11 Roelands, C. J. A., "*Correlational Aspects of the Viscosity-Temperature-Pressure Relationship of Lubricating Oils,*" Druk V. R. B.: Groninger, Netherlands, 1966.

12 Blok, H., "Inverse Hydrodynamics, "*Proceedings of the International Symposium on Lubrication and Wear,* McCutchan Publishing Corporation, 1964, pp. 1–151.

DISCUSSION

Y. P. Chiu[3]

The authors are to be congratulated for having performed the difficult and long awaited task of solving the elastohydrodynamic problems of point contacts. The authors attempt at developing a simple correlation between film thickness and the axis ratio of the contact is also praiseworthy.

It may be noticed in the authors' numerical example with constant values of dimensionless load $W = F/E/R_x{}^2$, the contact center becomes less flattened as the axis ratio (k) increases. This signifies a transition from EHD reguimes to a so called "Intermediate regime" (as defined by Dowson et al. ref. [15].[4]

As the value of k (or Ry) further increases under the constant load, the contact will shift to the "hydrodynamic regime" in which elastic deformation is negligibly small. This discussant therefore questions if it is sufficient to generate a simple film thickness formula for a general point contact without specifying what regime of lubrication it falls in.

The contact shape of a circular contact is predictable based on

Table 6 Tabulation of measured and calculated EHD film thicknesses in micrometers at $V = 1$ m/sec

Load, Newtons	Axis Ratio a/b	Oil No.	$\bar{U} \times 10^8$	Measured	Theory		
					Archard	(Modified) Kapitza	Cheng
22.2	4	1	3.91	.20	.16	.16	.21
22.2	4	2	18.4	.55	.51	.45	.66
22.2	4	3	59.3	1.30	1.22	.98	1.56
22.2	4	4	36.6	1.02	.85	.71	1.09
66.7	13.6	1	3.9	.20	.16	.17	.21
66.7	13.6	2	18.1	.55	.48	.46	.65
66.7	13.6	3	60.4	1.10	1.19	1.04	1.59
66.7	13.6	4	33.0	1.02	.76	0.69	1.02
111.2	4	1	3.7	0.16	.14	0.15	.20
111.2	4	2	18.1	.47	.44	.44	.57
111.2	4	3	46.1	.87	.89	.83	1.15
111.2	4	4	36.6	.78	.76	.71	.97
111.2	13.6	1	3.7	0.18	0.14	.16	.19
111.2	13.6	2	16.6	.51	0.44	.44	.59
111.2	13.6	3	55.7	.83	1.07	.98	1.44
111.2	13.6	4	31.6	1.02	0.71	.67	0.95

Table 7 Measured and calculated EHD film thickness in micrometers at $V = 10$ m/sec

Load (N)	Axis Ratio	Oil No.	$\bar{U} \times 10^8$	Measured	Theory			
					Archard	Kapitza (modified)	Cheng	Starvation Corrected
22.2	4	1	31.1	0.59	0.77	0.64	.99	0.61
22.2	4	2	152	.55	2.44	1.84	3.14	1.57
22.2	4	3	453	1.38	5.49	3.81	7.04	1.25
22.2	4	4	316	1.57	4.20	3.00	5.39	1.58
66.7	13.6	1	31.0	0.63	0.72	.66	0.97	0.64
66.7	13.6	2	129	1.46	2.08	1.72	2.78	1.55
66.7	13.6	3	413	1.57	4.94	3.74	6.90	1.76
66.7	13.6	4	285	1.50	3.74	2.92	5.0	1.69
111.2	4	1	28.8	0.59	0.63	.61	0.79	0.58
111.2	4	2	124	.87	1.87	1.61	2.40	1.22
111.2	4	3	295	1.02	3.53	2.86	4.53	.72
111.2	4	4	208	1.18	2.73	2.27	3.50	0.97
111.2	13.6	1	28.7	.55	0.66	0.63	.88	0.61
111.2	13.6	2	122	1.18	1.93	1.66	2.58	1.50
111.2	13.6	3	321	1.02	3.94	3.16	5.27	1.96
111.2	13.6	4	231	1.26	3.09	2.54	4.13	1.72

---

[3] Technology Service Division, SKF Industries, Inc., King of Prussia, Pa.

[4] Numbers [13–18] in brackets designate Additional References at end of discussion.

the discussant's [13] experimental correlation between contact shape and shape parameter (S) which is equal to the ratio of maximum isoviscous pressure ($q_{\max}$) to the maximum Hertzian pressure ($p_{\max}$) [14].

Many measurements of film thickness have been conducted via optical EHD rig in the discussant's laboratory using sapphire disks and having specially machined and honed 12.5 mm diameter steel rollers with crown radius of 49 mm and 330 mm yielding an axis ratio of 4.0 and 13.6 respectively. Four oils were tested i.e. (1) MIL-L-7808, (2) polyphenyl ether, (3) perfluoro ether and (4) naphthanic oil. The optically measured plateau film thickness were compared with theoretical film thickness of (1) Archard [7], (2) Kapitza modified for EHD conditions i.e. $q_{\max} = \alpha^{-1}$ [17] and (3) Cheng's formulas [8] using values of viscosity and density calculation at the temperature measured at the vicinity of the inlet and the values of pressure viscosity coefficient (α) obtained from the film thickness measurement.

Table 6 tabulates for a rolling speed of 1 m/s (40 in/s), the measured and calculated film thickness and values of $\bar{U} = \eta_0 v \alpha / R_x$ for the four oils and two axis ratios. The comparison between theoretical prediction and measured values are tolerably good. Table 7 tabulates similar data for high rolling speed of 10 m/s. It is shown that at high values of $\bar{U}$ ($>10^{-6}$), there is a significant reduction in film thickness from the calculated values due to severe inlet starvation as evidenced in the optical fringegrams. In the last column of Table 2, the results of a theoretical prediction based on a starvation theory (taking into account oil layer replenishment) in [17] and thermal reduction factor (by Cheng) are given to show better agreement between theory and experiment.

Also, it is of interest to point out that in an early report [18], the discussant found that at a constant maximum pressure, the measured film thickness at axis ratio (k) 1:4:13.6 are in the ratio 1: 1.28:1.32 as compared with 1:1.31:1.33 from the authors formula for H_c.

Additional References

13 Chiu, Y. P., and Sibley, L. B., "Contact Shape and New Newtonian Effects in Elastohydrodynamic Point Contacts," *Lubrication Engineering*, Feb. 1972.

14 Johnson, K. L., "Regimes of Elastohydrodynamic Lubrication," *Journal Mech. Engrg. Sc.*, Vol. 12, No. 1, 1970, pp. 9 16.

15 Dowson, D., and Whitaker, A. V., "The Isothermal Lubrication of Cylinders," *ASLE Trans.*, Vol. 8, No. 3, July (1965), pp 224–235.

16 McCool, J. I., et al. "Interim Technical Report on Influence of Elastohydrodynamic Lubrication on the Life and Operation of Turbine Engine Ball Bearings," *S K F Report No. AL74PO12*, submitted to AFAPL and NAPTC under AF Contract No. F33615-72-C-1467, Navy MIPR No. M62376-3-000007, July 1974.

17 Chiu, Y. P., et al, "Final Summary Report," Contract NOOO19-71-C-0425, S K F Report AL72PO1O 1972.

18 Chiu, Y. P., "Analysis and Prediction of Lubricant Film Starvation in Rolling Contact System," *ASLE Trans.*, Vol. 17, 1974, pp 22 35.

By P. M. Ku[6]

The authors have achieved, with great dexterity, a generalized isothermal solution to the EHD film pressure and film thickness problem for elliptic conjunctions—a problem of utmost importance but immense mathematical complexity. It is fitting that this work should have been associated with the name of Professor Dowson, who performed a similar feat for rectangular conjunctions just about two decades ago.

The 2-dimensional contour maps given in the Part II paper are very interesting indeed. However, the 3-dimensional renditions shown during the oral presentation, but not included in the preprint, are even more revealing. The authors might have been handicapped in this regard by the page restrictions now being placed by so many professional societies. But worthy things are always worth the cost of printing. It is suggested that these 3-dimensional plots be published in the authors' closure.

The 3-dimensional plots portray some intricate details not readily apparent from the 2-dimensional contours. For example, it appears that the pressure spike behind the main pressure peak in the central plane was more modest than that computed from the simple, exponential pressure-viscosity relationship for rectangular conjunctions. Could this be due to the use of the Roelands' expression in the work, or could it somehow be the result of the more complex flow involved? Further, for the case of an ellipticity parameter of 2, there appeared to be more than one pressure spikes in a plane parallel to the central plane. If so, what might be the reason? The authors' clarifications with respect to these details will be appreciated.

The comparison of film thicknesses in Fig. 5 of the Part II paper is interesting, but the use of curve H_A is puzzling. If H_A is the Archard-Cowking central film thickness, then it would appear more informative to compare it with the central film thickness found in this work. Also, the $H_{\min}$ in this work might properly be compared with the minimum film thickness found in some other work. Unpublished optical film thickness measurements made in this discusser's laboratory, using the ball-versus-flat arrangement with the ball size varied and at modest rolling and sliding velocities, showed that (a) the central film thickness varied approximately as that given by the Archard-Cowking formula for central film thickness, and (b) the minimum film thickness agreed almost exactly with the Westlake-Cameron formula for minimum film thickness [19].[7] The recent work of Ranger, Ettles, and Cameron showed still different relationships for central and minimum film thicknesses [20]. Of course, the bases of the cited relationships are all different, hence comparisons on strictly equal footing are difficult. Nevertheless, such comparisons may show just what the differences are and why, and lead to a better understanding of a very complex problem.

Finally, the most important ramification of this work is that a generalized solution is now available. A valuable extension to this work would appear to be solutions for the motion inclined at different angles to the ellipse, such as occurring in hypoid and spiral bevel gears. A special case of the problem is for the motion in the direction of the major axis of the ellipse, such as exists at the roller end of roller bearings. Analytical solutions of this type are still not available, and experimental measurements are very scanty.

The authors are to be congratulated for this landmark achievement. We look forward to further work for the isothermal case, and hopefully for the more difficult nonisothermal case as well.

Additional References

19 Westlake, F. J., and Cameron, A., "Interferometric Study of Point Contact Lubrication," *Elastohydrodynamic Lubrication, 1972 Symposium*, IMechE 1972.

20 Ranger, A. P., Ettles, C. M. M., and Cameron, A., "The Solution of the Point Contact Elastohydrodynamic Problem," *Proc. Roy. Soc. London*, 1975 (to appear).

Authors' Closure

The authors would like to comment on the discussion put forward by Dr. Chin. First he states that some of the results presented by the authors are in the rigid and intermediate region. The authors cannot agree with this point since great care was taken to assure that all data presented was in the elastic region. If the results for the high ellipticity ratio were in the rigid region they would not have agreed as well with the Dowson-Higginson line contact results. The approach used to make sure that the elastic region was maintained for all the ellipticity results is the following. Fig. 6 is obtained from Dowson-Higginson (reference [10]). Here we see for $G = 5000$ (which is very close to the

---

[6] Southwest Research Institute, San Antonio, Texas.

[7] Numbers in brackets designate Additional References at end of discussion.

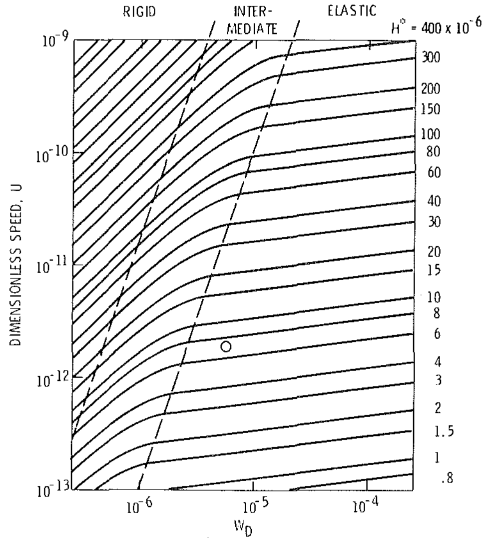

Fig. 6 Film thickness contours $[H=h_{min}/R_x)$ for $G = 5000$

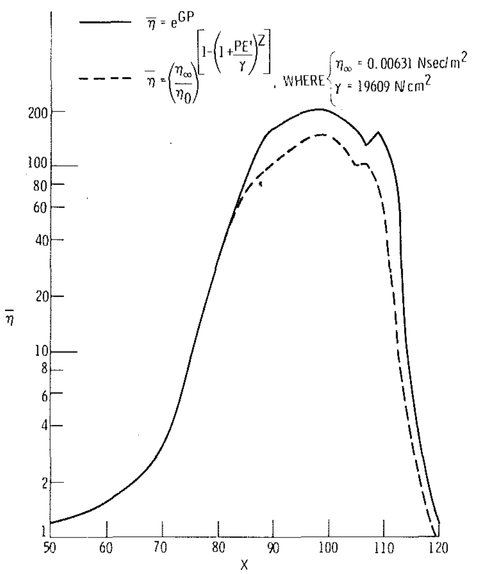

Fig. 7 Effect of two different viscosity formulas

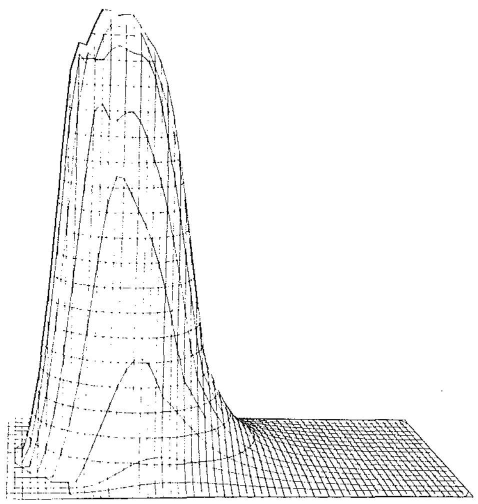

Fig. 8 Three-dimensional pressure contour for ellipticity parameter of six ($k = 6$)

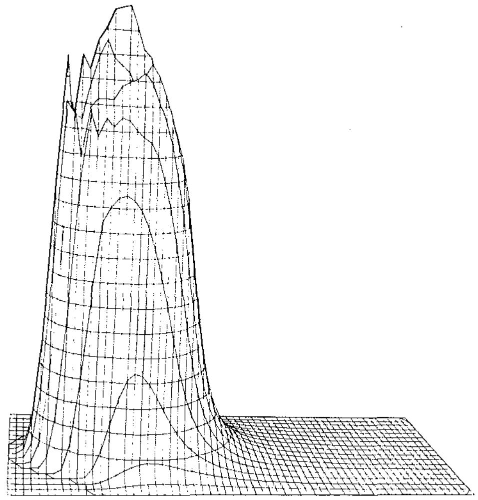

Fig. 9 Three-dimensional pressure contour for ellipticity parameter of two ($k = 2$)

case the authors have considered) and for various values of U and W_D the elastic, intermediate, and rigid regions are defined. The U used in Dowson-Higginson is exactly that used in this paper. The value of W_D used in this figure differs from that used in this paper and a tie between the two needs to be developed. The dimensionless load parameter as defined by Dowson-Higginson is

$$W_D = \frac{\bar{F}}{E'R_x}$$

where

$$F = \frac{\text{Force}}{\text{Unit length}}$$

Therefore in relating this to a point contact the dimensionless load parameter can be written as

$$W_D = \frac{F}{2aE'R_x}$$

Using this expression for $k = 8$ one finds that for the input conditions given in the paper the location of U and W_D is shown by a circle in Fig. 6. As can be seen, this is well into the elastic region. This proves that for the input conditions given in the paper for an elliptic parameter of eight ($k = 8$) the elastic region is maintained. For values of k less than eight the results move further into the elastic region.

The authors were most interested to see the close agreement between the ratios of Dr. Chin's measured film thickness at various ellipticity ratios and the predictions presented in the present paper.

The authors would like to thank Dr. Ku for his discussion to their paper. Dr. Ku has presented several interesting points. The first questions he asks is what would be the effect of using Roelands' formula for defining the effect of pressure on viscosity instead of the exponential pressure-viscosity relationship. For the given case of $k = 6$ the only change to the computer program was to use the exponential pressure-viscosity relationship instead of Roelands formula and observe the difference. Fig 7 shows the viscosity along the X axis while holding Y fixed near the axial center of the contact. From this figure it is seen that the pressure spike is somewhat suressed when using Roelands formula. Furthermore, the maximum pressure is higher using the exponential pressure-viscosity relationship.

The second point concerns the 3-D contour plots presented, however, not in the original text. These contour plots are shown in Fig 8 and 9. In Fig. 8 the ellipticity parameter is six ($k = 6$) and just the start of a pressure spike is seen to emerge. In Fig. 9 the ellipticity parameter is two and the pressure spike is fully developed. At this time it is not clear to the authors why the two spkes occur at the back end of the contact.