Developing Region of Laminar Jets With Uniform Exit Velocity Profiles

S. Eskinazi. Are we, academically oriented engineers, entitled to subscribe to the general philosophy of accepting methods and theories on the basis of "the end justifying the means" in a given analysis? We cannot say for sure, as we all have tendencies to use this form of justification from time to time. The work in this paper is no exception.

The converse argument of the "means justifying the end" is not always an acceptable logic either, and yet perfect rigor of mathematical and physical logic have often shown to be insufficient in describing reality, for lack of a total understanding of the problem.

Simply stated Messrs. Rankin and Sridhar have forced an existing analytical asymptotic solution for a laminar fully developed jet, by Schlichting [3], to be valid in the developing shear layers adjacent to the potential core of the jet in the near-field. Using the experimental measurements of axial velocity by Greene [2] they categorically show with Figs. 2-5 and 7-9 that their method works. Should anymore be said about it? Since the paper cites other references claiming similar successes in their respective methods of analysis, this attests to the fact that neither method possesses the generality and full adherence to the physical behavior of this flow field. Nevertheless, the arguments that will follow should not detract from the usefulness of the method proposed for some applications.

First, since only the axial component of the velocity is considered in this analysis, one of the following two physical arguments must hold: a) the derived solutions for the development region do not satisfy conservation of mass, or b) the radial component of the flow exists, but not bothered with, to give (d/dt + 1/ρu) equal on magnitude but opposite in sign to 2/ρuX. In either case, any function for u(x, r) will work as long as it is made to fit the experiments. Although the total volume flow rate in a circular jet increases slowly and linearly due to the entrainment, in this model the volume rate of flow in the developing shear region adjacent to the potential core increases very fast over and above the entrainment, because it incorporates more and more of the depleting potential core flow. As a matter of fact the volume flow rate in the developing shear layers grows as πα^2(1/4 - B^2). In percent to the amount of volume flow rate entrained, the loss of flow in the potential core is (1/4 - B^2)/8αX. On the basis of equation (19) this could be of the order of 1/8α, which could be very large.

In a similar way, momentum in the developing shear layer of the flow is not conserved, and yet the validity of Schlichting's solution adopted for the developing shear layer is based on conservation of axial linear momentum, in its total extent.

The consequence of this is that the angle of spread γ is not constant with x for a given Re as Schlichting's solution requires. Equation (6) admits this incongruity by stating that the integrated axial momentum is conserved for 0 < r < ∞ and not for the developing shear layer b < y < ∞.

For a kinetic energy balance, equation (7) is not complete. Energy being a scalar quantity, it must contain energy fluxes in all directions; namely, that of the radial velocity component, and especially the radial flux of kinetic energy in the axial velocity which may not be negligible near the nozzle where the axial changes of radial gradients are very large.

The significance of a virtual origin is in the establishment of a point of reference in the axial direction to which an asymptotic solution may be extended, upstream, with some validity. In the case of a constant angle of spread it is a much more valid concept. However, in the case of this problem, γ and x are functions of the axial coordinate x as evidenced by Figs. 8 and 9. Also, since x is measured from the mouth of the nozzle, in the direction of the mean axial flow, the referenced axial distance should have been (x - x) instead of (x + x), conforming to the concept of relative distance.

Furthermore, equation (4) is made to conform with the assumed boundary conditions stated after equation (11) by making X go to zero when X = 0, and as a consequence γ = 0. Why is this necessary? Because of entrainment the angle of the jet could be non zero at X = 0. The experiments in Figs. 8 and 9 may support this argument. Although equation (17) is an assumed relation, equation (18) is a consequence of these assumptions.

In the evaluation of the accuracy of the approximate solution over the "original" solution, the variance of X, is given as 0.000217 implying a small value. Since, by definition, X = x/d,Re, for Reynolds numbers close to the critical value, say 2000, the variance Δx/d is 0.000217 x 2000 = 0.43; meaning of the order of the radius. This is large in the initial stages of the development. The results in Figs. 5, 7, 8, and 9 are the most significant in giving support to the theory. The comparisons in Figs. 2, 3, and 4 would look good for other similar theories as well. Since the comparisons in Figs. 5, 7, 8, and 9 are apparently for Re = 100, the reader needs to know if the agreement remains as good at the larger Re's.

Finally, the comments raised, here, should not infer a criticism on the work effort presented as it may be useful in a number of applications. However, in view of the collective assumptions made in this work, the reader should exercise care in applying these results to flows with nonuniform nozzle exit velocity, and for the development of further analyses beyond the practical usefulness for which this work was intended.

A. Mitsunaga. It is noteworthy that a rather complicated flow, as that reported, can be treated by using a few approximate equations. In this investigation the velocity profile used in the development region is Schlichting's form, which is exact in the axisymmetrical developed region. Near the nozzle exit,
however, the velocity profile resembles a two-dimensional shear layer, inasmuch as the shear layer is thin compared to the potential core. This approximation has been applied in several studies related to turbulent jets. Abramovich1 for example, used a velocity profile which describes the smoothing out of a velocity discontinuity in the development region and the profile for an axisymmetrical jet in the developed region. Could the authors please comment on why they did not follow a similar approach in analyzing the development behavior of a laminar jet?2

### Authors' Closure

The authors would like to thank Professors S. Eskinazi and A. Mitsunaga for their respective discussions. In particular, we thank Professor Eskinazi for expounding the many assumptions that are required to obtain our simplified solution. A study of these assumptions, to determine why they work in this case, will certainly yield further insight into the physical behaviour of the flow field.

A solution for the radial component of velocity, consistent with continuity requirements, was not sought in connection with the present solution. Also, it is not the authors' intention to imply that a modification of Schlichting's radial velocities component would be valid in this case. Although a solution for the radial velocity may be found in the future, experimental verification would likely be difficult. In addition, it is important to note that the present expression for the axial velocity distribution was obtained analytically and not by curve fitting the experimental data.

The boundary condition that \( \gamma = 0 \) at \( x = 0 \) is not independently assumed in our analysis. It is inherent in the assumption of a uniform exit velocity profile. This can be seen by considering equation (5) which applies in the region \( B < R \). If \( \gamma = 0 \), \( U' = 0 \) for all \( R \) excluding \( R = B = 1/2 \). At this point \( U' \) is indeterminate. However, it can be shown to be equal to unity. If a non-zero value of \( \gamma \) is used, the shear layer has a finite width which contravenes our uniform profile assumption.

The authors feel that more information concerning the data points in Figs. 7, 8, and 9 should have been given. Lack of this information has resulted in the wrong impression that these assumptions, to determine why they work in this case, will certainly yield further insight into the physical behaviour of the flow field.

Regarding the virtual origin, we accept that the use of \( \gamma = 0 \) agrees with the concept of relative distance. However, if we stipulate that \( x_0 \) is always less than or equal to zero, we can use \( x + x_0 \). This procedure has also been followed by other investigators (reference [4], for example).

In response to Professor Mitsunaga's question concerning use of the twodimensional shear layer profile, the authors feel that the present profile is the better approximation over a larger portion of the jet development region. However, we agree that the two-dimensional shear layer profile would be the better approximation close to the nozzle exit.

### Turbulence Measurements in Boundary Layers Along Mildly Curved Surfaces

Ronald M. C. So.3 The authors are to be commended for an excellent job in their effort to obtain credible data on turbulent boundary layer development along mildly curved surfaces. These turbulence measurements are especially valuable since they substantiate the data of Meroney and Brandshaw [17] and complement the measurements of So and Mellor [12, 13] which were obtained on surfaces with fairly large curvature. In the opinion of this discussor, however, the discussion provided by the authors on their results is not very penetrating. As a result, their discussion fails to provide us with any new insights into the behavior of turbulent curved shear layers. This is especially true in view of the prior publication of Meroney and Brandshaw [17] on essentially the same experiments.

On the other hand, close examination of the present results and those reported earlier by the authors [15], Meroney and Brandshaw [17] and Ellis and Joubert [30] reveal that they all show that the flow is slowly approaching an equilibrium state. This is evidenced by the similarity displayed by the mean velocity and shear stress profiles in the flow far downstream of the entrance to the curved wall. If the flow is indeed approaching an equilibrium state, then a curvature parameter can be identified such that the equilibrium turbulent curved shear layer is a function of both this parameter and the pressure gradient parameter \( \beta = \delta^* (dp/dx)/\tau_w = constant \) (where \( \delta^* \) is the displacement thickness and \( dp/dx \) is the streamwise pressure gradient) first identified by Clauser [31, 32]. This was shown to be the case by So [33] recently. In the formulation, So [33] defined the pressure gradient parameter to read as \( \beta = \delta^* (dp/dx)_{A\tau_w} \) to avoid the confusion introduced by the pressure variation across the shear layer as a consequence of the "centrifugal" force created by the curvature of the streamline. This definition has the advantage that it reduces to Clauser's definition in the absence of curvature. The pertinent curvature parameter was shown to be given by \( \gamma = K \Delta \) where \( K \) is the surface curvature and \( \Delta \) is the defect displacement thickness for curved flows defined in a manner analogous to that proposed by Clauser [31, 32]. With the parameters thus defined, So [33] was successful in demonstrating that the governing two-dimensional equations of mean motion together with the eddy viscosity function for curved shear flows derived by So and Mellor [7] can be reduced to a single third order differential equations in terms of \( f(\gamma) \) and parametric in \( \beta, A \) and \( \gamma = u / U_w \). The function \( f \) is defined as

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