneity during such an increment the drawdown will be effected accordingly. For instance, local permeability variations due to a difference in compaction, aquifer thickness changes over short distances, random impermeable lenses *etc.*, all give rise to local erratic variations in the time-drawdown measurements. Hence, the drawdown increment with time might be a good indication of aquifer heterogeneity as explained already to some extent by Şen (1986).

The continuity and flow equations of any radial flow can be written as

$$\frac{\delta q(r,t)}{\delta t} + \frac{1}{r}\, q(r,t) = \frac{S}{m}\,\frac{\delta s(r,t)}{\delta t} \qquad (1)$$

and

$$q(r,t) = -k\,\frac{\delta s(r,t)}{\delta r} \qquad (2)$$

in which $q(r,t)$ and $s(r,t)$ are the specific discharge and drawdown at time t and radial distance r; S, k and m are constants of the storativity, hydraulic conductivity and the aquifer thickness, respectively. The combined solution of Eqs. (1) and (2) are presented by using the Boltzmann transformation by Şen (1989) which leads to

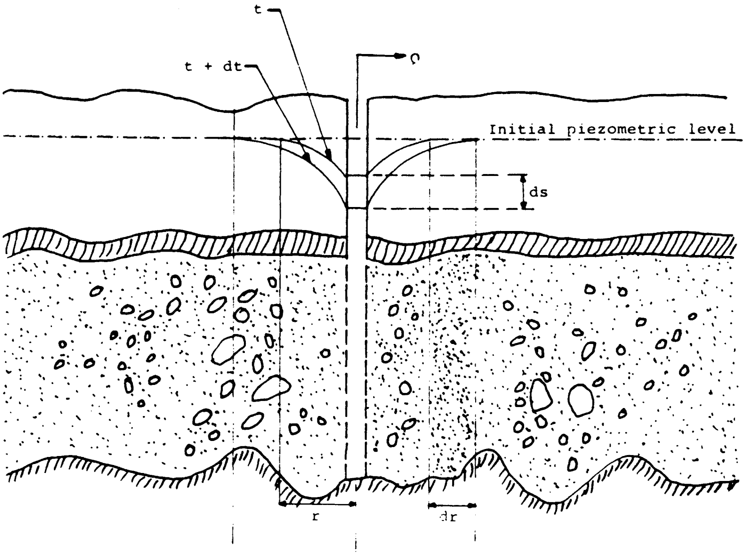

Fig. 2. Depression cone expansion.

$$q(r,t) = \frac{Q}{2\pi m r}\exp\left(-\frac{r^2 S}{4tT}\right) \tag{3}$$

in which $T = mk$ is the transmissivity and its substitution into Eq. (2) leads to

$$\frac{\delta s(r,t)}{\delta r} = -\frac{Q}{2\pi T r}\exp\left(-\frac{r^2 S}{4tT}\right) \tag{4}$$

This expression represents the radial expansion of the depression cone around the pumping well. It also indicates theoretically that the hydraulic gradient $\delta h(r,t)/\delta r$, is inversely and non-linearly related to the transmissivity as well as the storativity. Eq. (4) is the basis of the classical Theis equation provided that the aquifer is completely homogeneous and isotropic which give way to its integration from r to ∞ leading to

$$W(u) = \int_u^\infty \frac{e^{-x}}{x}\,dx \tag{5}$$

in which x is a dummy variable and the well function as well as the dimensionless time factor are

$$W(u) = \frac{4\pi T}{Q}s(r,t) \tag{6}$$

and

$$u = \frac{r^2 S}{4tT} \tag{7}$$

respectively.

However, if the aquifer modelled is not homogeneous then the integration procedure as applied above is not possible due to the expected spatial changes in *T* and/or *S*. In such situations, Eq.(4) can be used in obtaining the local parameter estimations. In order to obtain the temporal changes in the drawdown during an aquifer test one can differentiate Eq. (5) wiht respect to time according to Leibnetz's rule leading to

$$\frac{\delta s(r, t)}{\delta t} = \frac{Q}{4\pi t T} \exp\left(-\frac{r^2 S}{4tT}\right) \tag{8}$$

This expression is similar to Eq. (4) but it represents the expansion of the depression cone with time. The left hand side of Eq. (8) represents the slope of time-drawdown plot and theoretically it is clear that this slope is also inversely and nonlinearly related to the aquifer parameters. Although in Eq. (8) *T* and *S* are the only unknowns and the other terms are all known from an aquifer test performed in the field, unfortunately the solution of *S* and *T* is not uniquely possible. However, this difficulty can be overcome by the slope matching method (Şen 1986) which estimates the local changes in the transmissivity and storativity values.

Local deviations of drawdown measurements from the relevant type curve might mean local changes either in the transmissivity and/or in the storativity or in any other hydraulic property. Since, the transmissivity is a compound value of hydraulic conductivity, *K*, multiplied by the aquifer thickness *m*, their individual of simultaneous changes also effect the drawdown variations during a pumping test. This is tantamount to saying that even though the aquifer is homogeneous from hydraulic conductivity point of view, the changes in the aquifer saturation thickness with the radial distance from the well imply correspondingly changes in the transmissivity and storativity estimations. As mentioned earlier the time duration of such increases are also important. The basis of the slope method is to compare the time-drawdown slope as calculated from the field aquifer tests with the type curve slopes which are presented in table form by Şen (1986).

Application

The heterogeneity of aquifer is sought for the Saq sandstone formation which lies in the north western part of Saudi Arabia shown in Fig. 3. The surficial deposits covering the area consists mainly of silt and below with variable proportions of pebbles from quartz gravel and pebble of limestone and rocks of basement complex. the Tabuk formation is a thick sequence of shale, clay and sandstone units of marine and continental origin. Its thickness is about 500 m in the study area and age ranges from Ordovician to Devonian. At places the Tabuk formation provides confining layers for the Saq sandstone otherwise it is semipervious constituting leaky aquifer setup with the Saq formation. In this study, for aquifer tests, namely three leaky El, J29 and J25 and one confining A3, conditions are considered as the

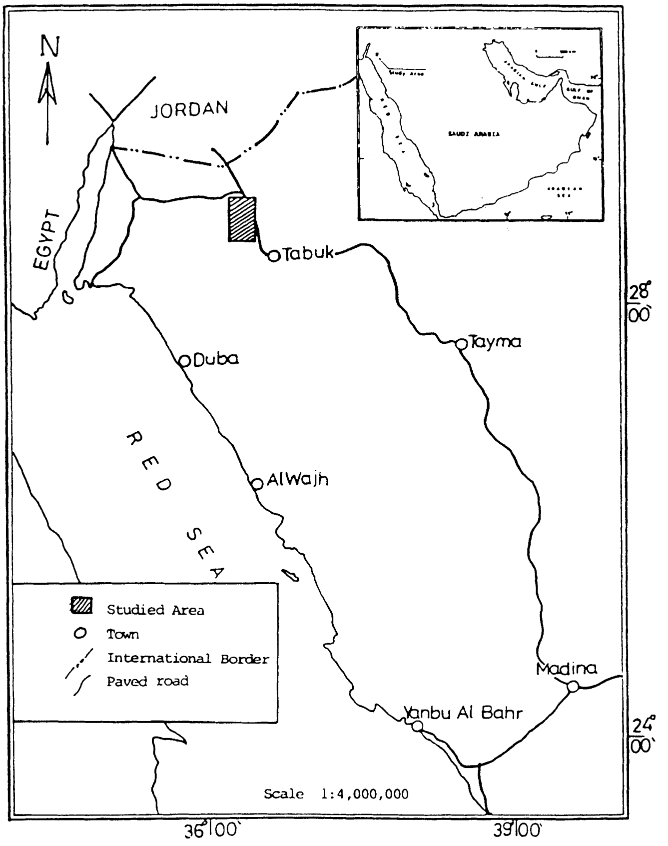

Fig. 3. Location of the study area.

field time-drawdown data which are presented in Figs. 4-7 with the most suitable type curve model matchings (Theis 1935 and Hantush 1956). It is obvious from these figures that there are erratic deviations from the type curves which indicate the heterogeneity of the Saq sandstone aquifer. These heterogeneities are accounted by the slope matching procedure and consequently series of storage coefficients S and transmissivities T are obtained with the results presented in Figs. 8-15. A general inspection of the main values in these figures reveals that the storativity and the transmissivity values have rather random patterns of ups and downs. At times significant deviations appear from the constant parameter values obtained from the type curve matching. Compared to other well locations A3 has less heterogeneity because the general fluctuations of the parameter values have relatively smaller amplitudes. The comparison of field data points and the type curve in Fig. 4 confirms this statement. Furthermore, these fluctuations occur around more or less constant trends. The storativity sample functions in all other well locations occur around rather constant levels whereas the transmissivity sample functions expose obvious trends in the forms of transmissivity decrease with

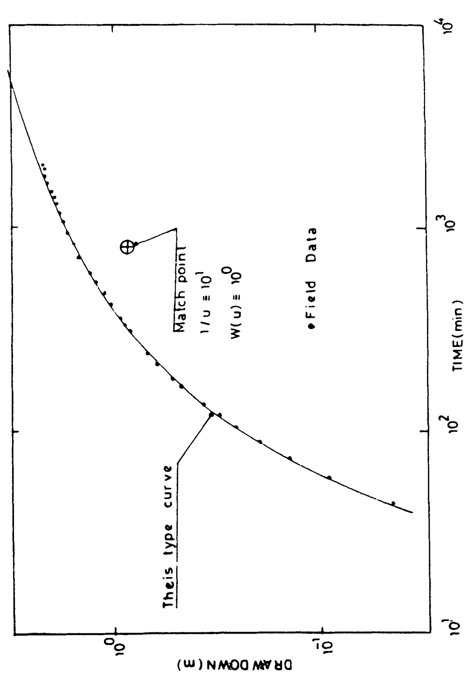

Fig. 4. Theis type curve matching for well A3.

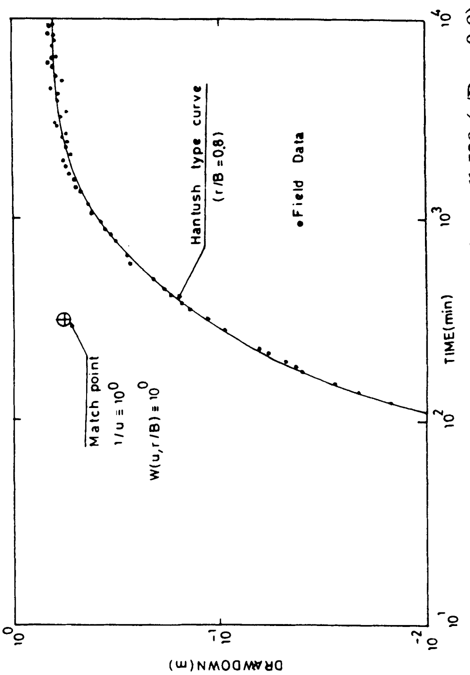

Fig. 6. Hantush type curve matching for well J29 (r/B ≡ 0.8).

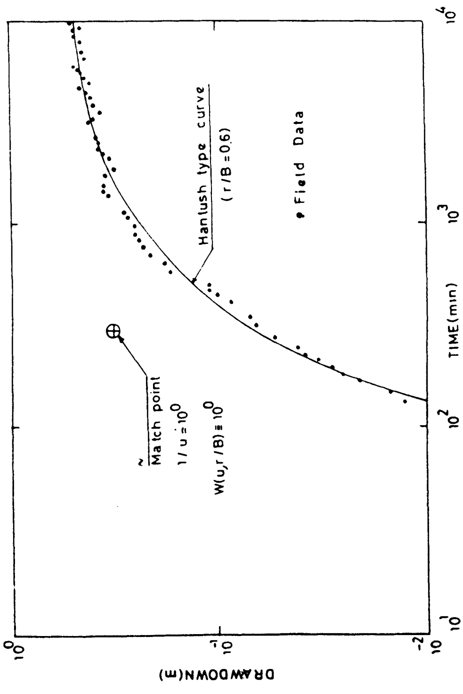

Fig. 5. Hantush type curve matching for well E1 (r/B ≡ 0.6).

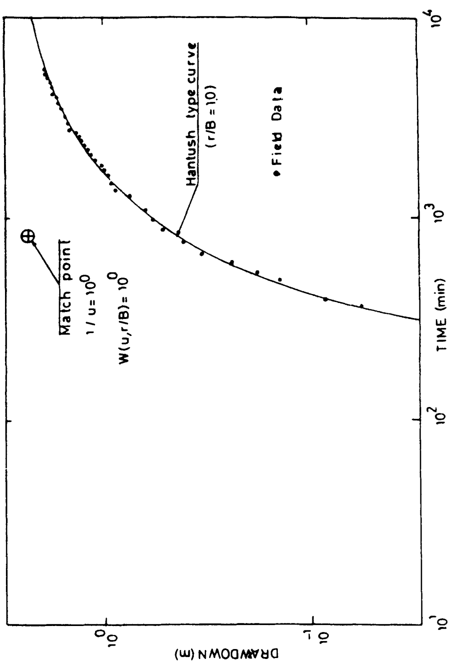

Fig. 7. Hantush type curve matching for well J25 (r/B ≡ 1.0).

time *i.e.* with the expansion of the depression cone. This is tantamount to saying that as the cone of depression expands the aquifer material covered is less permeable. This may physically be due to the fact that within the depression cone the groundwater velocity becomes greater toward the pumping well and therefore fine particles are washed out more and more in the well vicinity giving rise to the permeability increment toward the same well.

In order to be able to detect a trend more explicity, the moving average scheme is applied to each sample function and the results are shown on the same figures for moving average orders of 3 and 10. As the order increases the amplitude of fluctuations decrease and then it is possible to deduce whether the underlying trends are linear or non-linear. The physical significance of moving average method application is to be able to depict the hidden trends, jumps or functional spatial changes in the aquifer parameters around the pumping well. For instance, in Fig 10 the transmissivity trend is almost linear whereas the storativity sample function in Fig 11 has a non-linear trend which is in the form of a cycle. Initially, the storativity is low and it becomes relatively high at moderate times but decreases again at large times. The interpretation of such a trend might be that the aquifer material in the well vicinity as well as at large distances is composed of relatively small grains than at the moderate distances.

In Fig. 12 there is a trend of exponential decrease which is very obvious with the moving average of order 10. However, the storativity in Fig. 13 for the same aquifer indicates a composite pattern of trends where initially there appears a sharp decrease and then an increase of storativity takes place and after some stable level it decreases again.

The sample function in Fig. 14 indicates an initial exponential decrease which is followed by another phase of exponential decrease in the moving average values of order 10. Finally, the storativity sample function in Fig. 15 has stable trend.

Conclusions

Erratic deviations of the time-drawdown data plot from type curve indicate local heterogeneities within the aquifer material. Consequently, the aquifer parameter values found from the classical matching procedure provide overall estimations. Unfortunately, they do not furnish any clue about the heterogeneity of the aquifer material. However, the application of slope matching procedure leads to parameter sample functions which provide further opportunities in the aquifer material composition interpretations around the well. It is possible to detect quantitatively heterogeneities in terms of the trends, jumps, cycles and any other systematic changes in the aquifer parameters especially by means of the classical moving average procedure. It is hoped that future studies will shed more interpretive light on the aquifer parameter sample functions.

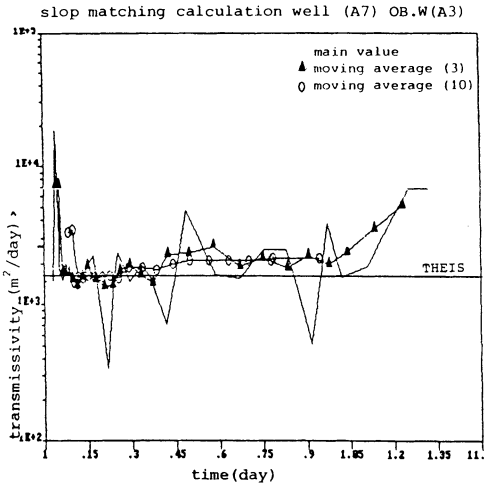

Fig. 8. Transmissivity sample function for well A3.

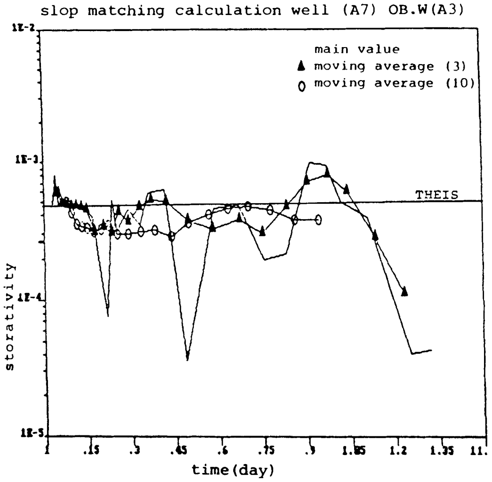

Fig. 9. Storativity sample function for well A3.

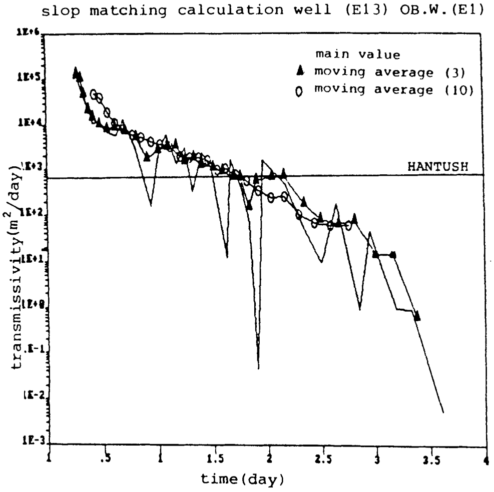

Fig. 10. Transmissivity sample function for well E1.

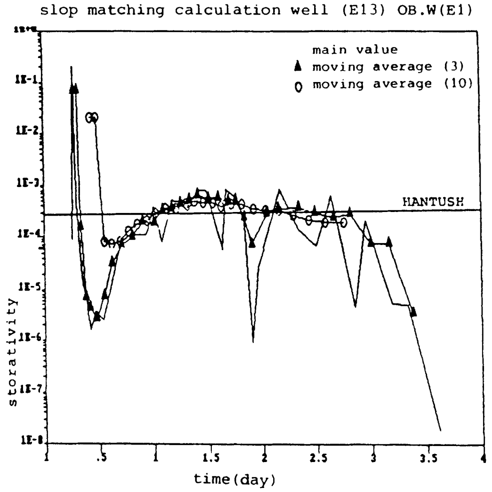

Fig. 11. Storativity sample function for well E1.

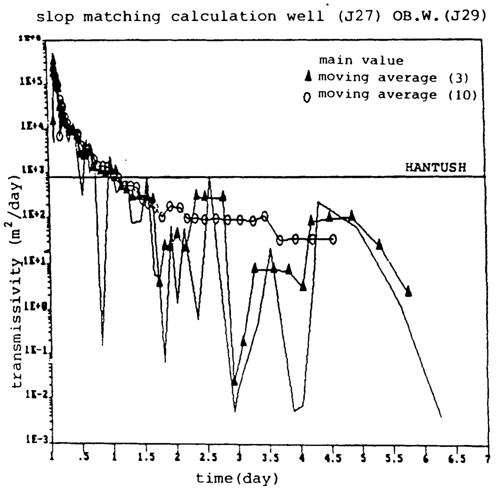

Fig. 12. Transmissivity sample function for well J29.

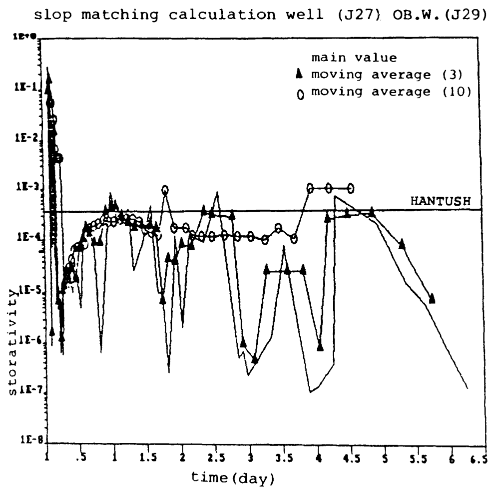

Fig. 13. Storativity sample function for well J29.

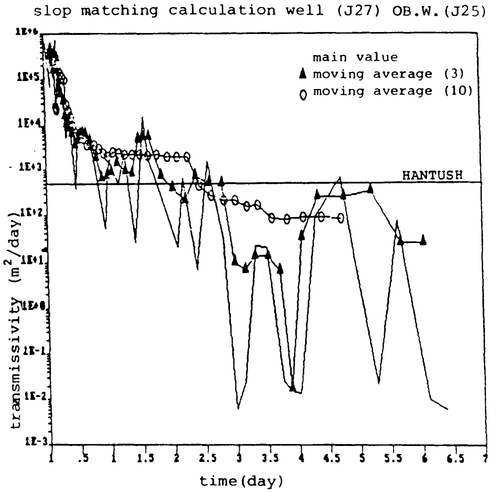

Fig. 14. Transmissivity sample function for well J25.

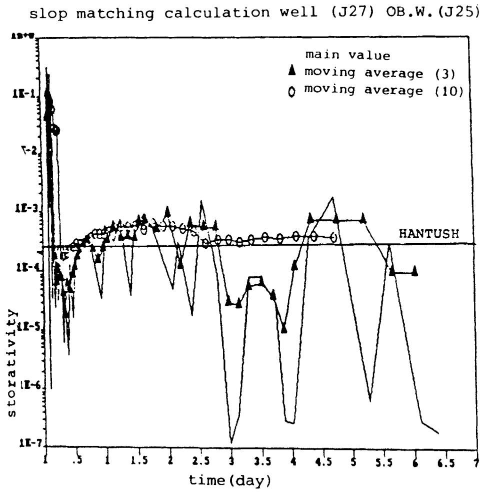

Fig. 15. Storativity sample function for well J25.

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