On the Effect of the Pauli Principle on the Collective Oscillations in Spherical Even Nuclei

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With the purpose of clarifying the effect of the Pauli principle on the collective oscillations in spherical even nuclei, the method which enables us to take into account one of the effects of the Pauli principle, the blocking effect, is developed in the framework of the "ideal boson" picture. The method developed in this paper is, in some sense, the refined treatment of the method proposed by Ikeda, Udagawa and Yamamura and independently by Hara. By taking account of this effect, agreement with the experimental facts becomes very well compared with the calculation based on the conventional random phase approximation.

§ 1. Introduction

Since Marumori, Baranger et al. proposed the microscopic theory of nuclear collective oscillations, the so-called quasi-particle random phase approximation, the nature of collective vibrational states of even-even spherical nuclei, especially the nature of first $2^+$ excited states of these nuclei, has been extensively investigated from the microscopic point of view. Fundamental idea of the theory is as follows: As is well known, after performing the Bogoliubov transformation in order to take into account the pairing correlation, we set up a two-quasi-particle normal mode in the framework of the new Tamm-Dancoff method. In this case, we must notice that the fundamental assumption in this treatment is to regard the average number of quasi-particles in the ground state as sufficiently small compared to the total number of states. On the basis of this assumption, the pair of quasi-particles is replaced by the "ideal boson" and the system is treated in the "ideal boson space".

Such a replacement inevitably leads to neglecting two main effects; the effect of the Pauli principle among the quasi-particles composing the different pairs and the dynamical effects due to the residual interaction (in the original Hamiltonian) omitted under the conventional random phase approximation. However, there are many cases in which the number of quasi-particles in the ground state calculated under the conventional random phase approximation is by no means small compared to the total number of states. In such cases, the results obtained under the random phase approximation are not reliable, and it is necessary to construct the method of taking the place of the conventional random phase approximation on the basis of the assumption that the average
number of quasi-particles in the ground state is not necessarily small compared to the total number of states, i.e. the ground state is highly correlated. In order to do this, we must pay attention to the fact that the effect of the Pauli principle will be very important apart from the dynamical effects.

On this subject, Ikeda, Udagawa and Yamaura and independently Hara proposed an interesting method. Their method owes its special interest to the facts that, in their theory, the number of quasi-particles in the ground state is explicitly taken up in terms of an expectation value with respect to the ground state, and what is better, the formalism is very analogous to that of the conventional random phase approximation. According to their opinion, the number of quasi-particles in the ground state introduced in their manner represents the effect of the Pauli principle. However, it is not necessarily self-evident that the effect of the Pauli principle due to highly correlated ground state can be "effectively" expressed only in terms of the number of quasi-particles in the ground state. Even if the effect adopted in their method corresponds to the effect of the Pauli principle, there is no guarantee that the effect acts only on the interaction part of the Hamiltonian. The effect may also act on the free part of the Hamiltonian. In this sense we are necessary to inspect the effect of the Pauli principle on the collective oscillations in nuclei in detail.

The main purposes of this paper are to clarify the physical meaning of the effect of the Pauli principle in the "ideal boson space" and to show that the effect of the number of quasi-particles in the ground state has a close connection with the effect of the Pauli principle under some approximation. At the same time, we will develop the method in which the number of quasi-particles in the ground state is considered from the standpoint of the effect of the Pauli principle, keeping a close contact with the conventional random phase approximation. In order to do this, the general theory, which has been proposed by one of the present authors (M. Y.) with Marumori and Tokunaga, is very powerful. Because, according to this theory, the two main effects, the effect of the Pauli principle and the dynamical effects, which are neglected under the conventional random phase approximation, can be evaluated correctly, in principle, in the framework of the "ideal boson space". As will be shown in §3, we can classify the effect of the Pauli principle in the "ideal boson space" into two types of the effects, the blocking effect and the exchange effect and we can understand that the effect considered in Ikeda et al. and Hara's method corresponds, in some sense, to the blocking effect. According to our treatment, the effect acts not only on the interaction part of the Hamiltonian but also on the free part of the Hamiltonian.

We shall start with a spherical $j-j$ coupling shell model Hamiltonian with

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4 On this subject we will show in §2, 2-3.

5 Hereafter, we will refer to it as (I).
the “pairing plus quadrupole” interaction. Following the discussion of the assumption of the conventional random phase approximation, we will recapitulate the method proposed by Ikeda et al. and Hara in terms of the “ideal boson” picture in § 2. In § 3, the physical meaning of the effect of the Pauli principle in the “ideal boson space” will be clarified and the method in which the number of quasi-particles in the ground state is taken up as the blocking effect will be developed. In order to see the effect adopted in Ikeda et al. and Hara’s method and the general features of our method, we will compare their method with ours and show numerical results on Te, Xe and Ba isotopes comparing with those of the other methods in § 4.

Throughout this paper, we will keep a close contact with the conventional random phase approximation and the method proposed by Ikeda et al. and Hara as much as possible. Therefore, hereafter we will call the conventional random phase approximation and Ikeda et al. and Hara’s method the “simple boson approximation” (the SBA) and the “extended boson approximation” (the EBA), respectively,*) following Hara’s paper.

§ 2. The simple and the extended boson approximation

2–1. The basic Hamiltonian

In order to avoid unnecessary complication, we will restrict ourselves to spherical even nuclei in which either the neutrons or protons are in a major closed shell and consider the system of even number of neutrons or protons in the upper unfilled shell consisting of several subshells.

Let us start with the spherical $j$-$j$ coupling shell model Hamiltonian with the “pairing plus quadrupole” interactions. To treat the pairing interaction, we use the well-known quasi-particle representation. Thus, we first perform the Bogoliubov transformation. In the transformed Hamiltonian, the various types of the interactions between quasi-particles which come from the quadrupole interaction still remain. However, in the present paper, we take into account only the terms which are retained in the SBA. Then, we can get the following Hamiltonian in terms of quasi-particle operators $a_\alpha^\dagger$ and $a_\alpha$:

$$H = H_0 + H_t,$$

$$H_0 = \sum_\alpha E_\alpha a_\alpha^\dagger a_\alpha,$$

$$H_t = -\frac{1}{4} \sum_{\alpha \beta \gamma \delta} \sum_{\nu \nu'} q_\alpha^\nu q_\beta^\nu q_\gamma^\nu q_\delta^\nu \epsilon_{\alpha \beta \gamma \delta}$$

$$\times \left( a_\alpha^\dagger a_\beta^\dagger \cdot s_{\gamma \delta} s_{\gamma \delta} a_\gamma^\dagger a_\delta^\dagger + 2a_\alpha^\dagger a_\beta^\dagger \cdot a_\delta a_\gamma + s_{\alpha \beta} s_{\alpha \beta} a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger a_\delta^\dagger \right),$$

where

(*) Hereafter, the quantities in the SBA and the EBA are designated by superscripts $S$ and $E$, respectively.
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\[ q_{\alpha \beta}^{\mu} = \langle \alpha | r^2 Y_{\mu M}(0\phi) | -\beta \rangle_{s_{-\beta}} = q_{ab} \langle j_\alpha m_\alpha j_\beta m_\beta | 2M \rangle, \]

\[ \xi_{ab} = \frac{1}{\sqrt{2}} (n_a v_b + v_a n_b). \] (2.1b)

The operators \( a_\alpha^\dagger a_\beta^\dagger \) and \( a_\alpha^\dagger a_\gamma \) satisfy the following commutation relations:

\[ [a_\alpha^\dagger a_\beta^\dagger, a_\alpha^\dagger a_\beta^\dagger] = (\delta_{\alpha\alpha'}\delta_{\beta\beta'} - \delta_{\alpha\beta'}\delta_{\beta\alpha'}) \]
\[ - (\delta_{\alpha\alpha'}a_\beta^\dagger a_{\beta'} + \delta_{\beta\beta'}a_\alpha^\dagger a_{\alpha'} - \delta_{\alpha\beta'}a_\beta^\dagger a_{\alpha'} - \delta_{\beta\alpha'}a_\alpha^\dagger a_{\beta'}), \] (2.2a)

\[ [a_\alpha^\dagger a_\gamma, a_\alpha^\dagger a_\beta^\dagger] = (\delta_{\alpha\alpha'} + \delta_{\beta\gamma}) a_\alpha^\dagger a_\beta^\dagger. \] (2.2b)

2-2. The simple boson approximation

As is well known, the fundamental assumption in the SBA is to regard the average number of quasi-particles in the ground state as sufficiently small compared to the total number of states, in other words, to neglect the terms \( \langle a_\alpha^\dagger a_\alpha \rangle \) and \( \langle a_\alpha^\dagger a_\beta \rangle \) on the right-hand side of the following equation:

\[ \langle [a_\beta^\dagger a_\alpha^\dagger, a_\alpha^\dagger a_\beta^\dagger] \rangle = (\delta_{\alpha\alpha'}\delta_{\beta\beta'} - \delta_{\alpha\beta'}\delta_{\beta\alpha'}) (1 - \langle a_\alpha^\dagger a_\alpha \rangle - \langle a_\beta^\dagger a_\beta \rangle), \] (2.3)

where \( \langle O \rangle \) denotes the expectation value of the operator \( O \) with respect to the true ground state. Therefore, as far as concerned with the low-lying states, we may adopt the following approximate commutators:

\[ [a_\beta^\dagger a_\alpha^\dagger, a_\alpha^\dagger a_\beta^\dagger] = \langle [a_\beta^\dagger a_\alpha^\dagger, a_\alpha^\dagger a_\beta^\dagger] \rangle \]
\[ = \delta_{\alpha\alpha'}\delta_{\beta\beta'} - \delta_{\alpha\beta'}\delta_{\beta\alpha'}, \] (2.4a)

\[ [a_\alpha^\dagger a_\gamma, a_\alpha^\dagger a_\beta^\dagger] = (\delta_{\alpha\alpha'} + \delta_{\beta\gamma}) a_\alpha^\dagger a_\beta^\dagger, \] (2.4b)

i.e. we may regard the operators \( a_\alpha^\dagger a_\beta^\dagger \) and \( a_\alpha^\dagger a_\gamma \) as

\[ a_\alpha^\dagger a_\beta^\dagger \rightarrow C_{\alpha\beta}^\dagger, \] (2.5a)

\[ a_\alpha^\dagger a_\gamma \rightarrow \sum_\beta C_{\alpha\beta}^\dagger C_{\gamma\beta}, \] (2.5b)

where the \( C_{\alpha\beta}^\dagger \)'s are the "ideal boson" operators satisfying the following relations:

\[ C_{\alpha\beta}^\dagger = - C_{\beta\alpha}^\dagger, \]

\[ [C_{\alpha'\beta'}, C_{\alpha\beta}] = \delta_{\alpha\alpha'}\delta_{\beta\beta'} - \delta_{\alpha\beta'}\delta_{\beta\alpha'}, \] (2.6)

\[ [C_{\alpha'\beta'}, C_{\alpha\beta}] = [C_{\alpha'\beta'}, C_{\alpha\beta}^\dagger] = 0. \]

Thus, the original Hamiltonian (2.1a) in this case is reduced to

\(^{*)\) The Greek subscript \( \alpha \) and the Roman subscript \( a \) mean the \( j-j \) coupling shell model state \( (n_\alpha^\dagger a^\dagger j_\alpha m_\alpha) \) and \( (n_a^\dagger a^\dagger j_\alpha m_\alpha) \) (except the magnetic quantum number \( m_\alpha \)) respectively and \(-a\) is obtained from \( a \) by changing the sign of the magnetic quantum number \( m_\alpha \). We also use the phase factor \( s_\alpha \) or \( s_M \) which means \((-1)^{a-m_\alpha}\) or \((-1)^{2-M}\).

\(^{**}\) The ground state of even nucleus has spin \( 0^+ \). Therefore, terms such as \( \langle a_\alpha^\dagger a_\alpha \rangle \) \((\alpha \neq \alpha')\) vanish.
This is just the fundamental Hamiltonian in the SBA and can be easily diagonalized by the well-known treatment.

Now, we must notice the following facts. As has already been discussed by Ikeda, Udagawa and Yamaura and independently by Hara, if the correlation between the pairs of quasi-particles is strong and, hence, the frequency of the vibration is low, the number of quasi-particles in the ground state will be large. In this case, the results derived under the SBA are contradictory to the original assumption of the SBA.

2-3. The extended boson approximation

Now, in order to see how the number of quasi-particles in the ground state has the effect on the collective vibration in nuclei (in connection with the effect of the Pauli principle), Ikeda, Udagawa and Yamaura and independently Hara adopted in their theory (the EBA) an approximation higher than that in the SBA on the commutators \((2\cdot2a)\) and \((2\cdot2b)\) between the pairs of quasi-particles and extended the SBA. Their theory is worth noticing, because in the EBA the effect of the number of quasi-particles in the ground state which is neglected in the SBA is self-consistently considered and the fundamental idea of the theory is a natural extension of the SBA.

We can make a formalism equivalent to the EBA by setting up the correspondences of the pair of quasi-particles and the "ideal boson" given from the following consideration. According to Ikeda et al. and Hara's point of view, the origin of invalidity of the SBA lies in the fact that, in the SBA, \(\langle a^\dagger_\alpha a_\alpha \rangle\) and \(\langle a^\dagger_\beta a_\beta \rangle\) on the right-hand side of Eq. \((2\cdot3)\) are neglected and, hence, the following approximate commutators must be adopted:

\[
[a^\dagger_\beta a_\alpha , a^\dagger_\alpha a^\dagger_\beta] = \langle [a^\dagger_\beta a_\alpha , a^\dagger_\alpha a^\dagger_\beta]\rangle \\
= (\delta_{\alpha\alpha'} \delta_{\beta\beta'} - \delta_{\alpha\beta'} \delta_{\beta\alpha'}) (1 - \langle a^\dagger_\alpha a_\alpha \rangle - \langle a^\dagger_\beta a_\beta \rangle),
\]

\[
[\alpha^\dagger_\gamma a^\dagger_\alpha a^\dagger_\beta] = (\delta_{\alpha\gamma'} \delta_{\beta\beta'} + \delta_{\alpha\beta'} \delta_{\beta\gamma'}) a^\dagger_\alpha a^\dagger_\beta.
\]

In consideration analogous to the SBA, we must inevitably regard \(a^\dagger_\alpha a^\dagger_\beta\) and \(a^\dagger_\gamma a^\dagger_\beta\) as \(C^\dagger_{\alpha\beta}/\sqrt{1 - \langle a^\dagger_\alpha a_\alpha \rangle - \langle a^\dagger_\beta a_\beta \rangle}\) and \(\sum_\gamma C^\dagger_{\gamma\beta} C_{\gamma\beta}\) in so far as we adopt the concept of the "ideal boson space". Taking notice of the relation

\[
\langle a^\dagger_\alpha a_\alpha \rangle = \langle \sum_\beta C^\dagger_{\alpha\beta} C_{\alpha\beta} \rangle \sim \frac{1}{2j_\alpha + 1} \langle \sum_\beta C^\dagger_{\alpha\beta} C_{\alpha\beta} \rangle = n_\alpha,
\]

we can get the following correspondences:
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\[ a_\alpha^+ a_\beta^+ \longrightarrow C_{\alpha\beta}^+ \sqrt{1 - n_\alpha - n_\beta}, \quad (2.10a) \]
\[ a_\alpha^+ a_\beta^+ \longrightarrow \sum_{\gamma} C_{\alpha\gamma}^+ C_{\gamma\beta}^+, \quad (2.10b) \]

which can be regarded as the extended relations of Eqs. (2.5a) and (2.5b). By using the above relations (2.10a) and (2.10b), the Hamiltonian in the EBA can be written down in terms of the "ideal boson" operators as follows:

\[ H^E = H_0^E + H_1^E, \]
\[ H_0^E = \sum_{ab} (E_a + E_b) \sum_{M} A^E_{1M}(ab) A^{E*}_{2M}(ab), \]
\[ H_1^E = -\frac{1}{2} \sum_{ab} q_{ab} \xi_{ab} \sqrt{1 - n_a - n_b} q_{cd} \xi_{cd} \sqrt{1 - n_c - n_d} \times \sum_{M} [A^E_{1M}(ab) s_{2M} A^E_{1M}(cd) + 2A^E_{1M}(ab) A^{E*}_{2M}(cd) + s_{2M} A^{E*}_{2M} (ab) A^{E*}_{2M} (cd)] , \]

(2.11)

where

\[ A^E_{1M}(ab) = \frac{1}{\sqrt{2}} \sum_{\alpha\beta} <j_\alpha m_\alpha j_\beta m_\beta|2M>C_{\alpha\beta}^+ . \]

(2.12)

This Hamiltonian can formally be diagonalized by the same treatment as in the case of the SBA:

\[ H^E = \sum_{e} \omega^E_e \sum_{M} X^E_{2M}(\sigma) X^{E*}_{2M}(\sigma), \quad (2.13) \]

where the \( \omega^E_e \)'s are the solutions of the following dispersion relation:

\[ 1 = 2\lambda \sum_{ab} q_{ab}^2 \xi_{ab} (E_a + E_b) (1 - n_a - n_b) \equiv \lambda F^E(\omega^E_e), \quad (2.14a) \]

and

\[ X^E_{2M}(\sigma) = \sum_{ab} [\phi^E_{\sigma, ab} A^E_{1M}(ab) - \phi^E_{\sigma, ab}^* A^{E*}_{2M}(ab)], \]

(2.14b)

\[ \phi^E_{\sigma, ab} = \frac{1}{N^E_\sigma} q_{ab} \xi_{ab} \sqrt{1 - n_a - n_b}, \quad \phi^E_{\sigma, ab}^* = \frac{1}{N^E_\sigma} q_{ab} \xi_{ab} \sqrt{1 - n_a - n_b}, \]

\[ N^E_\sigma = \left[ \sum_{ab} \left( \frac{q_{ab} \xi_{ab} \sqrt{1 - n_a - n_b}}{E_a + E_b - \omega^E_e} \right)^2 \right]^{1/2} \equiv N^E(\omega^E_e). \]

(2.15)

By using the ground state of the Hamiltonian (2.11), it is possible to perform the explicit calculation of \( n_\alpha \):

*) We treat the case \( J=2 \). So, the terms of \( J \neq 2 \) are neglected.

**) Here we have omitted the energy of the ground state.

****) Usually, the collective state corresponds to the state given by the lowest solution of this equation.
The set of Eqs. (2.14) and (2.16) is fundamental in the EBA. The $E2$ transition probability from the ground state to the collective state is also given with the aid of the above relations. The operator of $E2$ transition is written in the EBA as

$$
M^E(E2, M) = e \sum_{ab} q_{ab} \xi_{ab} \sqrt{1 - n_a - n_b} [A_{1,1}(ab) + s_{1M} A_{1-M}(ab)] .
$$

Then, using the matrix elements of the operators with respect to the ground state and the collective state, $B^E(E2; 0^+\rightarrow 2^+)$ is given by

$$
B^E(E2; 0^+\rightarrow 2^+) = 5e^2 \frac{P^E(\omega_E)}{N^E(\omega_E)^2} = 5e^2 B^E(\omega_E) .
$$

This is just the formalism equivalent to the EBA in the "ideal boson space".

As is clear from the above formalism, we can understand that in the EBA the effect of the number of quasi-particles in the ground state due to Eq. (2.3) is self-consistently taken into account and in this sense the EBA is a natural and formal extension of the SBA. However, from the view that the effect of the Pauli principle neglected in the SBA is taken into account, it seems for us not to be necessarily clear that the EBA is a natural extension of the SBA. For it cannot be asserted a priori that the effect of the Pauli principle can be expressed only in terms of the number of quasi-particles in the ground state.

In this sense, it will be necessary to inspect the effect of the Pauli principle in detail.

§ 3. The effect of the Pauli principle and the approximate diagonalization of the Hamiltonian

3-1. The physical meaning of the effect of the Pauli principle in the "ideal boson space"

As is clear from the above consideration, it is to be desired to clarify the physical meaning of the effect of the Pauli principle in the "ideal boson space". Let us start with the discussion as to what effects of the Pauli principle are neglected in the SBA. As an example, we consider the diagonalization of $H^S$ in a subspace of the free vacuum state and the "two-ideal boson" states. In this case, the secular equation contains the following types of the matrix elements of $H^S$:

$$
(0|H^S|0) = 0 ,
$$

$$
(\alpha\beta, \gamma\delta|H^S|\alpha'\beta', \gamma'\delta') = (E_\alpha + E_\beta + E_\gamma + E_\delta)
$$

$$
\times \{ (\delta_{\alpha\alpha'} \delta_{\beta\beta'} - \delta_{\alpha\beta'} \delta_{\beta\alpha'}) (\delta_{\gamma\gamma'} \delta_{\delta\delta'} - \delta_{\gamma\delta'} \delta_{\delta\gamma'})
$$

$$
+ (\delta_{\alpha\gamma'} \delta_{\beta\delta'} - \delta_{\alpha\delta'} \delta_{\beta\gamma'}) (\delta_{\gamma\alpha'} \delta_{\delta\beta'} - \delta_{\gamma\beta'} \delta_{\delta\alpha'}) \} ,
$$

(\alpha, \beta, \gamma, \delta)$

$$
(\alpha\beta, \gamma\delta|H^S|\alpha'\beta', \gamma'\delta') = (E_\alpha + E_\beta + E_\gamma + E_\delta)
$$

$$
\times \{ (\delta_{\alpha\alpha'} \delta_{\beta\beta'} - \delta_{\alpha\beta'} \delta_{\beta\alpha'}) (\delta_{\gamma\gamma'} \delta_{\delta\delta'} - \delta_{\gamma\delta'} \delta_{\delta\gamma'})
$$

$$
+ (\delta_{\alpha\gamma'} \delta_{\beta\delta'} - \delta_{\alpha\delta'} \delta_{\beta\gamma'}) (\delta_{\gamma\alpha'} \delta_{\delta\beta'} - \delta_{\gamma\beta'} \delta_{\delta\alpha'}) \} ,
$$

(\alpha, \beta, \gamma, \delta)$
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\[(\alpha^\beta, \gamma^\delta | H^s_i | 0) = (0 | H^s_i | \alpha^\beta, \gamma^\delta)^* \]
\[= -2\zeta \sum_{\alpha, \beta} q_{\alpha^\beta}^M \bar{q}_{\alpha^\beta}^M (-)^M q_{\gamma^\delta}^M \bar{q}_{\gamma^\delta}^M, \quad (3.1)\]

\[(\alpha^\beta, \gamma^\delta | H^s_i | \alpha'^\beta', \gamma'^\delta') \]
\[= -2\zeta \sum_{\alpha, \beta} q_{\alpha^\beta}^M \bar{q}_{\alpha^\beta}^M (-)^M q_{\gamma^\delta}^M \bar{q}_{\gamma^\delta}^M \left( \delta_{\alpha^\gamma} \delta_{\beta^\delta} - \delta_{\gamma^\alpha} \delta_{\delta^\beta} \right) \]
\[= -2\zeta \sum_{\alpha, \beta} q_{\alpha^\beta}^M \bar{q}_{\alpha^\beta}^M \left( \delta_{\alpha^\gamma} \delta_{\beta^\delta} - \delta_{\gamma^\alpha} \delta_{\delta^\beta} \right) \]
\[= -2\zeta \sum_{\alpha, \beta} q_{\alpha^\beta}^M \bar{q}_{\alpha^\beta}^M \left( \delta_{\alpha^\gamma} \delta_{\beta^\delta} - \delta_{\gamma^\alpha} \delta_{\delta^\beta} \right) \]
\[= -2\zeta \sum_{\alpha, \beta} q_{\alpha^\beta}^M \bar{q}_{\alpha^\beta}^M \left( \delta_{\alpha^\gamma} \delta_{\beta^\delta} - \delta_{\gamma^\alpha} \delta_{\delta^\beta} \right), \]

where \(|0\rangle\) and \(|\alpha^\beta, \gamma^\delta\rangle\) represent the free vacuum and the "two-ideal boson" states \(C_{ss} C_{ss} |0\rangle\), respectively. Here, we must notice the following two facts with respect to the above matrix elements:

(i) In the above matrix elements, there are some elements which are related to the states \(|\alpha^\beta, \gamma^\delta\rangle\) with \(\alpha = \gamma\) or \(\beta = \delta\). Properly speaking, these elements should vanish because the states \(|\alpha^\beta, \gamma^\delta\rangle\) with \(\alpha = \gamma\) or \(\beta = \delta\) must not exist for the reason of the Pauli principle.

(ii) Even if \(\alpha, \beta, \gamma\) and \(\delta\) are different from each other, the matrix elements connected with \(|\alpha^\beta, \gamma^\delta\rangle\), \(|\alpha^\gamma, \beta^\delta\rangle\) and \(|\alpha^\beta, \gamma^\delta\rangle\) have all different values. This fact is also contradictory to the Pauli principle, i.e. to the exchange character. It is clear that the origin is in the correspondence between the pair of quasiparticles and the "ideal boson" (2-5a) and (2-5b). Hereafter, we call the former and the latter in the above-mentioned two facts the blocking effect and the exchange effect, respectively.

Now, let us inquire into these two types of effects in more detail. In order to do this, it is important to inspect the difference between the pair of quasiparticles \(a^\dagger a\beta\) and the "ideal boson" \(C_{ss} C_{ss}\). The solution of this problem has already been worked out in principle, in (I). According to (I), this difference in the "ideal boson space" is given in the first approximation as follows:

\[Ua^\dagger a^\beta U^* - C_{ss}\]
\[= -\frac{1}{2} \left( 1 - \frac{1}{\sqrt{3}} \right) C_{ss} \sum_{\rho, \sigma} C_{\rho}^\dagger C_{\sigma}^\dagger - \frac{1}{\sqrt{3}} \sum_{\rho, \sigma} C_{\rho}^\dagger C_{\sigma}^\dagger + \text{***} \quad (3.2)\]

The above relation can also be rewritten in another form as

**See the footnote on page 341.**

***U is the operator connecting the "fermion space" to the "ideal boson space". The explicit expression for U is given in (I).**
Following the same procedure as in the above case, the operator $a_\gamma^\dagger a_\gamma$ can be written as

$$
U a_\gamma^\dagger a_\gamma U^\dagger = \sum_{\gamma} C_{\gamma 3} C_{\gamma 5}
$$

$$
= \left[ C_{\alpha \beta} - C_{\alpha \beta} \left( \sum_{\nu} C_{\alpha \nu} C_{\alpha \nu} + \sum_{\nu} C_{\beta \nu} C_{\beta \nu} - C_{\alpha \beta} C_{\alpha \beta} \right) \right] C_{\gamma 5} |0\rangle
$$

$$
= C_{\alpha \beta} C_{\gamma 5} |0\rangle \cdot \delta_{\alpha \beta \gamma 5} ,
$$

where

$$
\delta_{\alpha \beta \gamma 5} = 1 - \delta_{\alpha \gamma} - \delta_{\alpha \beta} - \delta_{\beta \gamma} - \delta_{\beta \delta} + \delta_{\gamma \beta} + \delta_{\gamma \delta} + \delta_{\delta \gamma} .
$$

We can see that the state $U a_\alpha^\dagger a_\beta U^\dagger C_{\gamma 5} |0\rangle$ vanishes in the case $\alpha = \gamma$ or $\beta = \delta$. However, we must notice that even if $\alpha$, $\beta$, $\gamma$, and $\delta$ are all different, the three states $U a_\alpha^\dagger a_\beta U^\dagger C_{\gamma 5} |0\rangle$, $U a_\alpha^\dagger a_\gamma U^\dagger C_{\beta \delta} |0\rangle$, and $U a_\delta^\dagger a_\gamma U^\dagger C_{\beta \gamma} |0\rangle$ are independent in the case of only the blocking effect. Therefore, strictly speaking, such states do not correspond to the fermion state. The operation of $U a_\alpha^\dagger a_\beta U^\dagger$, in the case that both effects are included, leads to

$$
U a_\alpha^\dagger a_\beta U^\dagger \cdot C_{\gamma \delta} |0\rangle = - U a_\alpha^\dagger a_\beta U^\dagger \cdot C_{\beta \delta} |0\rangle = U a_\alpha^\dagger a_\beta U^\dagger \cdot C_{\gamma \beta} |0\rangle
$$

$$
= \frac{1}{\sqrt{3}} \left( C_{\alpha \beta} C_{\gamma 5} - C_{\alpha \gamma} C_{\beta 5} + C_{\alpha \delta} C_{\beta 5} \right) |0\rangle ,
$$

which has the exchange character and of course has the blocking effect, so that it corresponds to the fermion state $a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger a_\delta^\dagger |0\rangle$. As was mentioned above,
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the "ideal boson" states with only the blocking effect do not correspond to the fermion states in the strict sense, nevertheless such states have one of the characteristic aspect of the Pauli principle. Therefore, hereafter we restrict ourselves to the discussion of the blocking effect.

Now, let us add the blocking effect to the Hamiltonian $H^8$. Substituting the blocking terms in Eqs. (3.3) and (3.4) into Eq. (2.1a), we can get the following Hamiltonian:

$$H^B = H^B_0 + H^B_\gamma,$$

$$H^B_0 = \frac{1}{2} \sum_{\alpha \beta} (E_{\alpha} + E_{\beta}) \{ C^\alpha_{\beta}C^\beta_{\alpha} - C^\beta_{\alpha}C^\alpha_{\beta} + \sum_{\nu} C^\alpha_{\nu}C^\nu_{\beta} - C^\nu_{\beta}C^\alpha_{\nu} \},$$

$$H^B_\gamma = -\frac{1}{2} \sum_{\alpha \beta \gamma \delta} \sum_{\nu} \frac{N^\alpha_{\nu \beta \gamma \delta}}{2} \{ C^\alpha_{\gamma}C^\gamma_{\alpha} + \sum_{\nu} C^\alpha_{\nu}C^\nu_{\gamma} - C^\gamma_{\nu}C^\nu_{\alpha} \} \times \left\{ \begin{array}{c} c^\alpha_{\gamma}C^\gamma_{\delta} - c^\gamma_{\delta}C^\alpha_{\gamma} + \sum_{\nu} c^\alpha_{\nu}C^\nu_{\delta} - c^\delta_{\nu}C^\nu_{\alpha} \end{array} \right\},$$

$$= s_{\gamma \delta} \{ C^\alpha_{\gamma}C^\gamma_{\alpha} + \sum_{\nu} C^\alpha_{\nu}C^\nu_{\gamma} - C^\gamma_{\nu}C^\nu_{\alpha} \} \times \left\{ \begin{array}{c} c^\alpha_{\gamma}C^\gamma_{\delta} - c^\gamma_{\delta}C^\alpha_{\gamma} + \sum_{\nu} c^\alpha_{\nu}C^\nu_{\delta} - c^\delta_{\nu}C^\nu_{\alpha} \end{array} \right\} + 2 \left\{ c^\alpha_{\gamma}C^\gamma_{\alpha} + \sum_{\nu} c^\alpha_{\nu}C^\nu_{\gamma} - C^\gamma_{\nu}C^\nu_{\alpha} \right\} \times \left\{ \begin{array}{c} c^\alpha_{\gamma}C^\gamma_{\delta} - c^\gamma_{\delta}C^\alpha_{\gamma} + \sum_{\nu} c^\alpha_{\nu}C^\nu_{\delta} - c^\delta_{\nu}C^\nu_{\alpha} \end{array} \right\} + \sum_{\alpha \beta \gamma \delta} \sum_{\nu} \frac{N^\alpha_{\nu \beta \gamma \delta}}{2} \{ C^\alpha_{\gamma}C^\gamma_{\alpha} + \sum_{\nu} C^\alpha_{\nu}C^\nu_{\gamma} - C^\gamma_{\nu}C^\nu_{\alpha} \} \times \left\{ \begin{array}{c} c^\alpha_{\gamma}C^\gamma_{\delta} - c^\gamma_{\delta}C^\alpha_{\gamma} + \sum_{\nu} c^\alpha_{\nu}C^\nu_{\delta} - c^\delta_{\nu}C^\nu_{\alpha} \end{array} \right\}.$$

As were given in the case of $H^8$ the matrix elements of $H^B$ related to the free vacuum and the "two-ideal boson" states are given in the following forms:

$$\langle 0 | H^B_0 | 0 \rangle = \langle 0 | H^8_0 | 0 \rangle,$$

$$\langle \alpha \beta, \gamma \delta | H^B_\gamma | \alpha' \beta', \gamma' \delta' \rangle = \langle \alpha \beta, \gamma \delta | H^8_\gamma | \alpha' \beta', \gamma' \delta' \rangle \cdot \delta_{\alpha \beta \gamma \delta} \delta_{\alpha' \beta' \gamma' \delta'},$$

$$\langle \alpha \beta, \gamma \delta | H^B_\gamma | 0 \rangle = \langle 0 | H^B_\gamma | \alpha \beta, \gamma \delta \rangle = \langle 0 | H^8_\gamma | \alpha \beta, \gamma \delta \rangle \cdot \delta_{\alpha \beta \gamma \delta},$$

$$\langle \alpha \beta, \gamma \delta | H^B_\gamma | \alpha' \beta', \gamma' \delta' \rangle = \langle \alpha \beta, \gamma \delta | H^8_\gamma | \alpha' \beta', \gamma' \delta' \rangle \cdot \delta_{\alpha \beta \gamma \delta} \delta_{\alpha' \beta' \gamma' \delta'},$$

From the factor $\delta_{\alpha \beta \gamma \delta}$ in the above matrix elements, we can easily verify that the blocking effect neglected under the SBA is taken into account in $H^B$ in the first approximation.\(^{9)}\) In particular, it should be emphasized that there are the blocking terms in $H^B_0$. This fact was very far beyond the consideration. Of course, in the EBA proposed by Ikeda et al. and Hara there does not exist the

\(^{9)}\) The meaning of the "first approximation" is as follows. From the above discussion, it is clear that, if $H^B$ is diagonalized in the framework of the free vacuum and the "two-ideal boson" states, the blocking effect is completely taken into account. However, in the case of including the "higher-ideal boson" states, the effect is partially taken into account.
effect corresponding to this effect in the free part of the Hamiltonian.\(^{*)}\) This blocking effect will play an important role in our method developed in the next sub-section.

3-2. The approximate procedure

We are now able to see how the blocking effect acts on the collective oscillations in even nuclei through the diagonalization of \(H^b\). However, in order to clarify the physical meaning of the effect taken into account in the EBA,\(^{**}\) we will develop one of the approximate methods in which the blocking effect is taken up in terms of the number of quasi-particles in the ground state, keeping close contact with the SBA or the EBA.

First, the blocking term in \(a^\dagger_\alpha a_\beta\) can be rewritten “effectively” as

\[
\text{blocking term} = -C^\dagger_{\alpha\beta} \left[ \sum_\nu C^\dagger_{\alpha\nu} C_{\alpha\nu} + \sum_\nu C^\dagger_{\beta\nu} C_{\beta\nu} \right] ,
\]

where the \(\sum_\nu C^\dagger_{\alpha\nu} C_{\alpha\nu}\)'s mean the expectation values of \(\sum_\nu C^\dagger_{\alpha\nu} C_{\alpha\nu}\)'s with respect to the true ground state and we further approximate as follows:

\[
\left\langle \sum_\nu C^\dagger_{\alpha\nu} C_{\alpha\nu} \right\rangle \sim \frac{1}{2j_\alpha + 1} \sum_\nu \left\langle C^\dagger_{\alpha\nu} C_{\alpha\nu} \right\rangle = n_\alpha .
\]

The above approximation and \(\sum_\nu \left\langle C^\dagger_{\alpha\nu} C_{\alpha\nu} \right\rangle\)'s mean the occupation probabilities of the quasi-particles in the ground state introduced by Ikeda et al. and Hara and play important roles in our method in the same way as in the EBA. Using the relations (3.10) and (3.11), the operator \(a^\dagger_\alpha a_\beta\) can be regarded as

\[
a^\dagger_\alpha a_\beta \rightarrow C^\dagger_{\alpha\beta} (1 - n_\alpha - n_\beta) .
\]

Through the same procedure as in the above case, we can get the correspondence of \(a^\dagger_\gamma a_\gamma\) in the following form:

\[
a^\dagger_\gamma a_\gamma \rightarrow \sum_\nu C^\dagger_{\gamma\nu} C_{\gamma\nu} (1 - n_\gamma - n_\nu) .
\]

Here, it must be emphasized that the blocking effect can be expressed in terms

\(^{*)}\) See Eq. (2.11).

\(^{**)}\) The reason why we rewrite the blocking term “effectively” or approximately as the form (3.10) is as follows:

\[
C^\dagger_{\alpha\beta} \left[ \sum_\nu C^\dagger_{\alpha\nu} C_{\alpha\nu} + \sum_\nu C^\dagger_{\beta\nu} C_{\beta\nu} - C^\dagger_{\alpha\beta} C_{\alpha\beta} \right] = C^\dagger_{\alpha\beta} \left[ \sum_\nu C^\dagger_{\alpha\nu} C_{\alpha\nu} + \sum_\nu C^\dagger_{\beta\nu} C_{\beta\nu} - 2C^\dagger_{\alpha\beta} \left\langle C^\dagger_{\alpha\beta} C_{\alpha\beta} \right\rangle + \sum_\nu C^\dagger_{\alpha\nu} C_{\alpha\nu} + \sum_\nu C^\dagger_{\beta\nu} C_{\beta\nu} \right] = C^\dagger_{\alpha\beta} \left[ \sum_\nu C^\dagger_{\alpha\nu} C_{\alpha\nu} + \sum_\nu C^\dagger_{\beta\nu} C_{\beta\nu} \right] ,
\]

where we assumed \(\left\langle C^\dagger_{\alpha\beta} C_{\alpha\beta} \right\rangle = \delta_{\alpha\beta} \left\langle C^\dagger_{\alpha\beta} C_{\alpha\beta} \right\rangle\), etc., and neglected terms such as \(\left\langle C^\dagger_{\alpha\beta} C_{\alpha\beta} \right\rangle\) and \(\left\langle C^\dagger_{\alpha\beta} C_{\alpha\beta} \right\rangle\) for the reason of clarifying the effect in the EBA and developing the method. However, it is by no means self-evident if the neglect of such terms is permissible. (See §5.)
of the number of quasi-particles in the ground state, $n_a$'s, in the framework of the above-mentioned approximate procedure and the effect is contained in $a_\gamma^\dagger a_\gamma$ or $\sum_\gamma C^\dagger_\gamma C_\gamma$ on an equal footing with taking into account in the case of $a_\alpha^\dagger a_\beta$.

Now, let us develop the method to correspond to the SBA or the EBA. With the aids of Eqs. (3·12a) and (3·12b), the Hamiltonian containing the blocking effect in our approximation written as

$$H^T = H_0^T + H^T,$$

$$H_0^T = \sum_{\alpha \beta} (E_\alpha + E_\beta) (1 - n_\alpha - n_\beta) \sum_{M} A^\dagger_{2M}(ab) A_{2M}(ab),$$

$$H^T = -\frac{1}{2} \sum_{\alpha \beta M} q_{\alpha \beta}^2 \overline{\varepsilon}_{ab} (1 - n_\alpha - n_\beta) q_{\alpha \beta}^2 \overline{\varepsilon}_{cd} (1 - n_\alpha - n_\beta)$$

$$\times \sum_{M} \left[ A^\dagger_{2M}(ab) s_{2M} A_{2-M}(cd) + 2 A^\dagger_{1M}(ab) A_{2M}(cd) + s_{2M} A_{2-M}(ab) A_{1M}(cd) \right],$$

(3·13)

where

$$A^\dagger_{2M}(ab) = \frac{1}{\sqrt{2 \sum_{\alpha \beta M}}} \sum_{\alpha \beta} \langle j_\alpha m_\alpha j_\beta m_\beta | 2M \rangle C_{\alpha \beta}^*.$$  
(3·14)

The Hamiltonian (3·13) can formally be diagonalized by the same treatment as in the case of the EBA. The result is as follows:

$$H^T = \sum_{\sigma} \omega_{\sigma} \sum_{M} X^\dagger_{M}(\sigma) X_{M}(\sigma),$$  
(3·15)

where $\omega_{\sigma}$'s are the solutions of the following dispersion equation:

$$1 = 2 \sum_{\alpha \beta} q_{\alpha \beta}^2 \overline{\varepsilon}_{ab} (E_\alpha + E_\beta) (1 - n_\alpha - n_\beta)^2 \omega_{\sigma}^2 = \chi F(\omega_{\sigma})$$  
(3·16a)

and

$$X^\dagger_{M}(\sigma) = \sum_{\alpha \beta} \left[ \psi_{\sigma, ab} A^\dagger_{2M}(ab) - \varphi_{\sigma, ab} s_{2M} A_{2-M}(ab) \right],$$

$$\psi_{\sigma, ab} = \frac{1}{N_\sigma} \frac{q_{ab}^2 \overline{\varepsilon}_{ab} (1 - n_\alpha - n_\beta)}{(E_\alpha + E_\beta) (1 - n_\alpha - n_\beta) - \omega_{\sigma}}, \quad \varphi_{\sigma, ab} = \frac{1}{N_\sigma} \frac{q_{ab}^2 \overline{\varepsilon}_{ab} (1 - n_\alpha - n_\beta)}{(E_\alpha + E_\beta) (1 - n_\alpha - n_\beta) + \omega_{\sigma}},$$

(3·16b)

$$N_{\sigma} = \left[ \sum_{\alpha \beta} \left( \frac{q_{ab}^2 \overline{\varepsilon}_{ab} (1 - n_\alpha - n_\beta)}{(E_\alpha + E_\beta) (1 - n_\alpha - n_\beta) - \omega_{\sigma}} \right)^2 \left( \frac{q_{ab}^2 \overline{\varepsilon}_{ab} (1 - n_\alpha - n_\beta)}{(E_\alpha + E_\beta) (1 - n_\alpha - n_\beta) + \omega_{\sigma}} \right)^2 \right]^{1/2} = N(\omega_{\sigma}).$$

As is well known, the eigenoperator $X^\dagger_{M}(\sigma)$ with the lowest eigenvalue $\omega_{\sigma}$ is

---

* Strictly speaking, the $n_a$'s correspond in the first approximation to the number of quasi-particles in the ground state.
just the creation operator of the "phonon" with angular momentum \( J=2 \) and its projection \( M \). By using the ground state of the Hamiltonian (3.13), we can perform the explicit calculation of \( n_a \):

\[
n_a = \frac{10}{2j_a + 1} \sum_{\sigma} \sum_{\sigma'} \varphi_{\sigma, \sigma'}^2 . \tag{3.18}
\]

The set of Eqs. (3.16) and (3.18) is fundamental in our method. These equations are coupled to each other through \( \omega_{\sigma} \) which may thus be obtained by solving them self-consistently. Such a self-consistent problem, however, is very involved and for the actual numerical calculation a further simplification is necessary. We, also, make the same simplification as was done by Ikeda et al.\(^*\) Then \( n_a \) becomes

\[
n_a = \frac{10}{2j_a + 1} \sum_{\sigma} \varphi_{\sigma, \sigma}^2 = n_a (\omega_{\sigma}) , \tag{3.19}
\]

where \( \omega_{\sigma} \) is the lowest solution of (3.16), i.e. the so-called "phonon" energy.

The \( E_2 \) transition probability from the ground state to the collective state is also given in the following form:

\[
B(E_2; 0^+ \rightarrow 2^+) = 5e^2 \frac{F(\omega)^2}{N(\omega)^2} = 5e^2 B(\omega) . \tag{3.20}
\]

The operator of \( E_2 \) transition in our case is given in the form

\[
\mathcal{M}(2E, M) = e \sum_{ab} q_{ab} \frac{\varphi_{\sigma, \sigma}}{\sigma} (1 - n_a - n_b) [A^+_a M(ab) + s_{2M} A_{a-M} (ab)] . \tag{3.21}
\]

This is just our formalism, in which the blocking effect is taken into account in terms of the number of quasi-particles in the ground state.

To conclude this section, it must be noted that if all the values of \( n_a \) are set equal to zero, the dispersion relation and the other quantities in our method are reduced to those derived under the SBA, and it might be necessary to assume that \( n_a \)'s are sufficiently small compared with unity, because the blocking effect in the Hamiltonian (3.10) is taken into account in the first approximation. Further, we must bear in mind that the Hamiltonian (3.10) does not contain the exchange effect and the dynamical effect, the types of the interactions neglected under the SBA. Of course, in the EBA the dispersion relation and the other quantities are also reduced to those derived under the SBA.

\(^*\) \( \varphi_{\sigma, \sigma} \)'s are in general expected to be small except for the one corresponding to the state with the lowest eigenvalue \( \omega_{\sigma} \) and therefore it is a good approximation to neglect these small contributions.
§ 4. Comparison with the extended boson approximation and discussion

4-1. The general features of the method and comparison with the extended boson approximation

It has been shown in the preceding section that the blocking effect can be expressed “effectively” in terms of the number of quasi-particles in the ground state and that it is possible to construct the method, in a form very analogous to the SBA, in which the effect of the number of quasi-particles in the ground state is self-consistently taken into account. This section will be devoted to a discussion of the general features of our method, in comparison with the SBA and the EBA. First, let us start with comparing Eq. (3.12a) with Eq. (2.10a). We can understand that in the EBA the blocking effect is “effectively” taken into account although the correspondence between the pair of quasi-particles and the “ideal boson” are slightly different in the two cases. The blocking effect always reduces the matrix element \( g_{\alpha \beta} \) by a factor \((1-n_a-n_b)\) as is clear from our basic Hamiltonian \( H' \). In other words, the blocking effect “effectively” reduces the strength of the quadrupole interaction. This is in the same situation as in the case of the EBA, though the factor of the reduction is slightly different from that in the EBA. However, the essential difference between our case and the EBA is in the correspondences (3.12b) and (2.10b).

In the EBA the blocking effect on \( \hat{a}_r \hat{a}_r \) or \( \sum C_{\gamma \delta} C_{\gamma \delta} \) is not taken into account; on the other hand, in our case the effect is contained in \( \hat{a}_r \hat{a}_r \) or \( \sum C_{\gamma \delta} C_{\gamma \delta} \) on an equal footing with taking into account in the case of \( \hat{a}_r \hat{a}_r \).

This fact reflects on the free part of the Hamiltonian \( H' \) or \( H'' \). In our case, the energy of the pair of quasi-particles \( (E_a + E_b) \) is reduced by the factor \((1-n_a-n_b)\). This reduction is of very interest and it can be physically understood in the following manner, together with the reduction of the interaction part of the Hamiltonian, \( H'' \). If any pair state \((a, b)\) is partially occupied in the ground state with the probability \((n_a+n_b)\), this state will be available for the excited states of the system only with the probability \((1-n_a-n_b)\) due to the blocking effect. Therefore, any matrix element will be reduced by the factor \((1-n_a-n_b)\).

It naturally follows that \( H'' \) and \( H' \) have the reduction factor \((1-n_a-n_b)\), respectively, each playing the role of reducing “effectively” the energy of the pair of quasi-particles and the strength of quadrupole interaction. It is also noticeable that the above two reduction effects play roles opposite to each other with respect to lowering the collective levels, i.e., the former has an action to lower the levels due to the reduction of the given energy of the pair of quasi-particles and the latter to raise the levels due to the reduction of the given strength of the interaction. Figures 1 and 2 represent the calculated values of \( F(\omega)^{\sigma_0} \)

\[\left(\omega_{\sigma_0}\right)\]
in Eq. (3.16a) and the average value $n_a$ in Eq. (3.19) as a function of $\omega$ for the nucleus Sn$^{116}$, respectively, together with the results based on the SBA and the EBA. The characteristic feature in the EBA is that the value of the lowest solution, $\omega$, of the dispersion relation is always larger than the corresponding one in the SBA if in both cases the same values of $\chi$ are assumed. This has been explained by the reduction factor in the interaction part of the Hamiltonian. On the other hand, according to our dispersion relation, if $\chi$ is very small, the value of the lowest solution, $\omega$, is smaller than that in the SBA, of course, the EBA, and as $\chi$ becomes larger, the value of $\omega$ in our method is larger than the corresponding one in the SBA and smaller than the one in the EBA; we must take an intermediate value of $\chi$ if we want in our method to get the same energy (for example, the experimental excitation energy) as those obtained in the other methods. (See Fig. 1.) This can be interpreted from the reduction effects both on the energy of the pair of quasi-particles and on the strength of the quadrupole interaction as follows. As mentioned above, these two reduction effects have the opposite actions with regard to lowering the levels. Therefore the action of the former is stronger than the latter when $\chi$ is very small, and our dispersion equation is placed above that of the SBA and vice versa when $\chi$ is large, although ours is never placed below that of the EBA because of the former effect. Next, we consider the occupation probabilities, $n_a$'s, in the ground state. We first notice the following fact: $n_a$'s in the EBA and in our method have the same dependence on the force strength $\chi$ and they are the monotone
increasing functions with respect to $\chi$. Therefore, as is clear from the discussion of the dependence of the dispersion relation on the force strength $\chi$, there is no necessity for taking the values of $n_a$'s in our case as large as those in the EBA, if we want to get the same excitation energy in our method as that in the EBA. We can also see from Fig. 2 that the lower the excitation energy $\omega$ is the more remarkable such a tendency becomes. Of course, this situation originates from the fact that in our method the energy of the pair of quasi-particles is initially reduced owing to the blocking effect, and this fact tells us that our ground state correlation is not so large as that in the EBA. Moreover, the following facts are interesting for the understanding of the structure of the theory. There exists $\chi_{\text{crit}}$ in the SBA which makes $\omega$ equal to zero and $\omega_{\text{crit}}$ in the EBA which corresponds to the infinitely large value of $\chi (=\infty)$. At such a point in the EBA, all the $n_a$'s are equal to 0.5. On the other hand, in our case, $\omega=0$ corresponds to the infinitely large value of $\chi$ and at this point all the $n_a$'s take the values equal to 0.5. However, we must keep in mind that the SBA, the EBA or our method is not applicable in the region where $\omega$ approaches to zero. Because the blocking effect is appreciable in such a region and this fact is contradictory to the original assumption of the method.

Now, we consider the blocking effect, i.e. the reduction effect on the $E2$ transition probability from the ground state to the collective state. This quantity essentially depends upon the two kinds of quantities, i.e. one is the reduced matrix elements of single particle quadrupole moments $q_{ab}\tilde{r}_{ab}(1-n_a-n_b)$'s (in our method) or $q_{ab}\tilde{r}_{ab}\sqrt{1-n_a-n_b}$'s (in the EBA) and the other is the forward and backward amplitudes. In the case of the EBA, the forward and backward amplitudes, $\phi_{ab}(\omega)$ and $\phi_{ab}(\omega')$, take the comparable values as those in the SBA when the excitation energy is taken as the same value in the two cases.

\textsuperscript{3)} It is impossible to give the general proof of this fact. We arrived at this conclusion as a result of the proof in the case of single $j$ shell model and actual numerical calculations in the several cases.

\textsuperscript{4)***} The blocking effect in the Hamiltonian $H^T$ to be diagonalized in our method is taken into account in the first approximation. (See the footnote on page 341.)

\textsuperscript{4)***} These reasons can be made clear from the following relations:

\begin{align*}
q_{ab}\tilde{r}_{ab}\sqrt{1-n_a-n_b} &\sim \sqrt{1-N}\cdot q_{ab}\tilde{r}_{ab} \\
q_{ab}\tilde{r}_{ab} &\sim q_{ab}\tilde{r}_{ab} \\
(E_a+E_0)\frac{(1-n_a-n_b)}{(1-n_a-n_b)+\omega} &\sim (E_a+E_0)\frac{1-n_a-n_b}{1-n_a-n_b+\omega} \\
N(\omega) &\sim N(\omega)(1-N),
\end{align*}

where $N$ denotes some average value of $(n_a+n_b)$'s. By using the above relations, we can get $\phi_{ab}\phi'(\omega)$ or $\phi_{ab}\phi'(\omega)\sim \phi_{ab}\phi'(\omega)$ or $\phi_{ab}\phi'(\omega)$ and

\begin{align*}
\phi_{ab}(\omega) &\sim \phi_{ab}(\omega)(1/N) \\
\phi_{ab}(\omega) &\sim \phi_{ab}(\omega)(1/N).
\end{align*}
Therefore the main effect on the $E2$ transition is based on the reduction due to the factor $q_{ab} \xi_{ab} \sqrt{1-n_a-n_b}$ in $\mathcal{M}^{E2}(E2, M)$ (Eq. (2.17)). In our case, on the other hand, $\psi_{ab}(\omega)$ or $\varphi_{ab}(\omega)$ is comparable to $\psi_{ab}^{a}(\omega/(1-N))$ or $\varphi_{ab}^{a}(\omega/1/(1-N))$ and so these amplitudes are smaller than $\psi_{ab}^{a}(\omega)$ and $\varphi_{ab}^{a}(\omega)$, respectively. Therefore, the $E2$ transition is reduced by not only the factor $q_{ab} \xi_{ab} (1-n_a-n_b)$ in $\mathcal{M}^{E2}(E2, M)$ (Eq. (3.21)) but also the amplitudes $\psi_{ab}(\omega)$ and $\varphi_{ab}(\omega)$. It is, of course, clear that this is also due to the reduction of the energy of the pair of quasi-particles. This double reduction is the reason why we have obtained the smaller value of $B(E2; 0^+ \rightarrow 2^+)$ than that in the EBA in the energy region which we are interested in. (See Fig. 3.) As was mentioned in §3, the values of the $n_a$'s are not so large and thus the experimental large $E2$ transition probability can still be well explained in our method.

4-2. Numerical results for Te, Xe and Ba isotopes

Lastly, we will show the isotope dependence of the calculated results based on our method. For this purpose, we carried out numerical calculations of some quantities for Te, Xe and Ba isotopes. In order to compare our results with the calculations of Tamura and Udagawa based on the SBA or Ikeda, Udagawa and Yamamura based on the EBA, additional approximations are made as were done by them. i) We treat the correlation between proton and neutron in the same way as that of Ikeda, et al. ii) In calculating the matrix elements of the quadrupole interaction, the harmonic oscillator wave function is used. iii) We employ the values of some parameters, i.e. single particle energy $\varepsilon_a$, the energy gap $\Delta$ and chemical potential $\lambda$, tabulated in the Appendix of the paper by Tamura and Udagawa.

We first present in Fig. 4 the force strengths for the cases of Te, Xe and

---

*\footnote{In the region where $\omega$ is smaller, as is shown in Fig. 3, $B^{E2}(\omega)$ is smaller than $B(\omega)$. It is due to the factor $q_{ab} \xi_{ab} \sqrt{1-n_a-n_b}$ in $\mathcal{M}^{E2}(E2, M)$, namely at the point $\omega_{crit}(=0)$ all the $n_a$'s are equal to 0.5 and the factor $q_{ab} \xi_{ab} \sqrt{1-n_a-n_b}$ vanishes.}

**\footnote{In the above section, the formulation was made in the case of the single closed shell nuclei. So a little modification of the formalism is necessary in the case of non single closed shell nuclei. However, the essential point is unchanged.}
Ba isotopes which correspond to the experimental values of $\omega$, together with the results of the SBA and the experimental value of $\omega(\omega_{exp})$. In these cases, the employed values of the force strength parameter $\tilde{\chi}$ are defined as follows:

$$\tilde{\chi} = \frac{5}{4\pi} \left( \frac{\hbar}{M\omega_0} \right)^2 A \chi^*$$

(4.1)

We can see from Fig. 4 that the force strengths of these nuclei take somewhat constant values ($\tilde{\chi} \sim 3.9$), contrary to the large fluctuation in the case of the SBA. In particular, the trend is found in the cases of very low excitation energy. Therefore it can be expected that we can fit the numerical results to the experimental values much better than those in the case of the SBA, starting with some constant value of $\tilde{\chi}$.

![Fig. 4. $\tilde{\chi}$ which corresponds to the experimental value of $\omega$ as a function of the neutron number $N$ for Te, Xe and Ba isotopes. In this figure, the results of the SBA and the experimental value of $\omega(=\omega_{exp})$ are given for comparison.](image)

The calculated results of $B(E2; 0^+ \rightarrow 2^+)$ for Te, Xe and Ba isotopes are shown in Table I, together with the results in other methods. These were obtained by using the experimental value of $\omega$. For the effective charge we employed the values, $e_p = 2$ and $e_N = 1$. The calculated values are smaller than $\omega_0$ in this case is the frequency of the harmonic oscillator used for the calculation of the matrix elements of quadrupole interaction and $A$ is mass number of nucleus.
those in the other methods as was mentioned in the above sub-section and have not so sharp dependence on the isotopes as those in the other methods have. Figure 5 shows the effective charge \( e_{\text{eff}} \) derived from the experimental values in the cases of our method and the SBA. These features show that the fit to the experimental data is generally good. Therefore we can also expect that the explanation of the experimental \( B(E2; 0^+ \rightarrow 2^+) \) is possible to some degree.

Table I. The reduced transition probability \( B(E2; 0^+ \rightarrow 2^+) \) in unit of the single particle value \( B(E2)_{\text{s.p.}} = 3.0 \times 10^{-1} A^{4/3} \times 10^{-49} e^2 \text{cm}^4 \) for the case of Te, Xe and Ba isotopes. Column two gives the values obtained from the present theory while column three or four the ones obtained from the SBA or the EBA respectively. Column five gives the experimental values. The values based on the SBA and the EBA and the experimental values are all the same as those listed in Table II in the paper of Ikeda et al.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>( B(E2) ) theor.</th>
<th>( B(E2) ) SBA</th>
<th>( B(E2) ) EBA</th>
<th>( B(E2) ) exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{52}\text{Te}^{130} )</td>
<td>27.6</td>
<td>54.5</td>
<td>41.5</td>
<td>29</td>
</tr>
<tr>
<td>( ^{52}\text{Te}^{122} )</td>
<td>27.1</td>
<td>50.5</td>
<td>39.3</td>
<td>35</td>
</tr>
<tr>
<td>( ^{52}\text{Te}^{124} )</td>
<td>25.0</td>
<td>42.1</td>
<td>34.5</td>
<td>21</td>
</tr>
<tr>
<td>( ^{52}\text{Te}^{126} )</td>
<td>21.8</td>
<td>33.4</td>
<td>28.8</td>
<td>28</td>
</tr>
<tr>
<td>( ^{52}\text{Te}^{128} )</td>
<td>18.0</td>
<td>24.4</td>
<td>22.2</td>
<td>22</td>
</tr>
<tr>
<td>( ^{52}\text{Te}^{130} )</td>
<td>12.1</td>
<td>14.9</td>
<td>14.7</td>
<td>18</td>
</tr>
<tr>
<td>( ^{54}\text{Xe}^{128} )</td>
<td>35.2</td>
<td>87.0</td>
<td>56.9</td>
<td></td>
</tr>
<tr>
<td>( ^{54}\text{Xe}^{130} )</td>
<td>32.9</td>
<td>66.2</td>
<td>49.3</td>
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</tr>
<tr>
<td>( ^{54}\text{Xe}^{132} )</td>
<td>28.4</td>
<td>48.5</td>
<td>39.8</td>
<td>24</td>
</tr>
<tr>
<td>( ^{54}\text{Xe}^{134} )</td>
<td>22.6</td>
<td>31.8</td>
<td>28.4</td>
<td>16</td>
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<tr>
<td>( ^{56}\text{Ba}^{130} )</td>
<td>15.9</td>
<td>19.4</td>
<td>18.2</td>
<td></td>
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<tr>
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<td>98.8</td>
<td>65.2</td>
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<td>21.0</td>
<td>25.6</td>
<td>23.9</td>
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</tr>
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</table>

Fig. 5. The values of the effective charge \( e_{\text{eff}} \), which are needed to give exact agreement of the theoretical \( B(E2; 0^+ \rightarrow 2^+) \) with experiments for Te, Xe and Ba isotopes.

\(^{4)} \) In this case, the effective charge \( e_{\text{eff}} \) is defined as \( e = e + e_{\text{eff}} \) and \( e_N = e_{\text{eff}} \).
§ 5. Concluding remarks

For the purpose of analysing the effect of the Pauli principle on the collective oscillations in spherical even nuclei and clarifying the physical meaning of the effect introduced by Ikeda, Udagawa and Yamaura and independently by Hara, we have developed the method which enables us to take into account one of the effect of the Pauli principle, the blocking effect, "effectively" in terms of the number of quasi-particles in the ground state in the framework of the "ideal boson" picture. Taking account of this effect, agreement with the experimental data is much improved over the earlier results based on the conventional random phase approximation. Our method is superior to the method developed by Ikeda, et al. and Hara, for the physical meaning of the effect adopted there is clear and the effect is taken into account consistently in our case. Moreover, if we further want to take into account the other effects, it seems for us to be difficult to extend Ikeda et al. and Hara's method in the framework of their point of view, on the other hand, in our case the extension is much easier.

In order to get a more detailed fit to the experimental data, it seems for us to be necessary to perform further calculations by more refined treatment on the blocking effect, as well as by taking into account the other effects, the exchange effect and the dynamical effects, although the calculations with the refined values of the single particle energy and the energy gap parameter are also necessary as was mentioned in paper of Ikeda et al. A clear evidence for this statement also seems to have been given by the latest calculations by Tokunaga, Marumori and one of the present authors (M. Y.) based on a kind of perturbation theory starting with the solutions of the random phase approximation as the zeroth-order approximation. Such calculations are now being made by the present authors.

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