DISCUSSION

Substantial discrepancies in collapse time exist between creep-buckling theories such as this one and experiments. Calculations based on rather simple creep-buckling hypotheses such as those proposed by Shanley [3] and Gerard [4] show up just as well or better in comparisons between theory and experiment as the more refined analysis, even though these simple hypotheses seem to have relatively little physical or rational basis. The analysis of this paper and of [1] and of the great majority of other papers on this problem (and there are many) are based on creep relations which are really only valid for creep under constant stress. That is, the constants in the creep relation (λ and k in this paper) are determined from constant stress-creep data. Actually, of course, the stress at some points in the column varies considerably as creep progresses. In [3] and in Libove’s papers, [5] and [6], an effort was made to account for this fact since the stress varies during creep of the column. Perhaps it is time to reappraise the creep relations that are being used in analyses of this type and determine whether or not they adequately represent the material behavior.

References


GUSTAV MESMER. It is well known (references [4, 5, 7] of this paper) that all systems of equipotential lines and the corresponding orthogonal streamlines can be considered to be isostatics (stress trajectories) of certain elastic plane-stress systems. These stress systems belong obviously to a peculiar limited class, fulfilling specific boundary conditions. This means that the trajectories of only a few exceptional cases of elastic problems can be considered to be potential lines or, which is equivalent, of the “square” type (differential parameters h1 and h2 being equal). To mention one important point: the “closed” type of internal isotropic points, very often appearing in stress trajectories, has no counterpart in the potential network. The method of the author can therefore be used with reliability only in some peculiar cases. The circular plate between two singular forces belongs to this group. Most real problems, for example, the problems of the simple tension and bending bar with holes or notches, do not belong to this class, and one cannot expect accuracy or reliability in the stress results based upon a partly similar but nonetheless wrong network of trajectories. Some comparative numbers of Table 3 of the paper are significant.

Author’s Closure

The author wishes to thank Professor G. Mesmer for his comments. As it was pointed out in the paper, and it was mentioned in the discussion, the necessary and sufficient condition for a network of isostatics of a plane-stress elastic system to be represented by a network of equipotentials and the corresponding stream lines is to be isometric. This restricts the rigorous application of this analogy only to those cases fulfilling this condition.

But, following Wegner, we can express the Cartesian stress components in a plane-stress elastic field by the following relations:

\[ \sigma_x = -r^2 R \cdot [F(x)] + R \cdot \left[ \int_{a}^{b} zF(x)dx + p \right] - R \cdot |N(x)| \]
\[ \sigma_y = r^2 R \cdot [F(x)] + R \cdot \left[ \int_{a}^{b} zF(x)dx + p \right] + R |N(x)| \]
\[ \tau_{xy} = r^2 R \cdot [F(x)] + I \cdot |N(x)| \]


Analysis of Plane Stress in Polar Coordinates and With Varying Thickness

V. S. MUSICK. In May, 1958, the writer submitted in partial fulfillment for a Master of Science degree in Engineering at Union College, Schenectady, N. Y., a thesis entitled, “On the General Solution of Variable Thickness Plane-Stress Problems in Polar Coordinates With a Special Solution of a Variable Thickness Stodala Type Disk With Nonuniform, Radial Boundary Loading.” The solution given in the present paper closely parallels the writer’s earlier work.

The author has observed that the characteristic equation

\[ \lambda^4 + 2(m - 2)\lambda^3 + (4 - 5m - 2n - 2m^2 + n^2)\lambda^2 - (m - 2) (m + vm + 2n^2)\lambda - n(4 + vm - 3m - vm^2 - n^2) = 0 \]  

is easily factorable for \( m = 2 \). In the writer’s investigation of this same characteristic equation, it was discovered, for \( n = 2 \) and \( n = 0.3 \), in the cases checked, the roots were real for \( 0 < m < 2 \) and for \( m = 2 \) some roots were found to be conjugate complex of the form:

\[ \lambda^4 = a_n + b_ni \]
\[ \lambda^3 = a_n - b_ni \]
\[ \lambda^2 = c_n + b_ni \]
\[ \lambda_1 = c_n - b_ni \]

Hence the form of equation

\[ R_n = (A_n r^{\lambda_1} + B_n r^{\lambda_2} + C_n r^{\lambda_3} + D_n r^{\lambda_4}) \]  

will change to

\[ R'_n = A_n r^{\lambda_1} \cos (b_n \ln r) + B_n r^{\lambda_2} \sin (b_n \ln r) + C_n r^{\lambda_3} \ln (b_n \ln r) + D_n r^{\lambda_4} \sin (b_n \ln r) \]

As a point in passing, one should make note of the fact that \( m > 2 \) represents a fairly high flare of cross section.

Direct Determination of Stresses in Plane Elasticity Problems Based on the Properties of Isostatics

2 Turbine Bucket and Rotor Engineering, Large Steam Turbine-Generator Department, General Electric Company, Schenectady, N. Y. Assoc. Mem. ASME.
where $R$ and $I$ denote the real and imaginary parts of two arbitrary analytic functions $F(z)$ and $N(z)$ of the complex variable $z$.

Therefore any plane-stress elastic field with no body forces can be considered as resulting from two independent isometric stress systems, one of which, defined by the function $N(z)$, is a harmonic stress system whose first invariant $2\sigma = (\sigma_1 + \sigma_2)$ is everywhere zero. The superposition of the two systems is possible only if certain criteria exist relating either the functions $F(z)$ and $N(z)$ or the difference of the principal stresses.

However, it may be noted that, in general, the contribution of the analytic function $N(z)$ on the resultant stress field is secondary and its influence in deviating the principal stress field corresponding to the analytic function $F(z)$ is negligible in regions far from singularities. This explains the isometric character of the field of isostatics in these regions. In cases of plane-stress fields containing singularities, these can be determined accurately from a photoelastic fringe pattern. The singularities correspond either to points of transition from one isostatic to its orthogonal or to isolated points belonging to one type of isostatic surrounded by the two extreme isostatics of the other family.

The determination of the isotropic points or lines on the boundaries or in the interior of the field allows the separation of the boundaries into two groups from which each belongs to one family of isostatics and the determination of the extreme isostatics. Each family of isostatics can be traced by using the extreme isostatics of the family as electrodes of application of the potential in the electrical analogy method. The parts of the boundaries corresponding to this family and not belonging to the extreme isostatics must have also electrodes. The potential of these electrodes is regulated to correspond to a limiting value of potential which makes this electrode an equipotential.

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Since the boundaries of the traced field of isostatics are compelled to correspond to those of the real problem, and since the greater discrepancies due to the function $N(z)$ are encountered in the vicinity of the singularities, which are forced to correspond to those of the rigorous problem by the placing of the electrodes, the deviations from the correct network are insignificant. Therefore the traced network of isostatics, by analogy, is in many cases more accurate than the corresponding network constructed by a purely graphical process on the basis of isoclinics which, moreover, are, in general, very badly determined photoelastically. As an example, we give in Fig. 1 the one family of isostatics traced by this analogy in the case of a circular ring subjected to a diametral compression because this case contains all kinds of singularities. For the sake of comparison, the same network of isostatics, traced graphically, is given in the book by M. M. Frocht, "Photoelasticity," vol. I, page 209.