DISCUSSION

the peripheral location of the intermittent thermal fluctuations shown in Fig. 6, which corresponds to the middle portion of the layer just described, the velocity fluctuations are not intermittent since bursts from the whole plate cover this region; but since only some of these bursts originate on the heated portion of the plate, thermal intermittency is observed. Moreover, the author's "base line" represents the energy of the cold fluid and all of the thermal intermittencies are due to the arrival of random high-temperature bursts and hence are one-sided.

The writers feel that the author's results are useful in two regards: (a) The work gives strong further evidence of the close relation between wall effects and the origin of turbulent bursts in the wake portion of the turbulent boundary layer. In particular, it tends to verify that the primary origin of bursts is in the wall layers and is not due to free-stream interactions. (b) The results suggest the use of thermal elements mounted in the wall as a means for tracking the history of one or more turbulent bursts originating in the wall layers. Such information should be of aid in describing the path of fluctuations and in determining the size and frequency of single bursts as well as the frequencies and angles with respect to the wall of the generating vortex system.

Slow Rotatory Motion of a Circular Disk About One of Its Diameters in a Viscous Fluid

E. M. SPARROW.1 The author is to be congratulated on his solution of an interesting problem in fluid mechanics. In the interests of a concise presentation, a definition of the parameter $a$ and a discussion of its characteristics have been omitted from the paper. Apparently, $a$ is a kind of eigenvalue; but it takes on a continuous spectrum rather than the discrete spectrum of values as is frequently encountered in boundary-value problems. Further remarks about the properties of $a$ would be of interest.

It would appear that equations (19) to (23) provide the velocity and pressure results corresponding to a given $a$. To achieve the final solution, integration over $a$ seems necessary, and this has been done for $w$ and $p$ in the plane $z = 0$. However, the integrations have not been carried out for $z \neq 0$; and in this sense, the solution is not complete. Whatever remarks the author may wish to make about the integrated results for the condition when $z \neq 0$ would be helpful.

Author's Closure

Professor Sparrow's comments are very much appreciated. His remarks about the parameter $a$ are quite correct. It may, however, be added that this method of solving the boundary-value problems is essentially the method of Hankel transforms.

The reason that the integration of the main results have been carried out only in the plane $z = 0$ is that the coupling of the disk is thereby completely determined. Second, the value of $w$ turns out to be given in an elegant closed form. The values of the pressure, as given by the relations (23) and (29) of the original paper, is

$$p = \frac{4 \sqrt{2} \mu \omega r^3}{\sqrt{\pi}} \sin \theta \int_0^\infty \alpha J_{1/2}(\alpha r)J_1(\alpha r)e^{-\alpha^2} d\alpha$$

The quantity $\sqrt{\alpha} J_{1/2}(\alpha r)J_1(\alpha r)$, can be expanded in the series in $a^3$

$$\alpha^{1/2}J_{1/2}(\alpha r)J_1(\alpha r) = \frac{1}{3} \left( 2 \frac{r}{\sqrt{\pi}} \right)^{3/2} \sum_{n=0}^{\infty} \left[ (2n + 3)(n + 2)(n + 1)F(-n, 3 + n; 5/2, 2; r^2, r^4) \times J_{2n+1}(\alpha r) \right].$$

Therefore

$$p = \frac{32 \mu \omega r^3 \sin \theta}{3\pi} \sum_{n=0}^{\infty} \left[ (2n + 3)(n + 2)(n + 1)F(-n, 3 + n; 5/2, 2; r^2, r^4) \times J_{2n+1}(\alpha r)e^{-\alpha^2} d\alpha. \right.$$

But

$$\int_0^\infty J_{2n+1}(\alpha r)e^{-\alpha^2} d\alpha = \frac{\sqrt{\pi} \left( \frac{z^2 + 1}{z^2} \right)^{n+3/2}}{\sqrt{2}}.$$ Substituting the relation (5) in (4), we have the complete evaluation of the pressure.

Incidently, now that the method of evaluation of the velocity components for the case $z \neq 0$, has been demonstrated in the foregoing analysis, one can investigate the shearing stresses experienced by the disk.

A New Theory of Elastic Sandwich Plates—One-Dimensional Case

J. M. FRANKLAND.2 For homogeneous plates, coupling of thickness-shear and thickness-stretch modes with the simpler forms of extensional and flexural vibration does not occur below very high frequencies. These phenomena are therefore of minor interest in the dynamic analysis of structures. If sandwich plates are used, however, this paper shows that the coupling can be of importance down to much lower frequencies. Sandwich structures subjected to acoustic excitation, for example, might show a more complicated behavior than a homogeneous structure. The subject deserves further study to make clear

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