Author's Closure

The author is grateful to Professor Rouleau for his Discussion; the author wishes to acknowledge that his contributions to both the areas of oil-impregnated and gas-lubricated porous bearings originated from the foundations laid by Professor Rouleau. The discussor makes two remarks: the author agrees with the first one that the Beavers-Joseph model has not been verified (experimentally) for the multidimensional, variable pressure gradient flow occurring in porous journal bearings. We are well aware that whenever a new model is proposed, the model is applied to other geometric configurations and so long as the theoretical predictions (based on the application of the model to the new configuration) do not differ from the performance obtained experimentally, we are prone to accept the model rather than reject the same. For example, Newton's law of shear viscosity could be thoroughly tested only through viscometric flows. However, there is no hesitation in applying it to different geometries though experimental verification is not possible in every case. As such, while experimental verification is most desirable, the absence of the same should not preclude the application of a new model to other geometric configurations.

While the author has his full appreciation for the paper by Rouleau and Steiner (Discusser’s reference [3]), it is not possible for him to accept the contention that a slip flow model is, in fact, unnecessary. The velocity slip occurring at the permeable bearing-film interface is governed by Darcy's law in their paper. This means that the velocity slip is governed by only a single physical property of the surface—permeability. As different from this, the Beavers-Joseph model envisages one more independent property of the surface \( \alpha \) governing the extent of velocity slip at the permeable boundary; \( \alpha \) is a dimensionless constant independent of the viscosity of the fluid but dependent on the structure of the permeable material. The value of \( \alpha \) could be experimentally determined for plane parallel flows by Beavers, Sparrow, and Magnuson (author’s reference [18]). The recent experimental work by Taylor [11] with grooved Perspex disks modeling a particular porous material and the companion theoretical work of Richardson [2] confirm the Beavers-Joseph model citing the possible asymptotic values for \( \alpha \) for the particular model chosen by them. Also, Saffman [3] analyzes the problem from the angle of statistical mechanics with a well-presented derivation of the Beavers-Joseph boundary condition. From all this evidence, the validity of the Beavers-Joseph model at least for plane parallel flows seems indisputable.

It is interesting to note that the extent of slip velocity at the permeable boundary given by Darcy law is the same as the one given by Beavers-Joseph model when \( \alpha = \alpha' \). This corresponds to the minimum slip velocity possible at the surface as per the Beavers-Joseph model and thus velocity slip derived from Darcy law can be treated as a special case of the Beavers-Joseph model (\( \alpha \to \alpha' \)). Thus it seems sensible to accept the validity of the model \( a \) priori for the multidimensional, variable gradient flows as well.

In this context, it is only right to point out the lack of experimental verification so far for the theoretical results of Rouleau and Steiner (Discusser’s reference [3]).

References


Axisymmetric Potential Flow Over Two Spheres in Contact

F. A. Morrison, Jr.

Contrary to the authors' claim, exact solutions are known for axisymmetric potential flow past two spheres. Solutions are available for flow past both separated and touching spheres.

The electric potential found by Morrison and Stukel [1], as part of an analysis of electrophoretic motion, is exactly analogous to the velocity potential describing axisymmetric irrotational flow past two separated equal spheres. This exact solution uses a bisphe-ric coordinate system.

Flow past two spheres in contact has been widely studied. Tangent sphere coordinates have been employed to obtain exact solutions for Stokes flow past rigid [2, 3] and fluid [4] spheres in contact. Majumdar's [5] study of the axisymmetric potential flow over two equal spheres in contact is of greatest importance here. Majumdar, using tangent sphere coordinates, found an exact solution for the stream function describing this flow.

The authors seek to solve this problem in terms of the velocity potential. Apparently the velocity potential has not been previously calculated. The authors' attempt is seriously in error, however, starting with their equation (4). Because the domain is semi-infinite in the coordinate \( \mu \), an integral representation rather than a series representation should be used. In place of equation (4), one should write

\[
\phi = \frac{U}{\nu}(\mu^2 + \nu^2) + (\mu^2 + \nu^2)^{1/2} \int_0^\infty [A(n) \cosh n\nu + B(n) \sinh n\nu] J_0(n\mu) \, dn
\]

Using this relation, the discusser has found an exact solution for the velocity potential describing axisymmetric potential flow over two equal spheres in contact. The analysis will be submitted for publication as a Brief Note. The result is

\[
\phi = \frac{U}{\nu}(\mu^2 + \nu^2) + (U/2\alpha)(\mu^2 + \nu^2)^{1/2} \int_0^\infty [2\mu - \mu \tanh \mu - \ln(2 \cosh \mu)] \sinh n\nu J_0(n\mu) \, dn
\]

References