
Reviewed by H. Saunders

The field of Boundary element (BE) continues to grow and now it has reached maturity. After the birth of integral equations, BE languished in the research laboratory for approximately 50 years. The advent of the fast digital computer gave it nourishment and impetus to grow. This opened up a wide avenue since the implementations of the discretization process arithmetically permitted numerical solution of suitable accuracy to be executed. This furnished greater impetus in developing new and improved boundary integral formulations. Beginning with the uniform potential flow over the surface, the formulations were able to compute the torsional rigidity and boundary shear stress for complicated cross sections. This continued into problem solving of electrostatic capacitance, biharmonic, and main boundary value problems of plane and linear elastostatics. In later years the terminology used in BE points out that there is a very common lifeline with finite elements (FE). As stated by the editor, "The emphasis will be on contributions which are self-contained and explain a different topic in sufficient detail for the analytical engineer or scientist to be able to understand the theory, in due course to write the relevant computer software. Another objective is to report work for direct application by the practicing engineer." Volume 1 of this series initiates a comprehensive investigation into present and future aspects of BE.

The book consists of eleven chapters, each written by a specialist in a particular field.

The initial chapter designated provides an approach based on the weighted residual and error approximations. This allows easy make-up of the governing boundary integral equations. It covers fundamentals of functional analysis, generalized Gauss' formula, and weighted residual schemes. It concludes with boundary integral formulation of Poisson and Navier's equations. The weighted integral becomes important when new problems arise where an integral statement is not at hand. The problems furnish a simple and very apt way of introducing BE method.

The next chapter, referred to as Chapter 1, reviews the theory of boundary integral equations and its evolution into the BE method. It contains Green's formula (single and double layer), boundary integral equations (single and double layers, direct formulations), Somigliana's formula and indirect vector formulations, i.e., Kupradze and Rizzo. The concluding section deals with logarithms, Q contours, 2D Green's formula, and plane elastostatics.

With the initial information on boundary elements under our belt, Chapter 2 covers applications in transient heat conduction. This includes integral formulation of heat conduction problems and the numerical solution of the integral equation, i.e., discretization and analytical and numerical techniques. The last section refers to time-marching procedures (I and II). The advantage of time-marching is the ability to use larger time steps than those employed in FE and finite differences; this permits good accuracy.

Chapter 3 is unique since it relates fracture mechanics in the thermoelastic state. Beginning with the integral equation formulation, it progresses to the transient heat condition, thermoelastic and stress intensity factor (modes I and II) computations. Examples are furnished of a rectangular plate with oblique edge crack and center crack with each subjected to heating shock and subregions which employ the block elimination procedure.

Chapter 4 deals with the application of BE method to fluid mechanics. Beginning with the history of the use of BE to fluid mechanics, this continues with references to aerodynamics and hydromechanics, porous media flow using Darcy's law and free boundary problems (flow over spillway and Bernoulli's equation). Continuing, we next encounter unsteady free boundaries (water waves), linear waves, and Stokes flow. BE method, once thought to be employed only in linear homogeneous equations with constant coefficients, can be applied to the more complicated problems. Its greatest asset is its computational efficiency plus additional advantages.

Chapter 5 continues with water wave analysis. Starting with the governing equations, it progresses to boundary element formulations via matrices plus special structural types, i.e., vertically integrated structures with vertical axes or plane of symmetry and rotational symmetry. These are all 3D problems that can be reduced to 2D, since each of the aforementioned can be integrated with respect to depth. The final section of this chapter presents well-known equations of motion of structure and illustrative examples of half and fully submerged cylinders, moored floating cylinders, elliptical cylinder and surface waves on a floating bottom.

The next chapter covers interelement continuity in BE method. Beginning with the boundary equation for continuous elements, we progress ahead to discretizations, modeling difficulties, and planar discontinuous elements. This leads directly to discontinuous elements for three-dimensional elements. The latter may be considered as partially discontinuous elements, single-edge discontinuity and two adjacent edge discontinuities. Examples are presented for steady state heat conduction in a bar and short cylinder. Caution must be used since discontinuities should be confined to geometric singularities (corner) and discontinuous or change type boundary conditions.

Chapter 7 discusses geomechanics with applications to circular tunnels excavated between two seams on a weak medium, discontinuity problems in tunnels, viscoplasticity of sand and rock structural systems. Chapter 8 continues with application to mining. Starting with the BE formulation, this
forges ahead into elastic-plastic natural behavior and combination of BE and FE. These chapters are most interesting and should be read by those interested in plasticity.

Chapter 9 deals with finite deflections of plates. Until previously most nonlinear problems pertaining to plates used simplified nonlinear differential equations. Examples of this are Berger's equation, von Karman's nonlinear differential equation for finite deflection of plates, shells, and others. Now, BE method becomes a very effective tool in dealing with the analysis for nonlinear as well as linear problems. Initiating the subject, the authors review the well-known Kirchhoff-Love equations referred to in Cartesian coordinates and von Karman equations. The integral equation for boundary elements are now the starting point. After casting the von Karman equations in matrix form, an iterative process is employed in integrating them numerically. Berger's equation neglects, without justification, the second invariant of the membrane stress in the potential energy expression for laterally loaded, homogeneous, isotropic thin plates. Nevertheless, this equation has been used extensively by investigators in dynamic and static problems for shallow shells, sandwich plates, and others. The Berger equations are then formulated in terms of integral equations and is thus available for numerical computation. The resulting numerical scheme and system of simultaneous equations are similar to those applied in linear bending. Proceeding from plates, we journey to nonlinear shell and sandwich plate/shell problems. Based upon the foregoing analysis, this opens up a new avenue in deriving integral formulations in studies other than Berger and von Karman field equations. This chapter comes to the point and should be read by those interested in plate and shell analysis.

The last chapter points out the advantage of using the Trefftz method cast in boundary integral equations. The prime purpose of his method is the investigation of approximate solutions obtained from the appropriate class of functions which satisfy exactly the differential equation but may not exactly satisfy the prescribed boundary conditions. The author develops the necessary recipes for the BE method and continues ahead to Green's formulas. Illustrations are provided using the latter in both the continuous and discontinuous fields. The chapter continues with the derivation of Laplace and reduced wave equation in both two and three dimensions on the T-complete system. The concluding section briefly mentions the Hilbert space formulation. This is a very good chapter and must be read carefully due to its mathematical content.

In summary, this is an excellent book on BE methods. Although not an elementary text, it does furnish a good understanding of the BE method. The reviewer would have preferred seeing chapters on BE method applied to a greater depth in acoustics and dynamics. Perhaps, this will be covered in later volumes. The reviewer recommends this book to those interested in the BE method.


This book is not meant for the "faint-hearted." The authors have written a book which proves in a methodical fashion the mathematical foundations of the finite element (FE) method. It is one of six volumes in this series. The book is essentially a mathematical text with very few illustrative examples. Two of the other volumes deal with solid mechanics and fluid mechanics. As stated by the authors, "The treatment is by no means comprehensive; rather the aim is to present only the basic mathematical properties of FE needed to understand how these methods work and how their convergence and stability properties can be established for linear elliptical properties. The subject presented here summarizes some of the important developments in the mathematical theory of FE completed since the 1970s." The authors accomplish this task but at times require extensive concentration and patience.

The book consists of five chapters.

Chapter 1 introduces the subject and shows how the modern theory of elliptic boundary-value problems (BVP) and the related FE approximations are based on the classical concept of variations. The authors begin with Banach spaces and continue to Sobolev spaces, Sobolov embedding theorem plus variational and approximation of variational BVP. Chapter 2 introduces reader to the finite element interpolation theory. The FE methods shows properties similar to the popular curve-fitting techniques and generally represents the function as a polynomial in the same sense as the Lagrange and Hermite interpolation methods. The interpolation of the functions is derived in Sobolev space. Some general properties of FE are considered as to assembly, global degrees of freedom, P-unisolvent elements, i.e., a unique function containing polynomials of degree P. The latter contains the Bogner-Fox-Schmidt rectangular element and the Hermite elements. Next, the affine family is considered where two finite elements of the same type are equivalent and can be mapped one on the other. The chapter concludes with the interpolation theory in Sobolev spaces. This is good chapter that requires intensive concentration.

Chapter 3 focuses on the FE approximation of elliptic BVP. This centers around the idea of sharpening up the rate of convergence of standard FE methods as the mesh parameter tends to zero. Beginning with standard error estimates, this leads to the question of how quasi-optimal estimation can be performed in lower order norms (Sobolev spaces.) Examples are furnished of the application of Laplace and biharmonic equations for elliptical BVP.

Stepping from the standard FE approximation of elliptical BVP, one investigates a class of "nonstandard" FE methods, i.e., mixed methods. The latter are important since they can be utilized in providing excellent examples of how the general approximate results (Chapter 1) and interpolation theory (Chapter 2) can be applied in studying the various types of FE methods. Mixed element methods have important applications in treating incompressible fluids, solids, and contact problems in elasticity. These are subjects of other volumes in this series. Moreover, FE methods are looked upon as an approximation theory for linear variational BVP problems with constraints. The saddle point problems and existence theory are some of the ideas of constrained minimization. They and Lagrange multipliers play important roles in the mixed methods. Our attention is next directed toward the approximation and error estimates of constrained problems. Then, the important Bakuška-Brezzi conditions are expanded upon. Examples in incompressible elasticity and fluid flow are next considered in the C' (conforming) approximations. The chapter concludes with a mixed method for fourth-order problems. This avoids the use of higher-order conforming FE.

The final chapter deals with another mixed method, i.e., hybrid methods. In previous chapters, the derivatives of the approximate solution must be continuous across interelement boundaries. Employing Lagrange multipliers, one can construct independent FE approximations on the interior of each element. Its various directional derivatives would be normal to the interelement boundaries. The author begins with some preliminaries, viz. partition of the domain into subdomains. The chapter continues with the hybrid variational principle. This sets the stage for the hybrid FE method. The chapter concludes with implementation of the derived hybrid FE accompanied by error estimates plus the stability of the hybrid methods.

As mentioned in the introduction, strict concentration is re-