keep rising by virtue of the momentum theorem. This is apparently what happens. Why it happens requires a deeper understanding of the physics of transitory stalls than we currently possess. A further study of transitory stall is underway by J. Ashjaee, J. P. Johnston, and the second author currently at Stanford. Hopefully, this will yield increased understanding.

With regard to the question on the method of "frozen dp/dx," the situation is similar. In particular, the displacement thickness grows rapidly enough in the zone of flow detachment, so that it significantly affects the pressure in the outer flow field. Since the pressure in the outer flow field affects the rate of growth of boundary layer displacement thickness, the result is that displacement thickness and longitudinal pressure gradient BOTH AFFECT EACH OTHER IN AN ONGOING WAY THROUGH DETACHMENT: In most classic boundary layer computing schemes and analysis, one computes the pressure gradient for the whole flow, then the displacement thickness for the whole flow using the last computed dp/dx values, then the pressure gradient, and so on. But this does not allow for the actual ongoing interaction between displacement thickness and dp/dx that occurs in the flow. Indeed, such a "cyclic" iteration may fail to converge; it will then predict *either:* (i) no separation; or (ii) unbounded growth of displacement thickness (depending on the direction from which convergence is approached). The method of "frozen dp/dx" circuvments this difficulty. It stabilizes the calculation by artificially maintaining the maximum adverse pressure gradient until the flow *separates*. This provides, in effect, a reasonable bound on the *location* of separation in cases where the separation is reasonably rapid. The method of "frozen dp/dx" should not be expected to give good details of flow field in the zone of detachment, merely a decent estimate of location; however, that is enough for some purposes. For details see reference [A]. Basically, then, the method of "frozen dp/dx" is a "fix." It is a relatively good fix for some purposes in that it is simple and gives good answers up to a known level of information. No other justification is known to the authors. A more fundamental way of resolving the problem is to use "simultaneous iterations;" see, for example, reference [B].

Additional References

A Cebeci, T., Mosinskis, C. J., and Smith, A. M. D., "Calculations of Separation Points in Incompressible Turbulent Flows," *J. Aircraft*, Vol. 9, No. 5, 1972, pp. 618–624.

B Ghose, S., and Kline, S. J., "Prediction of Transitory Stall in Two-Dimensional Diffusers," Report MD-36, Thermosciences Division, Dept. of Mechanical Engineering, Stanford University, Dec. 1976.

Reynolds Number Dependence of Skin Friction and Other Bulk Flow Variables in Two-Dimensional Rectangular Duct Flow[1]

J. LAUFER.[2] In this paper Dr. Dean formulates the turbulent channel flow problem in the same spirit as done by Coles for the boundary layer. It is based on two widely accepted empirical relations: the law of the wall and the law of the wake. The constants have been obtained on the basis of carefully selected experimental data; they were found to be very near those suggested by Coles for a turbulent boundary layer. This is actually not surprising since the channel flow could be considered very nearly as an equilibrium boundary layer with constant thickness. The result gives a reliable method for the calculation of the friction coefficient and for the mean velocity distribution over a wide range of Reynolds numbers.

[1]By R. B. Dean, published in the June, 1978, issue of the JOURNAL OF FLUIDS ENGINEERING, Vol. 100, p. 215.

[2]Professor and Chairman, School of Engineering, University of Southern California, Los Angeles, Calif., 90007.

Author's Closure

I would like to thank Professor Laufer for his comments on the paper which I appreciate very much, as his own work on two-dimensional duct flow nearly thirty years ago [7] was the first such paper on this subject and is widely regarded as a classic source of reliable data. The point he raises about the resemblance of the duct-boundary layer to an equilibrium boundary layer of constant thickness is an interesting one because, in fact, the values of Coles parameter Π may be zero or negative due to the pressure gradient requirements of the equilibrium boundary layer, which clearly affects the development of the outer layer. This is seen for example in the data of Herring and Norbury [41] where Π becomes zero and negative. This means that the separable variable form of the outer layer function $\Pi\ (\mathbf{x})/K(w(y/\delta))$ used by Coles [4], [20] becomes zero for all values of y/δ. This problem is avoided when the function $g\ (\Pi,\ y/\delta)$ is used instead, as proposed in [5] and used in the present paper, but it should be emphasized that there is a difference in philosophy between the two approaches. Coles [20] procedure for finding δ and U_{τ} by minimizing the r.m.s. deviation of the data from equation (9) using the sinusoidal form of $w(y/\delta)$ indirectly forces the data in favour of positive Π, which explains why there are very few "zero" or negative values of Π listed in [20]. However, it has been explained in [5] that the present approach allows the form of the wake profile to be described more accurately for different types of pressure gradient. In any event, the parameter Π has been shown in [4], [5], and [20] to be a very useful quantity for characterising turbulent boundary layers and the confirmation of its independence of viscosity in the present paper demonstrates its applicability to duct flows and other internal flows.

Additional Reference

41 Herring, H. J., and Norbury, J. F., "Some Experiments on Equilibrium Turbulent Boundary Layers in Favourable Pressure Gradients," *Journal of Fluid Mechanics*, Vol. 27, 1967, pp. 541-549.

The Effect of Taylor Vortex Flow on the Radial Forces in an Annulus Having Variable Eccentricity and Axial Flow[1]

J. FRÊNE.[2] The authors are to be congratulated for examining the effect of Taylor Vortex flow on the radial forces in an eccentric annulus formed by a stationary outer cylinder and a rotating inner cylinder with an axial flow.

On a plain bearing machine working under Taylor and turbulent conditions the discusser has recently [a1] obtained the

[1]By J. E. Coney and J. Atkinson, published in the June, 1978, issue of the JOURNAL OF FLUIDS ENGINEERING, Vol. 100, p. 210.

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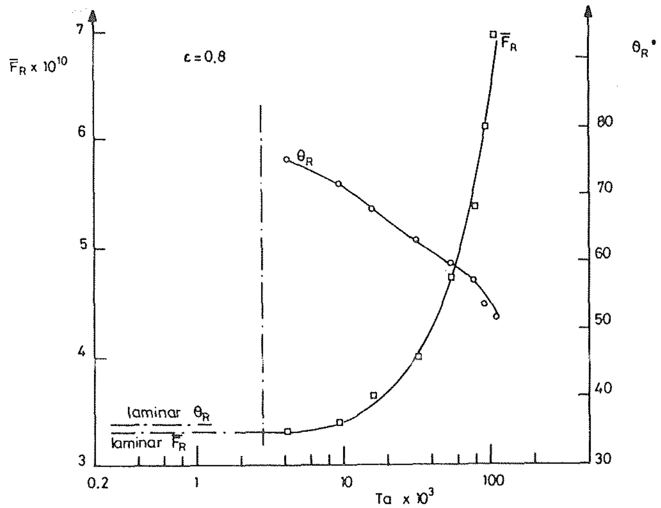

Fig. A1 F_R **and** θ_R **versus** T_a **for** $\epsilon = 0.8$ **and** $C/R = 0.00293$

eccentricity and the attitude angle for given loads and speeds. The bearing dimensions were as follows: journal diameter and bearing length are 80 mm which give a L/D ratio of 1. Clearance is 117 μm and the clearance ratio is 0.00293.

Typical results are given in Fig. A1 using the same dimensionless variables as the authors. In this figure the vertical dotted line gives the critical Taylor number based on the work of Frêne and Godet [a2]. It is interesting to note that the same tendencies are observed even though the dimensions are very different. These result in different values of Reynolds number for the same value of Taylor number. The mean Reynolds number based on the rotational speed of the inner cylinder varies between 40 and 1350 in the authors' case and between 500 and 6000 in the discusser's case.

In the case of $\epsilon = 0.9$ the authors show that immediately after the onset of Taylor vortex flow, the radial force $\bar{F}_R$ falls with T_a and then recovers; the discusser has not observed this process. This tendency could perhaps be explained by the effect of convective inertia forces.

Additional References

a1 Frêne, J., and Godet, M., "Plain Journal Bearings Operating Under Vortex and Turbulent Flow Conditions, Comparison Between Experimental and Theoretical Results," *Proceeding of Second Leeds Lyon Symposium*, "Superlaminar Flow in Bearings," Mechanical Engineering Publications 1977 paper Xiii pp. 194–198.

a2 Frêne, J., and Godet, M., "Performance of Plain Journal Bearings Operating Under Vortex Flow Conditions": ASME, Journal of Lubrication Technology, Vol. 96, No. 1, Jan. 1974, pp. 145–150.

G. S. RITCHIE.[3] The paper reports on experimental investigation of the effects, in the vicinity of the transition region, of nonlaminar flow on the pressures generated in a journal bearing configuration, when Taylor vortex flow rather than true turbulence is to be expected. The results have been compared, somewhat briefly, with, for the laminar flow the Sommerfeld long-bearing theory, and for the post-transition regime, the nonlinear Taylor vortex analysis of DiPrima and Stuart (reference [9] of the paper). The correlation is of necessity limited to the case where the axial flow is absent. This is also true of the present discussion, and furthermore, the numerical working on what follows is also restricted to small eccentricity so that small eccentricity ratio theory (as in reference [9]) is reasonably valid. It is unfortunately true that the smallest eccentricity used ($\epsilon = 0.5$) in the experiments in rather large in this context.

In the paper, no mention is made of fluid inertia effects which in view of the large clearance ratio ($\simeq 0.1$) would be expected to be comparable in magnitude to the Sommerfeld force. Taking the relevant results from reference [9] (although earlier workers have quantified them), for small eccentricity ratio (neglecting $0(\epsilon^2)$) and small clearance ratio (neglecting $0(\delta)$), in the laminar flow region, the force components on the rotor are given by equations 6.9 and 6.10 in the form

$$Fx = \frac{-\pi q_1 \mu l}{\delta^2} \frac{R_M}{5} \epsilon$$

$$Fy = \frac{-\pi q_1 \mu l}{\delta^2} 6\epsilon$$

where R_M to the applied approximation is the conventional "reduced" Reynolds number:

$$R_M = \frac{q_1 a \delta^2}{\nu} = \text{Re}\,\delta$$

Fx is the inertial force and is directed in the line of the eccentricity vector. Physically it is the consequence of the momentum changes necessary to accelerate the fluid through the narrow gap. It might be noted that the numerical factor 1/5, tends to 1/4 when the flow becomes fully turbulent due to the "squaring-off" of the turbulent velocity profiles (see Fritz, reference [A1]). Fy is the conventional Sommerfeld force, linearized in ϵ. (For $\epsilon = 0.5$, the numerical value is 3.0; the nonlinear value: $12\epsilon/(2 + \epsilon^2)(1 - \epsilon^2)^{1/2}$ is 3.08).

To gauge the relative importance of inertia and lubrication forces, we consider the experimental conditions at $\epsilon = 0.5$, $\text{Re}_a = 0$ (Fig. 3) at the quoted transition Taylor number 2900. $\text{Re} \simeq 170$; $R_M \simeq 17$ and the ratio Fx/Fy has the value 0.57. The theoretical attitude angle is then $\theta_R \simeq 60°$, compared with an experimental value $\theta_R \simeq 93°$. At Ta = 290 (i.e. one tenth of the observed transition) $\text{Re} \simeq 54$; $R_M = 5.4$; $Fx/Fy = 0.18$ and $\theta_R \simeq 80°$ compared with an experimental value $\theta_R \simeq 96°$. Agreement between experiment and theory, even in the laminar regime, is therefore not good.

So far as the nonlaminar regime is concerned reference [9] indicates that the effect of Taylor vortices is small in the limited range over which the theory may be expected to be valid. In the experiments, the rapidly decreasing attitude angle as the Taylor number increases indicates the increasing dominance of the force component in the line of the eccentricity. To a large extent this is to be expected, since the inertia force will increase according as the square of the speed, while the lubrication force (in the absence of any enhanced viscosity effects) increases in proportion to the speed. It is also possible that a contribution to the in-line force arises due to local enhanced viscosity in the initial 90° of the converging wedge as a consqeuence of the observable and predicted maximum in vortex activity in this region. It is, however, impossible to quantify this effect.

Overall the agreement between the experiments and present theories is disappointing for the range of parameters used, and it is not clear whether the theory or the experimental set-up is deficient. It would be interesting to calculate whether the test-rig distorts or deforms under the action the hydraulic pressures generated, particularly since the predicted forces tend to increase the eccentricity if any elasticity is present.

Such a problem existed during some experiments in the discusser's laboratory in 1970, on a somewhat similar geometry of test-rig ($L/D \simeq 4$, $\delta = 0.03, 0.11$) but with very much higher Taylor numbers ($6 - 15 \times 10^6$ for $\delta = 0.03$; $0.8 - 5 \times 10^8$ for

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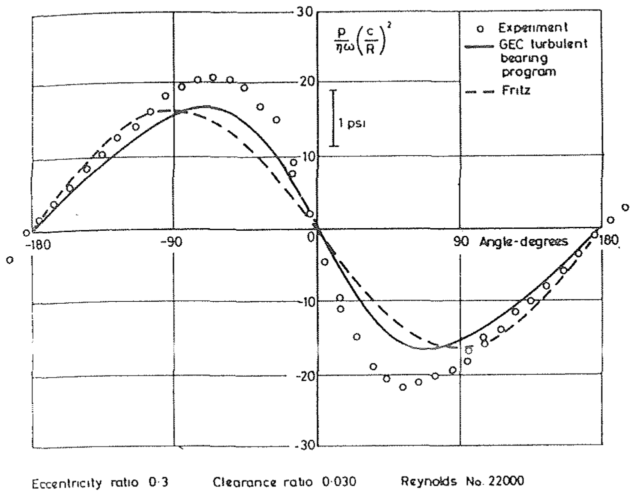

Fig. A1 Comparison of theoretical and experimental pressure distributions. Turbulent wedge

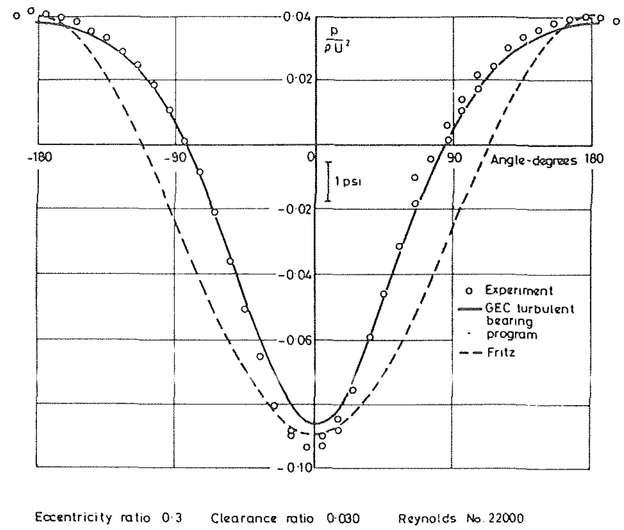

Fig. A2 Comparison of theoretical and experimental pressure distributions. Vertical component

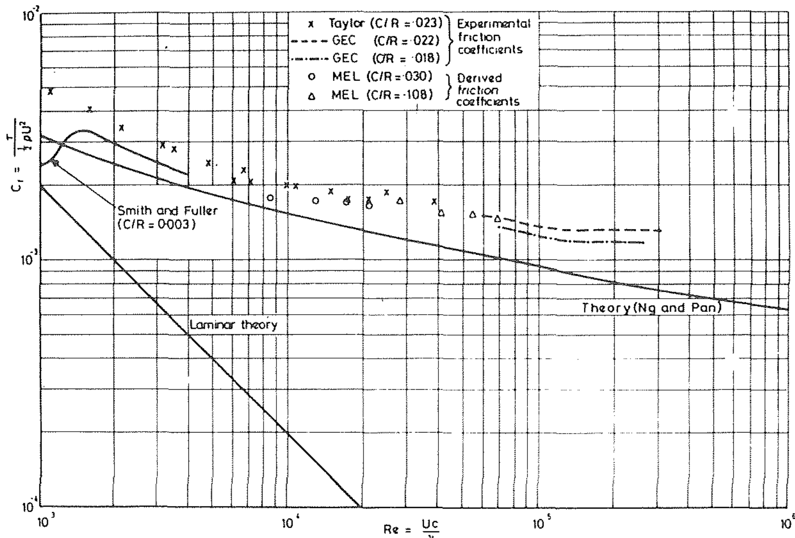

Fig. A3 Variation of friction coefficient with Reynolds number

$\delta = 0.11$). The results of these experiments have been briefly reported in [A2].

In the experiments, in-line and transverse pressure distributions were separated by taking the mean sum (for the in-line component) and half the difference (for the transverse component) of the pressures measured for either rotation direction. This resulted in distributions given in Figs. A1 and A2, where in theoretical estimates from the analysis of Fritz and predictions using an in-house turbulent journal bearing program are also given. The inertial pressure distribution (Fig. A2) is in excellent agreement particularly with the computer program results, since it is not linearized in eccentricity ratio as is the Fritz analysis. The lubrication pressures (Fig. A1) are slightly underestimated by either theory which however both use the Ng and Pan (reference [A3]) theory of turbulent flow. The similarity in shape of the experimental result with the computer program result however indicates that the turbulent model is good, but with too small a friction coefficient. In Fig. A3 the implied friction coefficients are shown in comparison with firstly the Ng and Pan theory and available direct friction coefficient measurements. It becomes apparent that, for any wide-gap geometry, the friction coefficient, probably due to the presence of Taylor vortex activity is indeed significantly above the Ng and Pan figure. As a corollary, for wide gap geometries, the lubrication force prediction will be given more accurately using an experimental value of the friction coefficient. It was concluded therefore that, far above transition, accurate prediction of pressure distributions was possible for a geometry of gap and conditions considerably removed from that for which the theory was devised.

Additional References

A1 Fritz, R. J., ASME *Journal of Basic Engineering*, Dec. 1970, p. 923.

A2 Ritchie, G. S., and Cowking, E. W., "Super Laminar Flow in Bearings," Mech. Eng. Pub., London, 1977.

A3 Ng., C-W, and Pan, C. H. T., ASME *Journal of Basic Engineering*, Vol. 87, No. 4, Sept. 1965, p. 675.

J. T. STUART.[4] 1. In the comparison of $\bar{F}_R$ (Discussion) for $\epsilon = 0.5$, $\delta = 0.1$, $\mathrm{Re}_a = 0$, an experimental value of 1.072 is compared with 1.048 from theory. It should be noted that this comparison may not be meaningful: in a later paper, the *Journal of Fluid Mechanics*, Vol. 87, 1978, pp. 209–231, Eagles, Stuart, and DiPrima have shown by a higher-order calculation that the nonlinear theory is restricted (especially for force calculations) to much smaller values of ϵ and δ.

2. Although the text (Discussion) quotes the Sommerfeld load $\bar{F}_{RS}$ as 0.486×10^6 for $\epsilon = 0.5$, Fig. 3 indicates a value near to 0.6×10^6. This is inconsistent with the error estimate of $\pm$ 5.7 % for $\bar{F}_R$.

3. Table 1 gives $T_{ac} = 2900$ for $\epsilon = 0.5$, $\mathrm{Re}_a = 0$, which is reasonably consistent with the theoretical value of about 2950. On the other hand, the text quotes $T_{ac} = 3754$ (Discussion), which is inconsistent with both of the above values.

Author's Closure

The authors were very interested in the confirmation of the trends of their experimental results given by Dr. Frene. However, they note that values of θ_R for the supercritical region only were given in Fig. A1. They would be glad to know if subcritical values exist and if they increase with T_a as do the authors' for $\epsilon = 0.9$.

Mr. Ritchie's contribution is of great value in that it draws attention to the effects of fluid inertia. Discussing the difference between theoretical results and the authors' experimental results, he mentions the possibility of rig distortion due to internal hydraulic pressure. From Fig. 1, it may be seen that the perspex outer cylinder is firmly bolted, at its outlet end, to the flange of the inner cylinder bearing housing but is only poorly supported, at its inlet end, by a flexible rubber tube. Hence it may be considered as a uniformly loaded cantilever. It is also assumed that the inner cylinder, being robustly constructed of steel, does not deflect. Taking the conditions giving the highest resultant force generated in the series of tests, viz. $\epsilon = 0.9$, $R_{ea} = 50$, $T_a = 251{,}000$ when $F_R = 79.05$N, the maximum deflection of the outer cylinder is estimated to be 0.05 mm. While this is small compared with the concentric gap width of 4.71 mm, it has significance with respect to the minimum gap if $\epsilon = 0.9$. Had the higher Taylor numbers of Mr. Ritchie's experiments been reached, it is agreed that this distortion would have been of importance. With respect to the increase in inside diameter of the outer cylinder due to hydraulic pressure, this has been shown to be negligibly small.

Regarding Professor Stuart's comments concerning the disparity in critical Taylor numbers, the authors feel that a misunderstanding exists which they would like to discuss through private correspondence.

Finally, the authors wish to thank the three contributors for the interest which they have shown in this paper.

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