

## DISCUSSION

In spite of the fact that the system of equations (3)–(5) is unstable, it might still be possible to utilize these results in a short duration impact because instabilities take time to grow. It is conceivable that a scheme could be devised in which the growth rate of the dependent variables are as slow as possible, i.e., make the positive real part of the characteristic root as small as possible so that its effects in a short duration impact are negligible.

## References

- 1 Routh, E. J., *Dynamics of Rigid Bodies*, Dover Publications, Inc., New York, 1955; reprint of 1892 edition of MacMillan & Co., Ltd., London.
- 2 Liapunov, M. A., "Problème général de la stabilité du mouvement" *Ann. Fac. Sci. Univ. Toulouse*, Vol. 9, 1907, pp. 203–469; *Ann. Math. Studies*, No. 17, Princeton University Press, Princeton, N.J., 1961.

## Authors' Closure

The authors are indeed pleased that Dr. Liu has provided analytical support to the experimental findings discussed in the paper. While it is important to show that the use of six linear accelerometers can lead to erroneous values of angular acceleration, it should be pointed out that the stability analysis provided by Dr. Liu considers a special configuration of six accelerometers and that it may not be valid for the general case of arbitrarily located accelerometers. In the latter situation, the governing equations become quite complex and a linearized perturbation analysis cannot definitely establish the instability of the system. In fact, the results in the discussion show that the solution is not asymptotically stable and are not sufficient to prove that it is unstable.

With regard to the growth of instability it should be noted that the problem at hand is the accurate determination of angular acceleration. If the system of equations is indeed unstable, then error is present as soon as  $t > 0$ . Furthermore, it is only in very special cases that one can insure a slow growth rate, since the characteristic roots are dependent upon the magnitude and sign (direction) of the angular velocity components of a rigid body in general three-dimensional motion.

Finally, the burden of proof of stability rests with the six-accelerometer user who must also identify quantitatively the time beyond which the errors become intolerably large.

## Scattering of Water Waves by a Pair of Semi-Infinite Barriers<sup>1</sup>

**G. Dagan.**<sup>2</sup> The independence of the transmission coefficient  $T$  upon the angle of incidence  $\alpha$  of the far wave, which represents one of the main results obtained by the author, seems to be in contradiction with simple physical facts.

Indeed, let us consider an incident wave with crest normal to the breakwaters, i.e., with  $\alpha = 0$ . In this case there is no scattering and  $\phi_s = 0$  (eq (7)) while  $\phi_i = A \exp(-ikx)$ . It is obvious that an exact solution for the wave propagated along the channel is  $\phi = \phi_i$  and the transmission coefficient is exactly  $T = 1$ . Hence, the solution for ( $T$ ) (equation (27)) cannot hold in this case and it is doubtful that it applies to other angles of incidence as well.

## Author's Closure

G. Dagan raised an important point on the uniform validity of the

<sup>1</sup> By P.L.-F. Liu and published in the December, 1975, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 42, No. 4, TRANS. ASME, Vol. 97, Series E, pp. 777–779.

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asymptotic solution; in particular, for the normal incidence, i.e.,  $\alpha \rightarrow 0$ . The author agrees with him that in this case the transmission coefficient should be exactly  $T = 1$ . Due to the fact that the scattered waves (equation (8)) were in the order of magnitude of  $O(\sqrt{kr})$  and were neglected in the  $\phi$  analysis, the transmission coefficient (equation (26)) could be best interpreted as  $T = 1 + O(\sqrt{ka})$ . In other words, as  $\sqrt{ka} \ll 1$  the transmitted waves between two breakwaters are indeed independent of the angle of incidence.

To include the effects of the angle of incidence, the scattered waves should be included and modifications on the inner solutions are needed. The author has not so far completed this study and would consider the question an open one.

## A Practical Two-Surface Plasticity Theory<sup>1</sup>

**A. Phillips.**<sup>2</sup> The author should be congratulated for a very interesting and stimulating paper. Three comments should be added to the author's presentation. The two most important ones are first that according to a large number of experimental results, by the reviewer and his coworkers, some of which have already been published [1–3]<sup>3</sup> and some of which are still in the process of publication [4], the theory of Mroz, at least as used in the general formulation of the present paper, does not agree with the experimental results. Not only the form of the yield surface changes with the motion but also the center of the yield surface does not move in the direction indicated by the author. The law of hardening of the yield surface is still not clear. The limit surface on the other hand could be considered to grow isotropically from the initial yield surface with its center remaining unchanged.

The second comment is that the stress-strain curves, the modeling of which is attempted by the author, include rate effects and therefore cannot be modeled very well by a plasticity theory. A theory of viscoplasticity is more likely to be successful. In particular, a theory of plasticity will represent the gross behavior of the stress-strain curves, while a theory of viscoplasticity will be able to represent the exact form of the curves.

The third comment is that the concept of the two surfaces plasticity theory has some previous history. It was considered first by this reviewer [5] and elaborated in a number of subsequent publications by him and his coworkers [6–8].

All three comments do not distract seriously from the achievement of the author in presenting a complete theory which, however, requires extensive improvements to become useful for the practice.

## References

- 1 Phillips, A., and Tang, J. L., "The Effect of Loading Path on the Yield Surface at Elevated Temperatures," *International Journal of Solids and Structures*, Vol. 8, 1972, pp. 463–474.
- 2 Phillips, A., and Kasper, R., "On the Foundations of Thermoplasticity—An Experimental Investigation," *JOURNAL OF APPLIED MECHANICS*, Vol. 40, TRANS. ASME, Vol. 95, Series E, 1973, pp. 881–896.
- 3 Phillips, A., Tang, J. L., and Ricciuti, M., "Some New Observations on Yield Surfaces," *Acta Mechanica*, Vol. 20, 1974, pp. 23–40.
- 4 Phillips, A., and Ricciuti, M., "Fundamental Experiments in Plasticity and Creep of Aluminum," *International Journal of Solids and Structures* (in press).
- 5 Phillips, A., and Sierakowski, R. L., "On the Concept of the Yield Surface," *Acta Mechanica*, Vol. 1, 1965, pp. 29–35.
- 6 Eisenberg, M. A., and Phillips, A., "A Theory of Plasticity With Non-coincident Yield and Loading Surfaces," *Acta Mechanica*, Vol. 11, 1971, pp. 247–280.

<sup>1</sup> By R. D. Krieg, and published in the September, 1975, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 42, No. 3, TRANS. ASME, Vol. 97, Series E, pp. 641–646.

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<sup>3</sup> Numbers in brackets designate References at end of Discussion.