Composite Models of Hadrons and

High Energy Photoproduction of Vector Mesons

Katsusada MORITA

Department of Physics, Kyoto University, Kyoto

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The possibility that high energy photoproduction of neutral vector mesons can be used to test the validity of composite models is discussed. Branching ratios for photoproduction of neutral vector mesons are calculated in terms of diffraction models from the view that hadrons are composite particles in the sense that they consist of some more fundamental objects. We found, as far as our investigations are concerned, that the quark and the three-triplet models are better candidates than others, but it is impossible to distinguish these models.

Leptonic decay modes of neutral vector mesons are discussed along the same lines.

§ 1. Introduction

Since the remarkable successes of $SU(3)$ symmetry several composite models of strongly interacting particles (hadrons) have been proposed in which the fundamental triplet or triplets (including singlet) are assumed to exist in nature and to create all the known hadrons by means of some yet unknown dynamical binding mechanisms. Such a fundamental triplet theory, we recall, involves one more assumption, that these triplets are represented by fields in a Lagrangian field theory. Even if some dynamical binding mechanisms were given as input information, one does not know how to describe the hadrons and their mutual interactions in terms of these basic fields. Moreover, what are the binding mechanisms? These are the extremely difficult problems, so we do not touch upon them here. Rather, we shall always assume that all the known hadrons are composite particles in the sense that they consist of some basic objects.

Then one may ask if, within the framework of the conventional field theory and using the existing experimental data, one is able to distinguish the so far proposed composite models without any specific dynamical mechanism and select one model out of others as the best candidate. Okubo has recently attacked this problem using the equal-time commutation relations of the current densities and concluded from the study of $\pi^0 \rightarrow 2\gamma$ decay and the $g_{\pi}/g_\gamma$ ratio that the quark model is the best candidate among others. On the other hand Ram has tried to apply the assumption of additivity of the basic two-body amplitudes to derive the hadron-hadron total cross section relations assuming hadrons to be composites.
of integral charge particles and to compare the results with those obtained assuming a quark structure.

In this paper we discuss the possibility that processes of the type

\[ \gamma + p \rightarrow V^0 + p, \]  

where \( V^0 \) is a neutral vector meson (\( \rho^0, \omega \) or \( \varphi \)), can be used to test the validity of composite models provided that diffraction mechanism is the dominant feature in reactions (A). This assumption is implied by the experimental fact\(^{4,5}\) that the differential cross section for photoproduction of \( \rho^0 \) (and perhaps \( \omega \)) meson (s) has a strong forward peak at high energies (above about 3 GeV). Two diffraction models will be considered: a photon dissociation model\(^{6,7}\) and a multiperipheral one.\(^8\) The former contains a matrix element of the electromagnetic current between the vacuum and a single neutral vector meson and the latter comprises that between two octets (one represents pseudoscalar meson and the other vector meson) and between octet and singlet (the electromagnetic interaction is treated in the lowest order in \( e \)). Therefore it is possible to use reactions (A) for detecting the unitary singlet component in the electromagnetic current.\(^1\)

After recapitulating briefly the essential features of the composite models so far proposed in \$2\), we discuss branching ratios for photoproduction of neutral vector mesons in terms of a photon dissociation model in \$3\) by using the additivity assumption\(^3\) in order to connect total cross sections for vector meson-nucleon scattering with those for pseudoscalar meson-nucleon scattering. A multiperipheral model will be treated in \$4\). Section 5 will be devoted to a discussion of the leptonic decays of neutral vector mesons. We summarize our results in the final section. In Appendix A the so-called rho-photon analogy\(^{22,36}\) together with its extension to the isoscalar part of the photon will be shown to lead to photon dissociation mechanism when applied to photon-induced reactions at high energies. Expressions for the electromagnetic current in terms of the basic fields will be given in Appendix B for each composite model.

\$2. Composite models of hadrons

Let us start with a brief review of composite models of hadrons in which the basic triplet field or fields (including singlet field) are assumed to exist. From group-theoretical point of view, the simplest model which can be used to construct hadrons is the following.

2-1. Quark model (hereafter referred to as the G-Z model)\(^9\)

One assumes in this model the existence of an \( SU(3) \) triplet \( q(q_1, q_2, q_3) \), called quarks, \( q_1 \) and \( q_2 \) forming an isodoublet and \( q_3 \) having no isospin. Meson and baryon states are supposed to be constructed as \( |qq\rangle \) and \( |qqq\rangle \), respectively. Despite having many attractive features, the quark model is not entirely satisfactory from a realistic point of view because a) the electric charges are not
integers, b) three quarks in the $s$-state do not form the symmetric representation of $SU(6)$ and c) the dynamical mechanism fails to realize only the zero-triality states as low-lying levels. One can consider models in which basic particles have integral charges by requiring more than just $SU(3)$ triplet as building blocks of hadrons.

2-2. Quartet model (hereafter referred to as M–H model)

This was suggested by Maki and Hara as a generalization of the Sakata model. Instead of taking $p, n, A$ and their antiparticles as fundamental particles, they assumed the existence of unitary triplet $t(t_1, t_2, t_3)$ with hypercharge $(1, 1, 0)$ and an unitary singlet $\chi_0$ with hypercharge 0. The baryons and mesons are considered to be built as $|t\bar{t}\chi_0\rangle$ and $|t\bar{t}\rangle$, respectively. There are two possible assignments of charge quantum number: One is the Sakata-type $t(1, 0, 0)$ and the other the Lepton-type $t(0, -1, -1)$; in both cases $\chi_0$ is neutral. For definiteness, we shall discuss only the Sakata-type charge assignment in this paper. The $SU(6)$ theory cannot be applied to baryon states because the baryon decuplet must have the structure $|t\bar{t}\bar{t}\chi_0\rangle$ in this model.

2-3. Tanaka model

Tanaka proposed a model in which the baryon was considered as consisting of three unitary triplet, hypothetical heavy leptons $t_1, t_2, t_3$ and a unitary singlet boson $B$ with baryon number +1. According to his scheme, the singlet boson $B$ can have the subhadronic charge (which was introduced according to a $U(3)$ description of the model) of +3, and each triplet heavy lepton correspondingly has subhadronic charge of −1. The baryon and boson are realized through the combination $|tttB\rangle$ and $|tt\rangle$, respectively, as the states of subhadronic charge = 0. Although a static treatment of the singlet boson $B$ inside a baryon leads to the approximate $SU(6)$ symmetry (this is a situation attractive in several respects), it is unlikely to expect that the simple additivity assumption holds for the baryon with a core. Therefore we do not discuss further this model in this paper.

2-4. Two-mixed-triplet model (hereafter referred to as the G–L–N–S model)

This was introduced by Gürsey, Lee and Nauenberg and independently by Schwinger. They considered two sets of triplets, one obeying Bose statistics with spin zero and one Fermi statistics of spin $\frac{1}{2}$. We shall call them $\alpha(\alpha_1, \alpha_2, \alpha_3)$ and $\beta(\beta_1, \beta_2, \beta_3)$, respectively. One way of constructing hadrons is to take the meson as the bound state of $\beta$ and $\bar{\beta}$ and the baryon as that of $\bar{\alpha}$ and $\beta$. This version of two-mixed-triplet model is equivalent to the quartet model in the sense that the former can be obtained from the latter by the transformation $\beta = \chi_0 t, \alpha = t$. There is, however, an essential difference between this model and others in connection with the composition of baryons—two-body in this model and three-body in others. Thus the well-known Levin-Frankfurt ratio $\sigma_{pp}/\sigma_{pp} = \frac{3}{4}$ cannot be derived from this model. Furthermore the presence of the boson contribution in the vector current would modify the beautiful
Adler-Weissberger relation so as to make the agreement with experiment less accurate. Nevertheless we will discuss the consequences of this model on the processes mentioned in the Introduction.

2-5. Two-triplet model (hereafter referred to as the B-N-H model)

This was introduced and discussed extensively by Bacry, Nuyts and Van Hove. Two fermion triplets \( T(T^+, T^0, T^{+}) \) and \( \Theta(\Theta^+, \Theta^0, \Theta^{+}) \), both carrying baryon charge one, are introduced to construct hadrons: meson \( =|TT\rangle \) or \( |\Theta\Theta\rangle \) and baryon \( =|TT\Theta\rangle \). One additional quantum number \( D \), called supercharge, is introduced in addition to the usual ones, \( Q \) and \( Y \), in such a way that, while the Nishijima-Gell-Mann relation is generalized in the form of
\[
Q = q_3 + \frac{1}{3} Y + D,
\]
all the known hadrons have \( D=0 \). This new quantum number as well as \( SU(3) \) are incorporated in the \( Sp(6) \) group, the symplectic group in six dimensions. Pseudoscalar meson octet and vector nonet are put in the representations \( (14, 8) \) and \( (21, 1+8) \), respectively, where the first number in the bracket indicates the dimension of the \( Sp(6) \) representation and the second one that of the \( SU(3) \) representation. Baryon octet and decuplet are assigned to \( (64, 8) \) and \( (56, 10) \), respectively. Beyond these known hadrons, the \( Sp(6) \) scheme predicts many new mesons and baryon resonances with \( D\neq 0 \).

2-6. Three-triplet model (hereafter referred to as the H-N model)

Table I. Properties of composite models. \( Q= \) charge, \( I= \) isospin, \( Y= \) hypercharge, \( N= \) baryon no., \( F= \) Fermi statistics, \( B= \) Bose statistics, \( B_8= \) baryon octet, \( P_{8(1)}= \) Pseudoscalar meson octet (singlet), \( V_{s(1)}= \) vector meson octet (singlet), \( \varphi_8= I=0 \) member of \( V_8 \), and \( \varphi_1= V_1 \).

<table>
<thead>
<tr>
<th>Symmetry</th>
<th>(A) Quark model ( SU(3) )</th>
<th>(B) Maki-Hara model ( U(1)\otimes U(3) )</th>
<th>(C) Two-mixed triplet model ( U(3)\otimes U(3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle symbol</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Q/e )</td>
<td>( q(q_1, q_2, q_3) )</td>
<td>( t(t_1, t_2, t_3) ) ( x_0 )</td>
<td>( a(a_1, a_2, a_3) ) ( \beta(\beta_1, \beta_2, \beta_3) )</td>
</tr>
<tr>
<td>( I )</td>
<td>( (2/3, -1/3, -1/3) )</td>
<td>( (1, 0, 0) )</td>
<td>( (1, 0, 0) )</td>
</tr>
<tr>
<td>( I_3 )</td>
<td>( (1, 1/2, 0) )</td>
<td>( (1/2, 1/2, 0) )</td>
<td>( (1/2, 1/2, 0) )</td>
</tr>
<tr>
<td>( Y )</td>
<td>( (1/3, 1/3, -2/3) )</td>
<td>( (1, 1, 0) )</td>
<td>( (1, 1, 0) )</td>
</tr>
<tr>
<td>( N )</td>
<td>( (1/3, 1/3, 1/3) )</td>
<td>( (1, 1, 1) )</td>
<td>1</td>
</tr>
<tr>
<td>Statistics</td>
<td></td>
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</tr>
<tr>
<td>( B_8 )</td>
<td>( (qqq)_8 )</td>
<td>( (t\bar{t}x_0)_8 )</td>
<td>( (\beta\bar{a})_8 )</td>
</tr>
<tr>
<td>( P_{8(1)}; \ V_{8(1)} )</td>
<td>( (q\bar{q})_{8(1)} )</td>
<td>( (t\bar{t})_{8(1)} )</td>
<td>( (\beta\bar{a})_{8(1)} )</td>
</tr>
<tr>
<td>(</td>
<td>\rho\rangle )</td>
<td>(</td>
<td>q\bar{q}q\bar{q}\rangle )</td>
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<tr>
<td>(</td>
<td>n\rangle )</td>
<td>(</td>
<td>q\bar{q}q\bar{q}\rangle )</td>
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<td>(</td>
<td>\pi\rangle )</td>
<td>(</td>
<td>q\bar{q}q\bar{q}\rangle )</td>
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<td>(</td>
<td>\Lambda\rangle )</td>
<td>(</td>
<td>q\bar{q}q\bar{q}\rangle )</td>
</tr>
<tr>
<td>(</td>
<td>\rho^0\rangle )</td>
<td>( (1/\sqrt{2})</td>
<td>q\bar{q}</td>
</tr>
<tr>
<td>(</td>
<td>\omega\rangle )</td>
<td>( (1/\sqrt{6})</td>
<td>q_1q_2 - q_2q_3 + 2q_3q_1</td>
</tr>
<tr>
<td>(</td>
<td>\omega_1\rangle )</td>
<td>( (1/\sqrt{3})</td>
<td>q_1q_2 + q_2q_3 + q_3q_1</td>
</tr>
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</table>
Composite Models of Hadrons and High Energy Photoproduction

### Table I

<table>
<thead>
<tr>
<th>Particle symbol</th>
<th>(D) Two-triplet model ( S_\phi(6) )</th>
<th>(E) Three-triplet model ( SU(3) \otimes SU(3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q/\pi )</td>
<td>((T^+T^0_T^0)^{\bar{\Theta}}\Theta^0, \Theta^+)</td>
<td>((t_I \xi_i, \xi_i\xi_i)\Theta^0, \Theta^+)</td>
</tr>
<tr>
<td>( I )</td>
<td>((\frac{1}{3}, \frac{1}{3}, 0)) ((\frac{1}{3}, \frac{1}{3}, 0))</td>
<td>((\frac{1}{3}, \frac{1}{3}, 0)) ((\frac{1}{3}, \frac{1}{3}, 0))</td>
</tr>
<tr>
<td>( I_8 )</td>
<td>((-\frac{1}{3}, \frac{1}{3}, 0)) ((-\frac{1}{3}, \frac{1}{3}, 0))</td>
<td>((-\frac{1}{3}, \frac{1}{3}, 0)) ((-\frac{1}{3}, \frac{1}{3}, 0))</td>
</tr>
<tr>
<td>( Y_3 )</td>
<td>((1/3, 1/3, -2/3)) ((-1/3, -1/3, 2/3))</td>
<td>((1, 1, 0)) ((0, -1))</td>
</tr>
<tr>
<td>( N )</td>
<td>((1, 1, 1)) ((1, 1, 1))</td>
<td>((1/3, 1/3, 1/3)) ((1/3, 1/3, 1/3))</td>
</tr>
</tbody>
</table>

**Statistics**

<table>
<thead>
<tr>
<th>( B_{8} )</th>
<th>( P_{8,1} ; V_{8,1} )</th>
<th>((TT\bar{\Theta})_{8_1})</th>
<th>((TT-\bar{\Theta})_{8_1})</th>
<th>((tt_I V_U t_V)_{a,1})</th>
</tr>
</thead>
</table>
| \( |p\rangle \) | \((1/2)|T^+T^0_T^0\Theta^0+T^0T^+\bar{\Theta}^0\Theta^0\rangle\) | \((1/\sqrt{6})\xi_i\xi_i\xi_i+\xi_i\xi_i\xi_i+\xi_i\xi_i\xi_i\) | (a) For the construction of baryon ground states, see reference 15).
| \( |n\rangle \) | \((1/2)|T^+\bar{T}^0_T^0\Theta^0-\bar{\Theta}^0\Theta^0\bar{T}^0T^+\rangle\) | \((1/\sqrt{6})\xi_i\xi_i\xi_i+\xi_i\xi_i\xi_i+\xi_i\xi_i\xi_i\) |
| \( |\pi^+\rangle \) | \((1/\sqrt{2})|\Theta^0\Theta^0+T^0T^+\rangle\) | \((1/\sqrt{3})\xi_i\xi_i\xi_i+\xi_i\xi_i\xi_i+\xi_i\xi_i\xi_i\) |
| \( |K^+\rangle \) | \((1/\sqrt{2})|\Theta^0\Theta^0+T^0T^+\rangle\) | \((1/\sqrt{3})\xi_i\xi_i\xi_i+\xi_i\xi_i\xi_i+\xi_i\xi_i\xi_i\) |
| \( |\rho^0\rangle \) | \((1/2)|\Theta^0\Theta^0+T^0T^0-T^0T^+\rangle\) | \((1/\sqrt{6})\xi_i\xi_i\xi_i+\xi_i\xi_i\xi_i+\xi_i\xi_i\xi_i\) |
| \( |\phi^0\rangle \) | \((1/\sqrt{2})|\Theta^0\Theta^0+T^0T^0-T^0T^+\rangle\) | \((1/\sqrt{6})\xi_i\xi_i\xi_i+\xi_i\xi_i\xi_i+\xi_i\xi_i\xi_i\) |
| \( |\omega^0\rangle \) | \((1/\sqrt{6})|\Theta^0\Theta^0+T^0T^0-T^0T^+\rangle\) | \((1/3)\xi_i\xi_i\xi_i+\xi_i\xi_i\xi_i+\xi_i\xi_i\xi_i+\xi_i\xi_i\xi_i\) |

Han and Nambu\(^{18}\) proposed a model for the low-lying baryons and mesons based on the three triplets with integral charges, \( t_I (\xi_i, \xi_i, \xi_i) \), \( t_U (\xi_i, \xi_i, \xi_i) \) and \( t_V (\xi_i, \xi_i, \xi_i) \). The charm number \( C \) is introduced as an additional quantum number: \( C = 1 \) for \( t_I \) and \( t_U \), \( C = -2 \) for \( t_V \), and \( C = 0 \) for all the hadrons composed of \( t_I \), \( t_U \) and \( t_V \). They then proposed a double \( SU(3) \) symmetry in which the large mass splittings between different representations are ascribed to one of the \( SU(3) \), while the other \( SU(3) \) is responsible for the mass splittings within a representation of the first \( SU(3) \). According to this scheme, the familiar baryon octet and decuplet and meson multiplets are realized as the states belonging to the singlet representation of the second \( SU(3) \). Thus the \( SU(6) \) symmetry can be easily realized with \( s \)-state triplets. Other aspects of the construction of the low-lying hadrons, however, are the same as those of the quark model.

For convenience, we summarized the relevant properties of composite models in Table I.

### § 3. Photoproduction of neutral vector mesons; photon dissociation model

In this and next sections we shall consider the following photo-reactions
in a few GeV region:

\[ \gamma + p \rightarrow \rho^0 + p, \quad (A_1)^{**} \]

\[ \gamma + p \rightarrow \omega + p, \quad (A_2) \]

\[ \gamma + p \rightarrow \varphi + p. \quad (A_3)^{**} \]

As is mentioned in the Introduction, our particular interest in reactions (A) stems from the fact that there is strong evidence\(^5\) that (A\(_1\)) and possibly (A\(_2\)) proceed mainly via a diffraction mechanism. Indeed, two experimental groups\(^5\) have reported their results with the conclusion that one-pion exchange cannot be the dominant mechanism for \(\rho^0\) and \(\omega\) productions and obtained a rough agreement with a diffraction or multiperipheral model proposed by Berman and Drell.\(^6\) In this model the main feature of reactions (A) may be understood by the known diffractive \(\pi-N\) scattering observed experimentally with the understanding that the absolute scale of cross sections for (A) must be reduced by a factor \(\alpha\), the fine structure constant, compared with that for \(\pi-N\) scattering. It should be noticed, however, that essentially the same conclusion could be reached by the photon dissociation model proposed by Ross and Stodolsky\(^7\) in which the incident photon is directly coupled to a neutral vector meson which is then scattered elastically on the proton, if, in the high energy region, \(V-N\) scattering has the properties analogous to the observed ones for \(P-N\) scattering (here, \(V\) and \(P\) indicate vector and pseudoscalar mesons, respectively). An attempt to make the detailed comparison of this model with experiment will have some uncertainties in the estimate of \(V-N\) scattering cross sections. However, in the light of \(SU(6)\) symmetry, or in the context of the so-called additivity assumption with spins of fundamental particles neglected, it is an easy

\[ \sigma(\gamma p \rightarrow \rho^0 p) \sim 0.4 \mu b \]

which must be compared with\(^9\)

\[ \sigma(\gamma p \rightarrow \omega p) \sim 15 \mu b \quad (2 \text{ GeV} < E_\gamma \leq 6 \text{ GeV}). \]

Harari\(^9\) pointed out that it seems hard to understand this suppression of \(\varphi\)-production within the framework of \(SU(3)\) symmetry and the usual \(\varphi-\omega\) mixing theory in a diffraction model used here if the photon is in an octet. However, it will be seen later on that this might be related to the very fact that the total \(K-N\) cross section is by about 25% smaller than the total \(\pi-N\) cross section at, say, \(p_L=6\ \text{GeV}/c.\)\(^9\) Thus we expect that, at high energies considered here, reactions (A) are controlled by the same production mechanism.
matter to relate $\sigma_T(VN)$ to $\sigma_T(PN)$ (essentially one has $\sigma_T(VN) = \sigma_T(PN)$). Although this approximation cannot be taken too seriously, what is to be emphasized here is that, if the dissociative production mechanism assumed here is the dominant one for all the reactions (A), since it involves the direct $\gamma-V$ coupling, it may be possible to detect the unitary singlet component in the electromagnetic current $j_\mu(x)$ by relating it to the production rates. On the other hand, whether the $SU(3)$ singlet term is present or not in $j_\mu(x)$ depends on which composite model of hadrons we consider. This will be revealed in Appendix B. Of course, if we assume only that the photon is purely in an octet but do not assume any composite model of hadrons, then the result except for the estimate of $\sigma_T(VN)$ would be the same as what we expect for the quark model in which $j_\mu$ consists only of octet.

With these understandings, in the following, we shall restrict ourselves to the processes of the type described in Fig. 1. A multiperipheral model will be treated in the next section. Our first task is then to find relations among the coupling constants of the direct photon-neutral vector meson transitions, which we shall denote by $\gamma_V$ for the $\gamma \rightarrow V$ transition ($V = \rho^0, \omega$ or $\phi$), on the basis of composite models of hadrons with the usual $\phi-\omega$ mixing hypothesis. The latter is generally expressed by

$$|\omega\rangle = \cos \theta |\omega_i\rangle - \sin \theta |\varphi_b\rangle,$$
$$|\varphi\rangle = \sin \theta |\omega_i\rangle + \cos \theta |\varphi_b\rangle,$$  \hspace{1cm} (3·1)

where $\omega_i$ and $\varphi_b$ are the unitary singlet and the $I=0$ octet vector mesons, respectively. Following Dashen and Sharp, we assume that the $\omega$ couple to photons with masses $m_\omega$ and the $\varphi$ with masses $m_\varphi$, thereby defining the $\gamma_V$'s in a phenomenological way by

$$\langle V^0(k) | j_\mu(0) | 0 \rangle = \gamma_V m_V^2 (2k_\mu(2\pi)^3)^{-1/2} \epsilon_\mu(k),$$  \hspace{1cm} (3·2)

where $k = (k, k_\mu)$ is the four momentum of $V^0$, $m_V$ the mass of $V^0$ and $\epsilon_\mu(k)$ the polarization vector. Then from (3·1) and (3·2), we find that

$$\gamma_\rho^2 : \gamma_\omega^2 : \gamma_\phi^2 = \begin{cases} 1 : \sin^2 \theta : \cos^2 \theta : 3 & \text{(G-Z, H-N)} \\ 1 : \frac{(\sin \theta + (1 + \epsilon) \sqrt{2} \cos \theta)^2}{3} : \frac{(1 + \epsilon) \sqrt{2} \sin \theta - \cos \theta)^2}{3} & \text{(M-H)} \\ 1 : \frac{(\cos \theta \sqrt{2} \sin \theta)^2}{6} : \frac{(\sin \theta + \sqrt{2} \cos \theta)^2}{6} & \text{(B-N-H)} \end{cases}$$  \hspace{1cm} (3·3)

*) Strictly speaking, here arises a difficult problem of violation of gauge invariance because photons under consideration are real. This has been only partially resolved for restricted class of model theories, e.g. for the universal vector meson theory of Sakurai and Gell-Mann. See references 6), 8), 2), 23) and the end of Appendix A for further information on this point.
the derivation of which will be given in Appendix B. The first relation was already derived by several authors\textsuperscript{19,25} from the octet property of the photon. This is the case for the G–Z model. The same relation holds also for the H–N model since, in this model, $\rho$, $\omega$ and $\varphi$ mesons couple only to the part of the electromagnetic current that transforms like components of $(8, 1)$ under $SU(3) \otimes SU(3)$. Two further points should be noted in (3·3); 1) in the M–H and G–L–N–S models, one arbitrary parameter $\epsilon$ has been introduced on account of the presence of the unitary singlet current, and 2) in the B–N–H model, although it has $SU(3)$ singlet current, $Sp(6)$ symmetry fixes uniquely the ratios between the $\gamma \nu$’s since $\omega_1$ and $\varphi_8$ belong to the same multiplet. Finally $SU(6)$ theory predicts $\epsilon = 0$.

As a next step we note that, in the photon dissociation model, the amplitude for reactions (A) is

$$T(\gamma p \rightarrow Vp) = \sum_{V'} T(V' p \rightarrow Vp) \quad (V = \rho, \omega, \varphi),$$

where $T(V' p \rightarrow Vp)$ stands for off-the-mass-shell invariant amplitude with $V'$ helicities $\pm 1$. (See Appendix A.) The sum on the RHS of this equation runs over

\begin{align*}
V' &= \rho, \omega, \varphi \text{ for G–Z, M–H and G–L–N–S models,} \\
V' &= \rho, \omega, \varphi \text{ and } Sp(6) \text{ singlet vector meson for B–N–H model,} \\
V' &= \rho, \omega, \varphi \text{ and vector mesons belonging to } (1, 8) \text{ for the H–N model.}
\end{align*}

In what follows we simply neglect inelastic transitions $V' \rightarrow V^\ast (V' \neq V)$. [See, however, references 7) and 26).] We have then\textsuperscript{30,7}.

$$T(\gamma p \rightarrow Vp) = \gamma \nu T(Vp \rightarrow Vp) \quad (V = \rho, \omega, \varphi).$$

In the high energy region considered here, we assume VN scattering to be helicity-nonflip in the diffraction domain so that helicity-flip amplitude makes no significant contribution to the elastic VN-cross section. On the other hand, the experimental $\sin^2 \alpha$ Adair-angle distribution for decay products of photo-produced $\rho$ indicates\textsuperscript{5} that the reaction $\gamma + p \rightarrow \rho^0 + p$ is dominated by the helicity-nonflip part of the amplitude at least in the near forward direction. We assume this is true not only for $\rho^0$ but also for $\omega$ and $\varphi$ productions. Thus we have for the photoproduction cross section of neutral vector mesons, in the high energy approximation, neglecting the off-shell effect,

$$\sigma(\gamma p \rightarrow Vp) = \sigma_{\nu} = \gamma \nu \sigma_{\nu d} (Vp), \quad (V = \rho, \omega \text{ or } \varphi) \quad (3·4)$$

\textsuperscript{5) When } $\sin \theta = 1/\sqrt{3}$, transitions $\rho$, $\omega \rightarrow \varphi$, $Sp(6)$ singlet in the case of the B–N–H model, $(1, 8)$ vector mesons in the case of the H–N model are forbidden since the latters always contain the $t_3$ component where $t_3$ denotes the third member of $SU(3)$ triplet while $\rho$ and $\omega$ do not. The transition $\rho \rightarrow \omega$ requires an isospin exchange whose contribution may well be much smaller than that of exchange with vacuum quantum number.
where \( \sigma_{el}(VP) \) is the elastic cross section for \( VP \) scattering at \( p_L \) (laboratory momentum of vector meson) \( \simeq E_\gamma \) (photon energy in the laboratory system). \(^*\)

At this stage we need further assumptions regarding elastic \( VP \) scattering; following Joos, \(^27\) we make them as follows:

1) High energy \( VP \) scattering has a strong forward peak and a constant total cross section analogous to the observed ones for, say, \( \pi-N \) scattering.

2) The slope of the forward peak is the same for all the \( V=\rho^0, \omega, \phi \), which we expect to be \( \sim 8-10 \mathrm{(GeV/c)}^{-1} \) from the observed \( \rho^0 \) photoproduction. \(^9\)

3) The forward elastic \( VP \) scattering amplitude is purely imaginary.

From 2) and 3), using the optical theorem, we immediately see that \( \sigma_{el}(VP)/\sigma_T^2(VP) \) is independent of the label \( V \). Thus, from Eq. (3.4), we arrive at the particularly simple results

\[
\frac{\sigma_\omega}{\sigma_\rho} = \frac{\gamma_\omega^2}{\gamma_\rho^2} \left[ \frac{\sigma_T(\omega p)}{\sigma_T(\rho p)} \right]^2 \quad \text{and} \quad \frac{\sigma_\phi}{\sigma_\rho} = \frac{\gamma_\phi^2}{\gamma_\rho^2} \left[ \frac{\sigma_T(\phi p)}{\sigma_T(\rho p)} \right]^2.
\]

(3.5)

In the approximation that \( SU(3) \) singlet exchange is dominant for \( VN \) scattering the following Freund-Harari ratios obtain

\[
\sigma_\rho : \sigma_\omega : \sigma_\phi = 9 : 1 : 2
\]

(3.6)

for the case of octet photon with \( \sin \theta = 1/\sqrt{3} \).

Rather we want to relate \( \sigma_T(VP) \) to \( \sigma_T(PN) \) by using the so-called additive decomposition assumption of the forward hadron-hadron scattering amplitudes, with spins of fundamental particles neglected. \(^{(Inclusion \ of \ the \ latter \ has \ been \ tried \ by \ some \ authors.} \)

Simple calculation shows\(^{**}\)

\[
\sigma_T(\rho^0 p) = \frac{1}{2} \Sigma_{\sigma p},
\]

\[
\sigma_T(\omega p) = \left\{ \left( \sqrt{2} \cos \theta + \sin \theta \right)^2 / 6 \right\} \Sigma_{\sigma p} + \left\{ \left( \cos \theta - \sqrt{2} \sin \theta \right)^2 / 6 \right\} \times \left( \Sigma_{Kp} + \Sigma_{Kn} - \Sigma_{\sigma p} \right),
\]

(3.7)

\[
\sigma_T(\phi p) = \left\{ \left( \sqrt{2} \sin \theta \cos \theta \right)^2 / 6 \right\} \Sigma_{\sigma p} + \left\{ \left( \sin \theta + \sqrt{2} \cos \theta \right)^2 / 6 \right\} \left( \Sigma_{Kp} + \Sigma_{Kn} - \Sigma_{\sigma p} \right),
\]

where \( \Sigma_{M_B} = \sigma_T(M^+B) + \sigma_T(M^-B) \) and we have assumed only \( SU(2) \) symmetry among the basic two-body amplitudes. We note first of all that the above-derived

\(^{*}\) Strictly speaking, one has

\[
\sqrt{(p_L)^2 + m_V^2} = E_\gamma - m_V^2/(2M_N) \quad (M_N = \text{nucleon mass})
\]

from which, for \( p_L \gg m_V \), one derives

\[
p_L \simeq E_\gamma - (m_V^2/2)(1/M_N + 1/p_L).\]

The approximation \( p_L \simeq E_\gamma \) includes an error of about 6% for the \( \rho^0 \) case at \( p_L = 6 \mathrm{GeV/c} \). One can safely neglect this error since, in practice, \( E_\gamma \) is determined only in some interval.

\(^{**}\) For usual mixing angle (\( \sin \theta = 1/\sqrt{3} \)), these are reduced to those obtained by Joos, \(^27\) giving \( \sigma_T(\rho^0 p) = \sigma_T(\omega p) = 27.4 \mathrm{mb} \) and \( \sigma_T(\phi p) = 13 \mathrm{mb} \) at \( p_L = 6 \mathrm{GeV/c} \) (we compare the LHS with the RHS of (3.7) at \( p_L = p_{L*} \) (laboratory momentum of pseudoscalar meson) in the high energy approximation).
relations hold for all the composite models since vector mesons are supposed to have the same structure except for spins as that of pseudoscalar mesons. It is also to be noted that, on the RHS of (3.7), only the sum of $\sigma_T(\bar{M}^+B)$ and $\sigma_T(\bar{M}^-B)$ appears because neutral vector mesons have odd charge conjugation parity ($C=-1$) so that only $C=+1$ systems can be exchanged in the crossed channel of elastic $V\bar{p}$ scattering whose contributions cancel out in the difference $\sigma_T(\bar{M}^+B)-\sigma_T(\bar{M}^-B)$. Finally one sees that $\sigma_T(\omega p)$ and in particular $\sigma_T(\varphi p)$ are rather sensitive to the $\varphi-\omega$ mixing angle.

Combining Eqs. (3.3), (3.5) and (3.7), we get

$$\frac{\sigma_\varphi}{\sigma_\omega} = C_{\rho_\varphi} \left\{ 3^{-1}(\sqrt{2} \cos \theta + \sin \theta)^2 + 3^{-1}(\cos \theta - \sqrt{2} \sin \theta)^2 \right\} \frac{\Sigma_{\kappa_\rho} + \Sigma_{\kappa_\rho} - \Sigma_{\Sigma_{\rho}}}{\Sigma_{\Sigma_{\rho}}}$$ \hspace{1cm} (3.8)

and

$$\frac{\sigma_v}{\sigma_\rho} = C_{\varphi_\rho} \left\{ 3^{-1}(\sqrt{2} \sin \theta - \cos \theta)^2 + 3^{-1}(\sin \theta + \sqrt{2} \cos \theta)^2 \right\} \frac{\Sigma_{\kappa_\rho} + \Sigma_{\kappa_\rho} - \Sigma_{\Sigma_{\rho}}}{\Sigma_{\Sigma_{\rho}}}$$ \hspace{1cm} (3.9)

where

$$C_{\rho_\varphi} = \frac{\bar{r}_{\rho_\varphi}^2}{r_{\rho_\varphi}^2} = \begin{cases} 3^{-1} \sin^2 \theta & (G-Z, H-N) \\ 3^{-1} (\sin \theta + \sqrt{2} \cos \theta)^2 & (M-H, G-L-N-S) \quad (3.10)* \\
6^{-1} (\cos \theta - \sqrt{2} \sin \theta)^2 & (B-N-H), \\
3^{-1} \cos^2 \theta & (G-Z, H-N), \\
6^{-1} (\sqrt{2} \sin \theta - \cos \theta)^2 & (M-H, G-L-N-S) \quad (3.11)* \\
6^{-1} (\sin \theta + \sqrt{2} \cos \theta)^2 & (B-N-H). \end{cases}$$

In Figs. 2 and 3, we have plotted the ratios of $\sigma(\varphi\rho \to \omega p)$ and $\sigma(\varphi\rho \to \varphi p)$ to $\sigma(\varphi\rho \to \rho^0 p)$ at $p_L = 6$ GeV vs. the $\varphi-\omega$ mixing angle $\theta$ by using Eqs. (3.8) – (3.11) and the experimental data\(^{29}\) on total cross sections of pseudoscalar mesons on nucleons. The experimental values indicated in Figs. 2 and 3 are taken from the recent works\(^{9,10}\) both by the CEA and DESY groups, some of which are quoted in references 7 and 19).

From Figs. 2 and 3 we see that 1) the G-Z and H-N models predict a slightly smaller ratio for $\sigma_\varphi:\sigma_\rho$** and a slightly larger ratio*** for $\sigma_\varphi:\sigma_\rho$ than

---

* For simplicity, we take $\epsilon=0$ in the cases of the M-H and G-L-N-S models as suggested by SU(6).

** We have neglected the one-pion exchange contribution to $\sigma_\varphi$ in our calculation. As mentioned at the beginning of this section, this may be justified for $\rho^0$- and $\omega$-productions by experiment and for $\varphi$ since $\varphi\varphi\varphi$ coupling constant is very small. We notice, however, that one-pion exchange cannot be completely neglected for $\omega$-production because the diffractive $\omega$-production is small compared with the $\rho^0$ case\(^{19}\). Thus if this contribution is included in $\sigma_\varphi$ and is assumed to be still small compared with diffraction production, good agreement of $\sigma_\varphi:\sigma_\rho$ with experiment will be achieved for the usual mixing angle $\theta \sim 36^\circ$.

*** Quite recently, Freund\(^7\) pointed out that this ratio can be reduced to the experimental one by taking account of SU(3) symmetry breaking effects on the $\tau-V$ links.
the corresponding experimental ones for the mixing angle $\theta \approx 40^\circ$ which may be compared with the usual value $\theta \approx 36^\circ$; 2) the M–H and G–L–N–S models may explain the low $\varphi$-production but predict the wrong $\sigma_\varphi : \sigma_\omega$ ratio and 2) the B–N–H model may explain the small experimental ratio for $\sigma_\varphi : \sigma_\omega$, but cannot account for the strong suppression of $\varphi$-production.

With sufficient confidence it is to be concluded that only G–Z and H–N models can explain the present experimental ratios both for $\sigma_\varphi : \sigma_\omega$ and $\sigma_\varphi : \sigma_\rho$. This conclusion will remain unchanged even if we analyse reactions (A) in terms of multiperipheral model which will be discussed in the next section.

We end this section by making some comments on the absolute magnitude of the high energy total $VN$ cross section. The additivity assumption, neglecting the spins of fundamental particles, leads to a prediction

$$\sigma_\tau(VN) = \sigma_\tau(PN),$$
\( P \) denoting pseudoscalar meson, which is also predicted by \( SU(6) \)\(^7\) and is roughly consistent with the estimate
\[
\sigma_r(\rho^0N) \approx 30 \text{ mb}
\]
given by Eisenberg et al.\(^4\) A much larger value was found by Ross and Stodolsky:\(^5\)
\[
\sigma_r(\rho N) = 50 \pm 5 \text{ mb}
\]
and by Drell and Trefil:\(^3\)
\[
66 \text{ mb} \leq \sigma_r(\rho N) \leq 94 \text{ mb}.
\]
More accurate determination of \( \sigma_r(VN) \) is important for testing the predictions of various symmetry schemes and ours as well.

**§ 4. Photoproduction of neutral vector mesons; multiperipheral model**

A photon dissociation mechanism used in the previous section is based upon the conjecture\(^6\) that real photons with sufficient energies can be dissociated into neutral vector mesons in matter with small momentum transfers while the same cannot be true in a free space simply because of energy-momentum conservation law and that these dissociations are probably the dominant inelastic processes at very high energies. Strictly speaking, however, processes under consideration do always violate the gauge invariance and some cancellation must occur in order to maintain the vanishing photon mass.

In order not to cast a cloud on our arguments given in § 3 because of these difficulties, we shall investigate the same problem as in § 3 in a multiperipheral model\(^8\) which does not suffer from any difficulty mentioned above. According to this model, cross sections for reactions (A) will be given by that for \( \pi-N \) scattering multiplied by factors representing coupling strengths of vertices \( VP\gamma \) and \( VVP \). See Fig. 4. Thus we may write\(^9\)
\[
\begin{align*}
\sigma_p & \propto \left( \frac{g_{\gamma\pi\pi}/4\pi}{12\Gamma_p/m_p} \right) \sigma_{el}(\pi N), \\
\sigma_{\omega} & \propto \left( \frac{g_{\gamma\omega\pi}/4\pi}{12\Gamma_p/m_p} \right) \sigma_{el}(\pi N), \\
\sigma_{\phi} & \propto \left( \frac{g_{\gamma\phi\pi}/4\pi}{12\Gamma_p/m_p} \right) \sigma_{el}(\pi N),
\end{align*}
\]
with irrelevant kinematical factors suppressed, where \( \Gamma_\rho \) is the width of the decay \( \rho \to 2\pi \) and \( g_{\gamma\pi\rho} \) and \( g_{\gamma\pi'\rho'} \) stand for the coupling constants corresponding to \( \gamma\pi V \) and \( V\pi V' \) vertices, respectively. In writing (4.1) we have neglected the transitions\(^{(*)}\) such as \( \gamma \to \varphi + \pi^0 \), \( Sp(6) \) singlet vector meson + \( \pi^0 \) in the case of the B-N-H model, \((1,8)\) vector mesons + \( \pi^0 \) in the case of the H-N model. From (4.1) one immediately derives

\[
\frac{\sigma_\omega}{\sigma_\rho} = \left( \frac{g_{\gamma\pi\rho}}{g_{\gamma\pi\omega}} \right)^2 = \frac{\Gamma_{\rho \to \omega \gamma}}{\Gamma_{\rho \to \omega \gamma}} \tag{4.2}
\]

and

\[
\frac{\sigma_\varphi}{\sigma_\rho} = \left( \frac{g_{\gamma\pi\rho}}{g_{\gamma\varphi\rho}} \right)^2 \left( \frac{g_{\gamma\pi\varphi}}{g_{\rho\varphi\rho}} \right)^2 = \frac{\Gamma_{\rho \to \varphi \gamma}}{\Gamma_{\rho \to \varphi \gamma}} \left( \frac{g_{\rho\varphi\rho}}{g_{\rho\varphi\rho}} \right)^2 \cdot \tag{4.3}
\]

Now for the present purpose, we need relations between the \( g_{\gamma\pi\rho} \)'s \((V = \rho^0, \omega, \varphi)\) to be derived from composite models. Actually we want to calculate the matrix element of the type \( \langle \pi^0 | j_\mu(0) | V^\rho \rangle \) under \( SU(3) \) symmetry. In the case of the G-Z model only two free parameters are involved since \( j_\mu \) is in an octet. The M-H and G-L-N-S models require one additional parameter representing the reduced matrix element of the unitary singlet current between two octets. In the case of the B-N-H model, since we do not know the relevant \( Sp(6) \) C-G coefficients, we must reduce \( Sp(6) \) symmetry to \( SU(3) \), leaving behind three free parameters just as in the cases of the M-H and G-L-N-S models. The H-N model does not differ from the G-Z one as far as we are not concerned with the representation \((1,8)\). Under those circumstances which we have two or three unknowns, we cannot say too much about the present problem so that we shall assume the validity of \( SU(6) \) symmetry and then we have \( \sin \theta = 1/\sqrt{3} \). In the light of \( SU(6) \) symmetry only one matrix element is unknown and, by eliminating it, we find the following relations among the \( g_{\gamma\pi\rho} \)'s: \(^{(*)}\)

1) G-Z and H-N models

\[
g_{\gamma\pi\rho} = (1/3)g_{\gamma\pi\omega}; \quad g_{\gamma\pi\varphi} = 0 \tag{4.4}
\]

2) M-H and G-L-N-S models

\[
g_{\gamma\pi\rho} = g_{\gamma\pi\omega}; \quad g_{\gamma\pi\varphi} = 0 \tag{4.5}
\]

3) B-N-H model

\[
g_{\gamma\pi\rho} = g_{\gamma\pi\omega} = 0; \quad g_{\gamma\pi\varphi} \text{ not determined unless one knows, e.g., } g_{\gamma\varphi\rho}. \]

Further, we know \( SU(6) \) symmetry forbids the decay \( \varphi \to \rho + \pi \) so that \( g_{\rho\varphi\rho} = 0 \). Collecting these results together with (4.2) and (4.3) we arrive at the final results

\[
\frac{\sigma_\omega}{\sigma_\rho} = \begin{cases} 
1/9 & \text{G-Z, H-N} \\
1 & \text{M-H, G-L-N-S} \\
0 & \text{B-N-H} 
\end{cases} \tag{4.6}
\]
and

\[ \frac{\sigma_v}{\sigma_p} = 0 \text{ for all models.} \quad (4.7) \]

Although the prediction (4.7) comes only from the fact that the coupling constant \( g_{\phi \pi} \) vanishes in the limit of \( SU(6) \) symmetry and \( \sigma_v \) is proportional to \( g_{\phi \pi}^2 \) in the multiperipheral model (so that reaction \( \gamma + p \to \varphi + p \) cannot be used to test the validity of the composite models), relations (4.6) can be checked experimentally. Essentially the same result was already obtained in § 3 if we put \( \sin \theta = 1/\sqrt{3} \) there. From the discussion given below (3.11) in the last section, the present experimental data definitely prefer the G–Z and H–N models.

§ 5. Leptonic decays of neutral vector mesons

There are processes for detecting directly the presence of the unitary singlet term in the electromagnetic current. One is the leptonic decay of the neutral vector meson, \( V^0 \to l^+ + l^- \), where \( l \) is a muon or electron, which should be actually mediated by the photon, and the other the radiative decay of the neutral vector meson, \( V^0 \to P^0 + \gamma, P \) being pseudoscalar. Both processes were discussed by Okubo who, however, restricted himself to the quark and the quartet models. Here, restricting ourselves to the decay modes \( V^0 \to l^+ + l^- \), we discuss the consequences of the composite models mentioned in § 2 and try to compare the results with experiment.

The decay width of \( V^0 \to l^+ + l^- \), is given by\(^{31,32} \)

\[ \Gamma(V^0 \to l^+ + l^-) = \gamma_v^2 \alpha \frac{m_v}{3} \left( 1 + \frac{2m_l^2}{m_v^2} \right) \left( 1 - \frac{4m_l^2}{m_v^2} \right)^{1/2}, \quad (5.1) \]

where \( \gamma_v \) is defined in Eq. (3.2) and the other notations are trivial. Since for both the electron and the muon the phase space factor, the product of the two brackets in (5.1), is equal to 1 within 0.2 \%, we shall set \( m_l = 0^+ \) in (5.1), finding

\[ \frac{\Gamma(\varphi \to l^+ + l^-)}{\Gamma(\rho^0 \to l^+ + l^-)} = \frac{m_{\omega}}{m_{\rho}} \frac{\gamma_{\omega}^2}{\gamma_{\rho}^2} \equiv \frac{\gamma_{\omega}^2}{\gamma_{\rho}^2}, \quad (5.2) \]

and

\[ \frac{\Gamma(\varphi \to l^+ + l^-)}{\Gamma(\rho^0 \to l^+ + l^-)} = \frac{m_{\phi}}{m_{\rho}} \frac{\gamma_{\varphi}^2}{\gamma_{\rho}^2} \equiv 1.34 \frac{\gamma_{\varphi}^2}{\gamma_{\rho}^2}. \quad (5.3) \]

If we now use the relations (3.3) between the coupling constants \( \gamma_{\rho}, \gamma_{\omega} \) and \( \gamma_{\varphi} \), we predict

\(^{4} \) This means that the decay widths for electron pairs or muon pairs for a given neutral vector meson should be identical.
As in the case of photoproduction of neutral vector mesons, we simply put $\epsilon = 0$.

For the values of $\theta$ in the interval $(0, \pi/4)$, we have shown in Figs. 5(a) and 5(b) how the ratios $\Gamma(\omega \rightarrow l^+ + l^-)/\Gamma(\rho^0 \rightarrow l^+ + l^-)$ and $\Gamma(\phi \rightarrow l^+ + l^-)/\Gamma(\rho^0 \rightarrow l^+ + l^-)$ are sensitive to the $\phi - \omega$ mixing angle. For the experimental values of these ratios we take the following as the best determination of the lepton-pair decay rates of $\rho^0$ and $\omega$:  

$$\Gamma(\rho^0 \rightarrow \mu^+ + \mu^-)/\Gamma(\rho^0 \rightarrow \pi^+ + \pi^-) = \left(0.44 \pm 0.21 \right) \times 10^{-4},$$  

with 

$$0.5 \times 10^{-4} \leq \frac{\Gamma(\omega \rightarrow e^+ e^-)}{\Gamma(\omega \rightarrow \text{all modes})} \leq 6 \times 10^{-4},$$  

from which 

$$0.06 \leq \frac{\Gamma(\phi \rightarrow l^+ + l^-)}{\Gamma(\rho^0 \rightarrow l^+ + l^-)} \leq 1.9$$  

for $\Gamma_\rho(\text{total}) = 124 \pm 4$ MeV and $\Gamma_\omega(\text{total}) = 12 \pm 1.7$ MeV. We have no significant information about $\Gamma(\phi \rightarrow l^+ + l^-)$. Quite recently, however, the work by Dubna group indicates that  

$$\Gamma(\phi \rightarrow e^+ e^-)/\Gamma_\phi(\text{total}) \leq 200 \times 10^{-5}$$  

for the above-quoted values of $\Gamma_\rho(\text{total})$ and $\Gamma_\omega(\text{total})$ and $\Gamma_\phi(\text{total}) = 4.0 \pm 1.0$ MeV, one finds  

$$\Gamma(\rho^0 \rightarrow e^+ e^-) = \left(4.8 \pm 0.17 \right) \text{keV},$$  

$$\Gamma(\omega \rightarrow e^+ e^-) = \left(5.8 \pm 0.29 \right) \text{keV},$$  

$$\Gamma(\phi \rightarrow e^+ e^-) \leq (8 \pm 2) \text{keV}.$$  

These must be compared only with the predictions of the G–Z and the H–N
models since (5·9) have been obtained under the assumption that the electromagnetic current is in an octet of \( SU(3) \).\(^{35} \)

For the particular value of \( \sin \theta = 1/\sqrt{3} \), we have computed the branching ratios using the estimate

\[
\Gamma_{\rho}^2 = \frac{g_{\rho\rho}^2}{g_{\rho\pi\pi}^2} \approx 0.42 \alpha, \quad (5·10)
\]

where \( g_{\rho\pi\pi} \) is the coupling constant for the decay \( \rho \to 2\pi \) with \( g_{\rho\pi\pi}^2/4\pi \approx 2.4 \) and \( \alpha \) is the fine structure constant. The equality (5·10) follows if we can write an unsubtracted dispersion relation for the pion electromagnetic form factor in the vector meson dominance model.\(^{21},^{36} \)

The results are as follows:

\[
\Gamma(\rho^0 \to l^+ l^-) \approx 5.7 \text{ keV},
\]

\[
\Gamma(\omega \to l^+ l^-) \approx \begin{cases} 0.64 \text{ keV} & (G-Z, H-N) \\ 5.8 \text{ keV} & (M-H, G-L-N-S) \\ 0 & (B-N-H), \\ 1.7 \text{ keV} & (G-Z, H-N) \\ 0 & (M-H, G-L-N-S) \\ 3.8 \text{ keV} & (B-N-H) \end{cases} \quad (5·11b)
\]

or

\[
\frac{\Gamma(\rho^0 \to l^+ l^-)}{\Gamma_{\rho} \text{(total)}} \approx (4.6 + 0.3 - 0.2) \times 10^{-5}, \quad (5·11a')
\]

\[
\frac{\Gamma(\omega \to l^+ l^-)}{\Gamma_{\omega} \text{(total)}} \approx \begin{pmatrix} 5.3 + 0.9 - 0.6 \\ 48 + 8 - 6 \end{pmatrix} \times 10^{-5} (M-H, G-L-N-S) \\
0 & (B-N-H), \quad (5·11b')
\]

\[
\frac{\Gamma(\varphi \to l^+ l^-)}{\Gamma_{\varphi} \text{(total)}} \approx \begin{pmatrix} 43 + 14 - 9 \\ 95 + 32 - 19 \end{pmatrix} \times 10^{-6} (M-H, G-L-N-S) \\
0 & (B-N-H). \quad (5·11c')
\]

(5·11a) is consistent with the experimental values (5·6). Accurate determination of these branching ratios would provide a critical test of the validity of composite models and in particular Eqs. (3·3) and (5·10).

§ 6. Summary and Discussion

We have investigated here the high energy photoproduction of neutral vector mesons in terms of diffraction models from the view that hadrons are composite particles in the sense that they consist of some more fundamental objects. Great
emphasis has been laid on the possibility that one can use these reactions to
detect the $SU(3)$ singlet component in the electromagnetic current or, in other
words, to test the validity of the composite models of hadrons proposed so far.
We found that the quark model of Gell-Mann and Zweig and the three-triplet
one of Han and Nambu are better candidates than others at the present ex­
perimental situation. Unfortunately, as far as we restrict ourselves to the low­
lying levels only, it is impossible to distinguish between these models, and the
same is true for the Maki-Hara and the mixed-two-triplet models. At this
point our conclusion differs from Okubo's. However, we are fully aware
that this can be traced to our approximation of replacing $(3\cdot 4a)$ by $(3\cdot 4b)$
and that only the part of the electromagnetic current that is responsible for $\rho^0$, $\omega$ and $\phi$ productions in this approximation was taken into account in our cal­
culation. If we believe in, say, the three-triplet model, then the missing part of
the photon which transforms like $(1, 8)$ under $SU(3) \otimes SU(3)$ should be ma­
terialized in matter in just the same way as the isovector part of the photon
is materialized as $\rho^0$ mesons, and the neutral vector mesons belonging to $(1, 8)$
should be created unless some damping mechanisms operate. The same reasoning
would be valid for the B–N–H model.

Under the assumption that the observed diffraction features of $\rho^0$-photo­
production emerge through the direct $\gamma-\rho^0$ coupling and the same mechanism
operates for $\omega$- and $\phi$-productions, we have calculated the branching ratios be­
tween the production rates of neutral vector mesons. These are sensitive to
the $\phi-\omega$ mixing angle and to the composite models of hadrons to be considered.
At the highest energy now available, the result was shown in Figs. 2 and 3.
Essentially the same result was also obtained in a multiperipheral model under
$SU(6)$ symmetry. Distinction between two diffraction mechanisms would be
apparent if we compute the high energy limits of photoproduction cross sec­
tions :

\[
\sigma_\rho : \sigma_\omega : \sigma_\phi = \left\{ \begin{array}{ll}
9 : 1 : 2 & (G-Z, H-N) \\
1 : 1 : 0 & (M-H, G-L-N-S) \\
2 : 0 : 1 & (B-N-H)
\end{array} \right.
\]

in a photon dissociation model and

\[
\sigma_\rho : \sigma_\omega : \sigma_\phi = \left\{ \begin{array}{ll}
9 : 1 : 0 & (G-Z, H-N) \\
1 : 1 : 0 & (M-H, G-L-N-S) \\
1 : 0 : 0 & (B-N-H)
\end{array} \right.
\]

in a multiperipheral model with $SU(6)$ symmetry. An independent test of the
latter would be provided by accurate determination of the decay widths such
as $\Gamma_{\omega \rightarrow \pi \pi}$, $\Gamma_{\rho \rightarrow \pi \pi}$, $\Gamma_{\phi \rightarrow \pi \pi}$, $\Gamma_{\phi \rightarrow \rho \pi}$. An important test of the former is to see whether
the present strong suppression of $\phi$-production persists at higher energies or not.
We have also presented the branching ratios of leptonic decay modes of neutral vector mesons. Figures 5(a) and 5(b) show their sensitivity to the \( \phi-\omega \) mixing angle and also to composite models.

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**Appendix A**

*Extension of the Gell-Mann and Zachariasen model*\(^{37}\) to photoproduction at high energies

It is shown here that photon dissociation mechanism for photoreaction at high energies is a natural consequence of the Gell-Mann and Zachariasen model.\(^{39}\)
Our argument is based on field theory in the Heisenberg picture although it may be given within the framework of dispersion theory.

The $S$-matrix element of photon initiated reactions from hydrogen target under consideration is

$$
\langle f(-) | p, \gamma (k, \epsilon^{(\gamma)} (k)) (+) \rangle = \frac{(2\pi)^4 \delta^4 (P_i - P_f)}{\sqrt{2 \omega_x (k) (2\pi)^3}} \langle f(-) | j_{\mu} (0) | p \rangle \epsilon^{(\gamma)} (k), \tag{A·1}
$$

where $P_i$ and $P_f$ denote the total four-momentum of initial and final states, respectively, $k, \omega_x (k) = |k|$ and $\epsilon^{(\gamma)} (k)$ momentum, energy and polarization vector of incident photon, respectively, and $j_{\mu}$ is the electromagnetic current density. We are concerned here only with photons of energy larger than, say, a few GeV where the diffraction dissociation of the photon may actually be possible in matter.\(^5\)

Let us put $j_{\mu}$ in the form

$$
j_{\mu} = a j_{\mu}^{(\gamma)} + b j_{\mu}^{(\rho)} + c j_{\mu}^{(\sigma)} + dj_{\mu}',
$$

where $j_{\mu}^{(\gamma)}$ is current of the $\gamma$-type, $j_{\mu}'$ residual term which has different transformation properties from the first three terms under $SU(3)$, and $a, b, c,$ and $d$ are numerical constants depending on which model used and will be determined in Appendix B.

Now according to Gell-Mann and Zachariasen,\(^{36}\) for any states $n, m$, we may write the matrix element of the electromagnetic current in terms of that of the corresponding vector meson current in the form

$$
\langle n | j_{\mu}^{(\gamma)} (0) | m \rangle = \frac{-\gamma \delta m_i^2 (1 + \epsilon \delta \theta \gamma)}{t - m_i^2} \langle n | (-1)^{\delta t} J_{\mu}^{(\gamma)} (0) | m \rangle, \tag{A·3}
$$

$$(i = 3, 8, 0)
$$

$$
\langle n | j_{\mu}' (0) | m \rangle = \frac{-\gamma m_i' \epsilon}{t - m_i'^2} \langle n | J_{\mu}' (0) | m \rangle, \tag{A·3}'
$$

with $t = (P_i - P_m)^2$, where $J_{\mu}^{(\gamma)}$ and $J_{\mu}'$ are the source currents coupled to neutral vector mesons:

$$
(\Box + m_i^2) V_{\mu}^{(i)} (x) = J_{\mu}^{(i)} (x) \quad (i = 3, 8, 0 \text{ corresponding to } \rho, \sigma, \omega) \tag{A·4}
$$

$$
(\Box + m_i'^2) V_{\mu}' (x) = J_{\mu}' (x) \tag{A·4}'
$$

and $\gamma$ stands for the effective coupling constant for transitions $\gamma \leftrightarrow$ neutral vector meson. In Eq. (A·3), a factor $(-1)^{\delta t}$ appears because of our phase convention (see Table I) and $\epsilon$ represents possible non-degeneracy of octet and singlet vector mesons according to the Wigner-Eckart theorem. $SU(6)$ symmetry predicts $\epsilon = 0$.

Noting that the effective coupling strength $\gamma_V$ of the direct $\gamma$-$V$ transition for physical $V = \rho, \sigma, \varphi$, are defined by
\[ \langle \rho | a_{j_{\rho}}^{(\rho)}(0) | 0 \rangle = -\frac{\gamma_{\rho} m_{\rho}^2}{t - m_{\rho}^2} \langle \rho | J_{\rho}^{(\rho)}(0) | 0 \rangle, \quad (A\cdot 5a) \]

\[ \langle \omega | (b_{j_{\omega}}^{(\omega)}(0) + c_{j_{\omega}}^{(\omega)}(0)) | 0 \rangle = -\frac{\gamma_{\omega} m_{\omega}^2}{t - m_{\omega}^2} \langle \omega | J_{\omega}^{(\omega)}(0) | 0 \rangle, \quad (A\cdot 5b) \]

\[ \langle \varphi | (b_{j_{\varphi}}^{(\varphi)}(0) + c_{j_{\varphi}}^{(\varphi)}(0)) | 0 \rangle = -\frac{\gamma_{\varphi} m_{\varphi}^2}{t - m_{\varphi}^2} \langle \varphi | J_{\varphi}^{(\varphi)}(0) | 0 \rangle, \quad (A\cdot 5c) \]

\[ \langle V' | d_{j_{\omega}}^{(\omega)}(0) | 0 \rangle = -\frac{\gamma_{V'} m_{V'}^2}{t - m_{V'}^2} \langle V' | J_{\omega}^{(V')}(0) | 0 \rangle, \quad (A\cdot 5d) \]

where

\[
\left( \Box + m_{\psi}^2 \right) V_{\omega}(x) = J_{\omega}^{(V)}(x) \quad (V = \rho, \omega, \varphi, V') \quad (A\cdot 6)
\]

and using the usual prescription of \( \varphi - \omega \) mixing, (3.1) in § 3, we find

\[
\begin{align*}
\gamma_{\omega} &= \alpha \gamma_{\beta}, \\
\gamma_{\varphi} &= -b \cos \theta (m_{\varphi}^2/m_{\omega}^2) + c \sin \theta (1 + \epsilon) (m_{\varphi}^2/m_{\omega}^2) \gamma_{\beta}, \\
\gamma_{V'} &= \{ b \sin \theta (m_{\varphi}^2/m_{\omega}^2) + c \cos \theta (1 + \epsilon) (m_{\varphi}^2/m_{\omega}^2) \} \gamma_{\beta}.
\end{align*}
\]

Introducing (A·3) together with (A·7) into (A·1) gives for \( t = k^2 = 0 \)

\[ \langle f(-) | p, \gamma(k, \epsilon^{(V)}(k))(+) \rangle = \sum_{\gamma = \rho, \omega, \varphi, V'} \gamma \langle f(-) | p, V(k, \epsilon^{(V)}(k))(+) \rangle. \quad (A\cdot 8) \]

Of course, the \( V \)'s in this sum are off-shell and have only states with helicities \( \pm 1 \).

A particular case in which we are interested in § 3 is to take

\[ | f \rangle = | p, V \rangle: \]

\[ \langle p, V(-) | p, \gamma(k, \epsilon^{(V)}(k))(+) \rangle = \sum_{\gamma = \rho, \omega, \varphi, V'} \gamma \langle p, V(-) | p, V(k, \epsilon^{(V)}(k))(+) \rangle. \quad (A\cdot 9) \]

Neglecting \( \nu \leftrightarrow V \) transitions leads to a very simple result

\[ \langle p, V(-) | p, \gamma(k, \epsilon^{(V)}(k))(+) \rangle = \gamma \langle p, V(-) | p, V(k, \epsilon^{(V)}(k))(+) \rangle. \quad (A\cdot 9)' \]

These are essentially the equations upon which some authors\(^6,7\) are based in order to discuss photoproduction of neutral vector mesons in terms of photon dissociation model.

The final remark will be related to the problem of gauge invariance in the case of conversion of real photon into neutral vector mesons. The only gauge invariant interaction Hamiltonian is, apart from coupling constant,

\[ \int d^4 x F_{\mu\nu}(x) (\partial_{\nu} V_{\nu}(x) - \partial_{\mu} V_{\mu}(x)), \]

which, by integration by parts, is reduced, using the Lorentz condition and
neglecting surface integral, to
\[
\int d^4x \nabla A_\mu(x) \cdot V_\nu(x).
\]
In momentum space this reads
\[
k^2 A_\mu(k) V_\nu(k),
\]
where \(k\) is four momentum of the photon. Real photon requires \(k^2 = 0\) which states that real photon cannot be converted into virtual neutral vector mesons unless gauge invariance were violated. This is an intuitive interpretation of the arguments given by Feldman and Mathews.\(^{23}\) On the other hand, our interaction Hamiltonian equivalent to Eq. (A·3) is
\[
\sum_{r=p, n, q} \gamma_r m_r^2 \int A_\mu(x) V_\nu(x) d^4x
\]
which is obviously not gauge invariant since the gauge transformation \(A_\mu \rightarrow A_\mu + \partial_\mu A\) with \(\nabla A = 0\) introduces, after integration by parts, an extra term \(\partial_\mu V_\mu(x) \cdot A(x)\) which does not vanish unless \(\partial_\mu V_\mu(x) = 0\). This last condition is consistent with the Heisenberg equation only if \(J_\mu^{(v)}\) be conserved.\(^{21,20}\) In general, however, \(J_\mu^{(v)}\) is not conserved so that an additional term will be present on the RHS of Eq. (A·3) in order to maintain the conservation law of \(j_\mu\). Remembering that gauge invariance is related to the conservation of the electromagnetic current, the assumption of dominance on the RHS of Eq. (A·3) over this additional term does not necessarily require that the corresponding effective interaction (A·10) must be gauge invariant. This is essentially the viewpoint adopted by Berman and Drell.\(^8\)

**Appendix B**

*Relations between \(\vec{r}_p, \vec{r}_n\) and \(\vec{r}_q)*

From Eq. (A·7) in Appendix A, we find relations between \(\vec{r}_p, \vec{r}_n\) and \(\vec{r}_q\) once we know the values of constants \(a, b, c\) and \(d\). For this purpose, we shall write down the electromagnetic current in terms of the basic fields for each composite model.

1) G–Z model
\[
j_\mu(x) = \frac{ie}{2} \vec{q}(x) \gamma_\mu (\lambda_5 + (1/\sqrt{3}) \hat{\lambda}_8) q(x),
\]
where \(q = [q_1, q_2, q_3]^{*}\) and \(\lambda_5\) and \(\lambda_8\) are Gell-Mann’s matrices.

2) M–H model

\(^{*}\) By \(a = [a_1, \ldots, a_n]\) we mean a column vector
\[
a = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}.
\]
\( j_\mu(x) = (i\varepsilon/2) \bar{\psi}(x) \gamma_\mu (\lambda_3 + (1/\sqrt{3}) \lambda_8 + \sqrt{2/3} \lambda_0) \psi(x), \) \hspace{1cm} (B\cdot2)

where \( t = [t_1, t_2, t_3], \) \( (\lambda_3)_{ij} = \sqrt{2/3} \delta_{ij} \) \((i, j = 1, 2, 3), \) and a charge assignment of Sakata type is employed.

3) G-L-N-S model

\[ j_\mu(x) = \left( i\varepsilon/2 \right) \{ \alpha^+(x) \gamma_\mu (\lambda_3 + (1/\sqrt{3}) \lambda_8 + \sqrt{2/3} \lambda_0) \alpha(x) \]
\[ + \bar{\beta}(x) \gamma_\mu (\lambda_3 + (1/\sqrt{3}) \lambda_8 + \sqrt{2/3} \lambda_0) \beta(x) \}, \]

where \( \alpha = [\alpha_1, \alpha_2, \alpha_3] \) and \( \beta = [\beta_1, \beta_2, \beta_3]. \)

4) B-N-H model

\[ j_\mu(x) = \left( i\varepsilon/2 \right) \{ \varphi(x) \gamma_\mu (A_i + (1/\sqrt{3}) A_8 - (1/\sqrt{6}) A_9 + \sqrt{3/2} A_0') \varphi(x) \}, \]
\[ \varphi = [\varphi_1, \ldots, \varphi_8] = [T^a, T^b, T^c, \Theta^a, \Theta^b, \Theta^c] \]

and

\[ A_i = \begin{pmatrix} \lambda_i & 0 \\ 0 & -\lambda_i \end{pmatrix} (i = 3, 8, 0), \]
\[ A_0' = \begin{pmatrix} \lambda_0 & 0 \\ 0 & \lambda_0 \end{pmatrix}, \]

0 being \( 3 \times 3 \) null matrix and \( A_i (i = 0, 1, \ldots, 20) \) the infinitesimal generators of \( Sp(6) \) group.

5) H-N model

\[ j_\mu(x) = \left( i\varepsilon/2 \right) \{ \hat{\xi}(x) \gamma_\mu (\lambda_3 + (1/\sqrt{3}) \lambda_8) \otimes \sqrt{3/2} \lambda_0' \]
\[ + \sqrt{3/2} \lambda_0 (\lambda_0' + (1/\sqrt{3}) \lambda_0') \} \hat{\xi}(x), \]

where

\[ \hat{\xi} = [\hat{\xi}_1, \ldots, \hat{\xi}_8] \]

\[ \lambda_0' = \lambda_0, \lambda_0' = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{pmatrix}. \]

These expressions directly give the values of constants \( a, b, c, d \) in Eq. (A\cdot2) which are collected in the next table. The residual terms \( j_\mu' \) in Eq. (A\cdot2) which are present in the last three models are not relevant to our discussion in § 3 since they do not couple to \( \rho', \omega, \varphi. \)

---

\(^{3}\) A direct product of two \( 3 \times 3 \) matrices \( A = (a_{ij}) \) and \( B = (b_{ij}) \) is here defined by \( C = A \otimes B, \)
\[ C = (c_{ij}; m) = (a_{ij} b_{jm}) \text{ where} \]
\[ C = \begin{pmatrix} c_{01:11} & c_{02:11} & c_{03:11} \\ c_{11:11} & c_{12:11} & c_{13:11} \\ c_{21:11} & c_{22:11} & c_{23:11} \end{pmatrix}. \]
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Table. Values of $a$, $b$, $c$ and $d$ in (A·2)

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-Z</td>
<td>1/2</td>
<td>$1/2\sqrt{3}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>M-H</td>
<td>1/2</td>
<td>$1/2\sqrt{3}$</td>
<td>$1/\sqrt{6}$</td>
<td>0</td>
</tr>
<tr>
<td>G-L-N-S</td>
<td>1/2</td>
<td>$1/2\sqrt{3}$</td>
<td>$1/\sqrt{6}$</td>
<td>$-a$</td>
</tr>
<tr>
<td>B-N-H</td>
<td>1/2</td>
<td>$1/2\sqrt{3}$</td>
<td>$-1/2\sqrt{6}$</td>
<td>$-a$</td>
</tr>
<tr>
<td>H-N</td>
<td>1/2</td>
<td>$1/2\sqrt{3}$</td>
<td>0</td>
<td>$-a$</td>
</tr>
</tbody>
</table>

a) Residual terms in these models consist of two or three terms which have different transformation properties for different model.

Although symmetry breaking through mass terms may be taken into account in Eq. (A·7), we shall neglect simply its effect on $\gamma_v$ and set $m_i^2/m_v^2=1$ ($i=1,8$ and $V=\varphi, \omega$). Then, from the above table and Eq. (A·7) with this simplification, we find immediately the relation (3·3) between the $\gamma_v$'s.

References

8) S. M. Berman and S. D. Drell, Phys. Rev. 133 (1964), B791.
18) M. Y. Han and Y. Nambu, Phys. Rev. 139 (1965), B1006.