Analysis of Proton-Proton Scattering at High Energy

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Phenomenological analysis of proton-proton elastic scattering above a few GeV/c is made in order to have the idea as to whether the strong repulsive core exists or not at such high energy and how strong the repulsive core can be if it exists. We make the simplified assumption that the nucleon-nucleon interaction is described by a short range repulsive core surrounded by a rather long range absorptive medium and calculate the scattering amplitude by the distorted wave Born approximation. The analysis is performed in two cases: (I) The repulsive core is described by a hard core. (II) It is described by a soft core of the Gaussian shape.

It is quite difficult to explain the high energy data without modifying the strong repulsive core established at low energy.

§ 1. Introduction and summary

It is established by extensive analyses of proton-proton scattering from low to 300 MeV energy region that there exists a strong repulsive force in the inner region of nuclear forces, at least in the $^1S_0$ state. It seems likely that there is the similar repulsive force in the other states. This repulsive force is phenomenologically described by "hard core", which has infinite strength. Recently, several authors have investigated the nucleon-nucleon scattering in order to clarify the "softness" and the shape of such repulsive core.

On the other hand, from the phase shift analysis of proton-proton scattering at high energies, it has been known that below 1 GeV the absorption effects in the inner region are very weak but above 1 GeV they become stronger with increasing energy. The optical model analysis of proton-proton scattering above 10 GeV indicates an almost complete absorption.

However, it has been an open question whether the strong repulsive core established in the low energy is masked by the strong absorption due to inelastic channels, or a mechanism producing the repulsive core is transformed into another to cause the absorption. The main purpose of this paper is to examine nucleon-nucleon scattering phenomenologically above a few GeV/c to have the idea as to whether the strong repulsive core exists or not and how strong the repulsive core can be if exists at these energies.

In order to carry out the above purpose, we make the simplified assumption
that the proton-proton interaction is described by a short range repulsive core surrounded by a rather long range absorptive medium and calculate the scattering amplitude by the distorted wave Born approximation (the absorption model). That is, the contribution from the repulsion to the scattering amplitude is given through real phase shifts while that from the absorption is taken into accounts through the absorption coefficient obtained from the forward scattering.

The results obtained in this paper are summarized as follows: Case I). When the repulsive core is described by the hard core radius $2/M$ ($M$ is the nucleon mass) established from the analyses of low energy nucleon-nucleon scattering is too large to consistent with experiment for $P_L>6\text{GeV}/c$, where $P_L$ is the incident proton momentum in laboratory system. The core radius $1/M$ seems to be rejected in the similar way for $P_L>10\text{GeV}/c$. Thus the hard core allowed to exist may be such that the core radius decreases exponentially as $P_L$ increases.

Case II). When the soft core of the Gaussian shape, $V(r)=V_0 \exp(-r^2/R^2)$, is assumed, $R>2\text{(GeV}/c)^{-1}$ with $V_0>4\text{GeV}$ may contradict with experiment at $P_L\approx 10\text{GeV}/c$. For $R>1\text{(GeV}/c)^{-1}$, $V_0>4\text{GeV}$ seems to be rejected at $P_L\approx 20\text{GeV}/c$.

In § 2, the method of the analysis is shown and the absorption coefficient is calculated from the experimental data. The case of the hard core is studied in § 3 and that of the soft core is in § 4. Finally in § 5, concluding remarks are given.

§ 2. Formulation

In this section, formulation of the analysis is given by means of the absorption model.

If we neglect the effects of the Coulomb interaction and of the spin, the scattering amplitude and the differential cross section for elastic scattering are given respectively as follows:

\[ f(\theta) = \frac{1}{2ik} \sum_{l=0}^{n} (2l+1) (1-e^{i\delta_l}) P_l(\cos \theta), \quad (1) \]

\[ \frac{d\sigma}{d|t|} = \frac{\pi}{k^2} \frac{d\sigma}{d\omega} = \frac{\pi}{k^2} |f(\theta)|^2, \quad (2) \]

where, $k$ and $\theta$ denote the momentum and the scattering angle in the center of mass system, respectively, $P_l$ is the Legendre polynomial of the $l$-th order, $-t$ is the four-momentum transfer squared and $\delta_l$ is the complex phase shift of the $l$-th partial wave. This is explicitly written as

\[ \delta_l = \delta_l^R + i\delta_l^I, \quad (3) \]

*We take the unit $\hbar=c=1$.\]
where \( \delta_r \) and \( \delta_i \) denote the real part and the imaginary part of the phase shift respectively. Using Eq. (3), we can rewrite Eq. (1) as

\[
f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1) \left[ (1-e^{-\delta_i}) + e^{-\delta_i}(1-e^{2\delta_i}) \right] P_l(\cos \theta).
\] (4)

The absorption coefficient for the \( l \)-th partial wave is defined by the imaginary part of the phase shift:

\[
a_l = 1 - e^{-\delta_i}.
\] (5)

We determine \( a_l \) from the cross section for diffraction scattering at forward angles.\(^9\) It is well known that the forward differential cross section at high energy is of the form

\[
\frac{d\sigma}{dt} = A \exp[\alpha t], \quad (-t < 1 \text{GeV/c}^2),
\] (6)

and that its main part comes from the imaginary amplitude.\(^9\) Neglecting the contribution from the real part of the phase shift in Eq. (4), we get \( f(\theta) \) at forward angles as follows:

\[
f(\theta)_{\text{forward}} = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1) a_l P_l(\cos \theta).
\] (7)

From Eq. (6), we derive the following expression for \( a_l \):

\[
a_l = \frac{k^2}{\sqrt{\pi}} \sqrt{A} \exp[-\alpha k^2] \int_{-1}^{1} e^{i\alpha x} P_l(x) dx.
\] (8)

<table>
<thead>
<tr>
<th>( P_0 ) (GeV/c)</th>
<th>( \alpha ) (GeV/c)(^{-2} )</th>
<th>( A ) mb/(GeV/c)(^2 )</th>
</tr>
</thead>
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<tr>
<td>4</td>
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</tr>
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<td>26</td>
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<td>73.83</td>
</tr>
<tr>
<td>30</td>
<td>10.13</td>
<td>73.17</td>
</tr>
</tbody>
</table>

Here, the parameter \( A \) of Eqs. (6) and (7) is determined from the total cross section by using the optical theorem in order to ensure the unitarity.

The values of \( A \) and \( \alpha \) used in our analysis are listed in Table I. \( \alpha \) is also plot versus the incident momentum in comparison with the experimental data in Fig. 2-1. The absorption parameter \( a_l \) calculated by Eq. (8) is shown as function of the impact parameter \( b = (1/k) \sqrt{l(l+1)} \) in Fig. 2-2. Ambiguities in determining \( \alpha \) are discussed in § 3.

For convenience of calculation of the cross section at large angles, we rewrite Eq. (4). Using Eqs. (6) and (7):
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Fig. 2-1. The parameter $a$ of Eq. (6) is plotted versus the incident momentum in the laboratory system in comparison with the experimental data.\(^1\)

\[
\frac{d\sigma}{d|t|} = \frac{1}{4k^2} \left[ \sum_{l=0}^{\infty} (2L+1) \frac{2(1-a_l)\tan \delta_l^R}{1 + \tan^2 \delta_l^R} P_l(\cos \theta) \right]^2 + \left[ \sum_{l=0}^{\infty} (2L+1) \frac{2(1-a_l)\tan \delta_l^R}{1 + \tan^2 \delta_l^R} P_l(\cos \theta) - \frac{2k^2}{\sqrt{\pi}} \sqrt{\lambda} e^{-\lambda R} \right]^2. \tag{9}
\]

Let us take the real phase shift into consideration. First, in the case of the hard core potential, the real phase shift is given by the following expression:

\[
\tan \delta_R^l = \frac{j_l(kR_e)}{n_l(kR_e)}, \tag{10}
\]

where, $j_l$ and $n_l$ are the spherical Bessel and Neuman functions of the $l$-th order respectively and $R_e$ is the radius of the hard core.

Secondly, in the case of the soft core potential with the Gaussian shape of $V = V_0 \exp\left[-\frac{r^2}{2R^2}\right]$, the $T$-matrix in the Born approximation is given by

\[
T_l^B = -\sqrt{\frac{\pi}{16}} V_0 MkR^2 e^{-\lambda R^2}(i)j_l\left(\frac{1}{2}ikR^2\right). \tag{12}
\]

The Born amplitude does not always satisfy the unitarity condition. This is guaranteed by adopting the $K$-matrix formalism:\(^6\)

\[
e^{i\delta_l^R} = \frac{1+iT_l^B}{1-iT_l^B}, \tag{13}\]

which is equivalent to put

\[
\tan \delta_l^R = T_l^B. \tag{14}\]

Using Eqs. (11) ~ (14), effects of a long-range (attractive) force is also estimated in § 4.
§ 3. Hard core repulsion

In this section, numerical results in the case of assigning the hard core potential to the short range repulsive force are shown and discussed.

3-1 Numerical results

The real part of the phase shift is calculated from Eq. (10). Figures 3-1 and 3-2 show the \( l \)-dependence of the quantity

\[
R_l = (2l+1) \frac{(1-a_l) \tan \delta_l^R}{1 + \tan^2 \delta_l^R},
\]

which gives the effect of each partial wave on the real amplitude. From these

\begin{align*}
\text{Fig. 3-1} & \quad R_l = (2l+1) \frac{(1-a_l) \tan \delta_l^R}{1 + \tan^2 \delta_l^R} \\
\text{Fig. 3-2} & \quad R_l = (2l+1) \frac{(1-a_l) \tan \delta_l^R}{1 + \tan^2 \delta_l^R}
\end{align*}

\begin{align*}
\text{Fig. 3-3} & \quad \frac{d\sigma}{dt} \text{ in c.m.} (\text{GeV/c}^2) \\
\text{Fig. 3-4} & \quad \frac{d\sigma}{dt} \text{ at } \theta = 85^\circ \text{ in the c.m. system vs. both } P_L \text{ and } k_{\text{c.m.}}. \text{ The experimental one is that at } \theta = 90^\circ. \text{''}
\end{align*}
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figures, it is found that only a few partial waves play the decisive roles for the differential cross section. Note that the effective angular momentum at which $|R_l|$ take the maximum value is equal to $kR_c$.

The differential cross section calculated by Eqs. (9) and (10) is shown in Fig. 3-3 as a function of the four momentum transfer squared. At high energies, the calculated differential cross section at large angles is almost isotropic.

To see the energy dependence of the calculated differential cross section, the differential cross section $d\sigma/d|t|$ at $\theta_{c.m.}=85^\circ$ of the center of mass system is plotted versus energy in Fig. 3-4. It is well known that the measured cross section at large angles decreases exponentially with increasing energy. However, the calculated one does not show such a strong energy dependence.

3-2 Discussion

The above results indicate that the hard core with the radius of order $(\text{GeV/c})^{-1}$ is not consistent with the existing experimental data. The validity of this indication is investigated below.

The estimate of the absorption coefficient has some ambiguities arising from the following three factors.

1) Errors of experimental data which give rise to ambiguities of $\alpha$.
2) The contribution from the imaginary amplitude for $|t|>1(\text{GeV/c})^2$.
3) The effects of the real amplitude at the forward angles.

The ambiguities (1) and (3) are examined by changing the $\alpha$ by $\pm 10\%$ from the one adopted in § 2. The calculated values for the differential cross section are shown in Figs. 3-5 and 3-6. From this figure, it can be seen that the ambiguities (1) and (3) change the large angle scattering only by a factor and so do not invalidate the above indication.

The uncertainty (2) can be serious only for low partial waves. Therefore the $a_l$'s with small $l$ are changed arbitrarily in so far as they reproduce the forward differential cross section within the experimental errors. Since the strong absorption effect makes the contribution of the real amplitude due to the hard core decrease, the $a_l$'s with small $l(\leq l_b)$ are taken larger values than
the one adopted in § 2, especially, several of them are put equal to one (the complete absorption).

Figures 3-7 and 3-8 show the examples, where all $a_i$ for $l \leq l_0$ are put equal to one. The strong absorption indeed masks the hard core, but the cross section at large angles cannot be smaller than the experimental value. This can be reasonably interpreted by the diffraction pattern. Figures 3-7 and 3-8 also show the correction by changing $a_i$ into $a_i'$ at large angles:

$$\frac{d\sigma'}{dt} = \frac{\pi}{4K^2} \sum_{l=0}^{\infty} (2l+1) (a_i - a_i') P_l (\cos \theta)|^2$$

(16)

It is quite difficult to guess a modification which just happens to mask the hard core effect without giving rise to large diffraction pattern.

The antisymmetrization for the amplitude should be taken into account because of the $p-p$ scattering. Since the antisymmetrization is important only at large angles it is taken into account for the real amplitude. Figure 3-9 shows that the antisymmetrization makes the differential cross section increase at most by a factor 2.

Thus we have the conclusion as stated in § 1.
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Fig. 3-9. $dσ/d|t|$ by taking account of the antisymmetrization.

§ 4. Soft core repulsion

In this section, the soft core potential of the Gaussian shape (11) is investigated.

4-1 Numerical results

The real parts of the phase shifts are calculated from Eqs. (12) and (14). Figures 4-1 and 4-2 are the $l$-dependence of the quantity $R_1$ defined in (15), which gives the contribution of each partial wave to the real amplitude.

In comparison with the case of hard core, the higher partial waves are effective. The $l$ of the effective partial wave increases with increasing energy.

The differential cross section calculated by Eqs. (9), (12) and (14) is

Fig. 4-1. $(2l+1)((1-a_1)\tan \delta_l^{R_1}/(1+\tan^2 \delta_l^{R_1}))$ vs. angular momentum for $R=1(\text{GeV}/c)^{-1}$.

Fig. 4-2. $(2l+1)((1-a_1)\tan \delta_l^{R_2}/(1+\tan^2 \delta_l^{R_2}))$ vs. angular momentum for $R=2(\text{GeV}/c)^{-1}$.
Fig. 4-3. $\frac{d\sigma}{dt}$ vs. $-t$ (GeV/c)$^2$ for $V_0=1$ (GeV). The partial waves are taken into account up to $l=23$.

Fig. 4-4. The same as Fig. 4-3 but $V_0=2$ (GeV).

Fig. 4-5. $\frac{d\sigma}{dt}$ at $\theta=85^\circ$ vs. both $P_L$ and $R_{\text{cm}}$. The experimental one is at $\theta=90^\circ$.\(^{13}\)
shown in Figs. 4-3 and 4-4 as function of the four momentum transfer squared.

Roughly speaking, the shape of the differential cross section is governed by the core range and the incident energy, while the absolute value of it is governed by height of the core.

In Fig. 4-5, the differential cross section \(d\sigma/d|t|\) at \(\theta_{\text{c.m.}}=85^\circ\) of the center of mass system is compared with the experimental data. The energy dependence is stronger than that in the hard core case. The smaller core range has the weaker energy dependence, which is characteristic of the Gaussian type potential.\(^\dagger\) Therefore once the calculated cross section exceeds the experimental one at a certain energy, such a soft core should be excluded above the energy even if the calculated cross section is smaller than the experimental one there.

4-2 Conclusion and discussions

The ambiguities of the analysis can be investigated quite in parallel to the

\(^\dagger\) Cottingham et al.\(^{11}\) have pointed out that the strong energy dependence of the large angle differential cross section for proton-proton scattering is understood by either a Gaussian-like potential or a potential with stronger singularity at zero point.
hard core case. We only cite the similar figures.

Effects due to a long range (attractive) potential outside the soft core are examined and shown in Fig. 4-10. The long range potential is assumed to have the Gaussian type with the $2\pi$-exchange range for simplicity of calculation. It is found that such an attractive potential affects only the differential cross section at intermediate angles because of the longness of the range.

Thus we may conclude as in § 1.

§ 5. Concluding remarks

An investigation for essentially the same purpose as ours has been performed by the Waseda Group. They have analysed the proton-proton interaction at high energies using the Klein-Gordon equation with the hard core. There, the absorption effect has been taken into accounts through the optical potential. Their results are in good agreement with ours for the case of the hard core repulsion.
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Our conclusions in this paper are qualitative. In order to draw more quantitative conclusion, we will have to look for more accurate experimental data including such as the polarization data on the nucleon-nucleon scattering.*

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* Analysis more akin to the phase shift analysis than ours have been made recently.14)