mounted installation further furnishes the assurance that such circumstances would not be esoteric happen-stances. Thus is is evident that the "perturbation of an easy nonlinear solution" is a convenient approach to treat this class of problem. The solution pertains to the steady-state motion, which corresponds to the asymptotic trend obtained with the step-by-step integration method. Insofar as this may be the only result of interest, there is no wasted effort. The two parts of the solution, induced harmonic excitation and perturbed fluid film forces, can be documented once for all and be subsequently utilized to analyze any rotor system in which such a damper installation exists. As an improvised technique addressing a specific aspect of damper characteristics, it is most certainly a more efficient way to deal with the limited problem. The usefulness of the step-by-step integration method, however, would not be totally eliminated; it would always be relegated to investigate all such other aspects of rotor dynamic problems for which a specific method of solution has not yet been devised.

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DISCUSSION

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The increased use of fluid film dampers in high-speed rotating machinery has prompted several recent investigations. These have pointed out the importance of understanding the nonlinear behavior or the damper, particularly as it relates to force attenuation and transition through resonance. This paper, together with the work of Mohan and Hahn [10], represent in my opinion, some of the more serious attempts to analyze the nonlinear dynamics of load-carrying fluid films. Of particular interest is that the work departs from "brute force" direct integration schemes which seem to have been overused in the last few years despite their tendency to provide little understanding and minimal design information. The analytical approach, on the other hand, offers insight into the effect of system parameters on performance and can therefore be a valuable design tool. That is not to say that this latter is without the usual drawbacks associated with nonlinear analysis. Certainly, it falls short of predicting the effect of initial conditions (e.g., blade loss situations) and guaranteeing the uniqueness of the solution. Therefore, my remarks are not intended to promote one method over the other, but rather to emphasize their complimentary nature.

A question that arises whenever analytical work is presented is its accuracy in predicting the physical system. Have the authors verified the analysis, particularly that the details of the excellent experimental work of Tonnesen [12] was available to them? Rolling element bearings in applications such as those being considered tend to exhibit nonlinear stiffness characteristics. Will the authors' comment on the feasibility of incorporating such a feature in their model and the effect it would have on orbit eccentricity and the occurrance of "doubly-looped" orbits?

Authors' Closure

The authors are grateful to have recieved Dr. Ezzat's thoughtful comments and kind words.

The choice of methodology to solve problems in nonlinear mechanics is for the most part a matter of personal taste. The authors agree with Dr. Ezzat that one should keep the option open and evaluate the relative merits of the specific circumstances. So far as squeeze-film dampers are concerned, it is felt that the basic understanding is still lacking regarding the cavitation or film-rupture process, not only for extreme loads but also when violent transients are involved. In this respect, the limitation of analytical methods is self-evident, but the deficiency would not be overcome by resorting to a numerical integration scheme.

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The primary conclusions of the present analysis are that the eccentric operation of the squeeze-film damper would cause a self-generated harmonic excitation, which consists predominantly the second harmonic, and that, in the event of film rupture, there is a tendancy to oppose the eccentricity of the orbit center. When this work was started, it was hoped that an exhaustive comparison with Tonnesen's experimental data [12] would be possible. Unfortunately, it is rather difficult to examine the feature of harmonic excitation because the system dynamics of the experimental setup does not lend itself readily to analytical treatment. A moderate amount of data on static lift was available. Correlation with analytical prediction can be evaluated in terms of the index.

$$I_0 = (\Delta e_0)_{test}/(\Delta e_0)_{theo.}$$

A perfect correlation would be indicated by $I_0 = 1.0$. Δe_0 is the shift in the orbit center from the preset position. $(\Delta e_0)_{\text{theo.}}$ is calculated from equation (29) assuming "half-film" and with the small ϵ_0 approximation, that is

$$F_{(0)} = \frac{2(3 + \epsilon_1{}^2)}{\pi(1 - \epsilon_1{}^2)^3}$$

Since the orbit is elliptical, the average between the amplitudes parallel and perpendicular to the preset displacement is used to calculate ϵ_1.

A total of 58 eccentric orbit data are available. Four data points were not used because these are very small orbits, $\epsilon_1 < 0.1$, and the shift of orbit center cannot be accurately determined. For the 54 cases considered, the mean value of I_0 is 0.794 and the corresponding standard deviation is 0.223. The highest and lowest values are 1.509 and 0.265, respectively. This result is gratifying in view that ϵ_0/ϵ_1 exceeded unity in 10 cases, for which the analysis is not expected to hold. As stated in the papers, static lift vanishes in the "full-film" approximation. Thus the correlation appeared to indicate that, on the average, film-rupture was somewhat less extensive than the "half-film" approximation.

It should be quite feasible to apply the present method of analysis to consider the nonlinear character in the Hertzian contacts of rolling element bearings. In the presence of simultaneous static radial and rotating loads, similar effects such as orbit center shift and second harmonic excitation should occur. Since there is relatively little damping in common rolling element bearings, second harmonic excitation can be quite prominent in the event of its coincidence with a natural frequency, and "doubly looped" orbits would be observed in such circumstances.