Dynamic optimal control for groundwater optimization management with covariates

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ABSTRACT

It is well known that obtaining optimal solutions for groundwater management models with covariates is a challenging task, especially for dynamic planning and management. Here, a theory and method of dealing with mutual-feed joint variation in groundwater management models is described. Specifically, an equation expressing the inherent connection between covariates and groundwater level was developed. This equation was integrated into a mathematical simulation model of groundwater, after which a groundwater dynamic optimization management model with covariates was constructed using the state transition equation method and solved with differential dynamic programming algorithms. Finally, the above theory and method were applied to a hypothetical groundwater system. For the same groundwater system, a groundwater management model with covariates was developed and the results of the two optimization methods were found to be nearly identical, which validated the theory and methods put forth here.

Key words | covariate, differential dynamic programming, groundwater management model, mutual-feed joint variation

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INTRODUCTION

In groundwater dynamic prediction, the groundwater level is solved based on specified boundary conditions, discharge, precipitation, evaporation and irrigation. Most groundwater dynamic models that have been developed to date provide a single output from multiple inputs. However, such models cannot reflect actual systems objectively under certain conditions, such as groundwater systems with evaporation and spring discharge. As a result, increasing the groundwater level leads to increasing groundwater evaporation and spring discharge, while dropping the groundwater level leads to the spring discharge and groundwater evaporation decreasing or disappearing (Zhang 1992). During numerical simulation and optimal management of the groundwater system, factors describing the exchange capacity of the system and the surrounding environment are referred to as covariates, such as the exchange between surface water and groundwater, spring discharge, groundwater discharge to drain and evapotranspiration. The characteristics of covariates are as follows: their amounts are dependent on the state variable of the groundwater system, i.e. groundwater level; the covariates and groundwater level interact with each other. In other words, the level of groundwater determines the amount of covariates, which in turn influence the level of groundwater through consumption of groundwater storage within the groundwater system. In addition, amounts of covariates, such as spring discharge, groundwater discharge to drain and evapotranspiration, become zero when the groundwater level declines below a specified reference elevation. This particularity makes the treatment method in the simulation model and management model of the groundwater system different from artificial pumping or recharge. The interactions among artificial pumping or recharge, groundwater level and the covariates are referred to as a relation of mutual-feed joint variation.

In a groundwater simulation model with covariates, the above problem has been solved by iteration and the Cauchy boundary condition (de Lange 1999). In addition, some software programs have the ability to solve this problem.

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However, in groundwater management models with covariates, the artificial pumping or recharge, groundwater level and covariates are all unknown during the prediction stage. Moreover, these three variables inherently affect one another, resulting in management models that are more difficult to solve. Lu (1994) discussed the optimization management of a karst groundwater system with a large karst spring and made an initial attempt to solve the mutual-feed joint variation by increasing the decision variance and constraining the equation; however, this model is not generalized for all variables. Li (2008) established a groundwater management model with covariates to different variables, which provided an ideal solution for the problem of mutual-feed joint variation. It should be noted that Li constructed the groundwater linear programming management model with covariates using the response matrix method.

In essence, groundwater systems are complex dynamic systems. The natural conditions and human activities are variable, especially when the groundwater will be exploited over the long term as a water supply resource. Thus, groundwater resource management models must be amended over time to ensure their reliability and accuracy (Shu & Wang 2005). Research regarding this topic can be found in Yakowitz (1982), Georgakakos & Vlatsa (1991), Lee & Kitanidis (1991) and Andricevic (1993). However, the currently available dynamic programming methods are not perfect, and dimension disaster occurs under high dimension conditions. At present, the method that is most commonly used in groundwater resources dynamic programming management models is differential dynamic programming (Jacobson & Mayne 1970). This method is an advanced algorithm in multi-dimensional dynamic programming that overcomes increased computation by avoiding discrete state variables and decision variables. Accordingly, it provides a feasible analytical algorithm for large-scale groundwater resources management models, as well as those with multiple time intervals or for unstable systems (Wang & Shu 2005).

Jones *et al.* (1987) used the Constrained Differential Dynamic Programming algorithm (CDDP) for unsteady, nonlinear groundwater management problems. Due to the stage-wise decomposition of the CDDP, the dimensionality problems associated with embedding the hydraulic equations in the management model as constraints were significantly reduced. Chang *et al.* (1992) applied CDDP to optimize the remediation design of time-varying pumping rates while considering only the operational cost. Culver & Shoemaker (1992) found that time-varying policies were more cost-effective than time-invariant policies. The CDDP used herein is a modification of SALQR that has been shown to be efficient in solving time-varying problems. Culver & Shoemaker (1993) presented a quasi-Newton differential dynamic programming approach that can further reduce the computational effort associated with the dynamic management problem. Chang & Hsiao (2002) integrated CDDP and the Genetic Algorithm (GA) to overcome the problem of simultaneously considering both the fixed costs of well installation and the operating costs of time-varying pumping rates. Chang *et al.* (2007) integrated GA and CDDP to design a pump–treat–inject system. The proposed model considered both the cost of installing wells (fixed cost) and the operating cost of pumping, injection and water treatment. Chu & Chang (2009a) applied the ANFIS and CDDP approaches to remediation design. Chu & Chang (2009b) also integrated an artificial neural network (ANN) and CDDP as a simulation–optimization model. The model they developed saved a considerable amount of computational time when solving large-scale problems. Bauser *et al.* (2010) presented a dynamic optimal control approach for the management of drinking water well fields, which handles the problem online, in real time and in a real-world case study.

Recently, there have been many theoretical studies conducted to evaluate the relation of mutual-feed joint variation in the groundwater management model. Before this study is developed, the embedding method and response matrix have been used in building a groundwater management model with covariates and the problem has been successfully solved. However, documents about the treatment of a groundwater dynamic programming management model with covariates and treatment of a groundwater system management problem using the theory and methodology of the relation of mutual-feed joint variation have not been published, and this is the problem that this paper wants to solve.

Here, a dynamic optimization groundwater management model containing covariates is described, which is solved by the dynamic differential programming method. The theoretical method developed here was then applied to a hypothetical groundwater system. Additionally, to confirm the results of the theoretical model, we composed an optimization groundwater management model that combined the covariates using the embedding method and was solved using the Lingo software (Don *et al.* 2006; Xie & Xue 2005). The model developed here can provide a technical method of solving the mutual-feed joint variation in actual groundwater systems.

THE MATHEMATICAL EXPRESSION OF COVARIATES

Spring discharge

We can consider the spring discharge to be proportional to the difference between the groundwater level and the elevation of the spring:

$$Q_s = \begin{cases} C_s(h - h_s) & \text{if } h > h_s \\ 0 & \text{if } h \leq h_s \end{cases} \quad (1)$$

where Q_s is the spring flow (m^3/d), C_s is the scale factor (m^2/d), identified by the field data of the spring flow measurement, h is the groundwater level (m) and h_s is the surface elevation of the spring (m).

The amount of river and groundwater exchanged

The formula for the exchange of river water and groundwater is

$$Q_r = \begin{cases} C_r(h - h_r) & \text{if } h > Z,\ h > h_r \\ C_r(h_r - h) & \text{if } h > Z,\ h < h_r \\ C_r(h_r - Z) & \text{if } h \leq Z \end{cases} \quad (2)$$

where Q_r is the exchange flux between river and groundwater (m^3/d), C_r is the leakage coefficient (m^2/d), h_r is the water level of the river (m), h is the groundwater level (m) and Z is the elevation of the river bottom (m).

Evaporation

Evaporation of phreatic water and the groundwater level are negatively correlated. When the groundwater level reaches the surface, evaporation reaches the maximum; therefore, when the groundwater level is equal to or lower than the evaporation limit of the water level (the groundwater level at which evaporation is zero), the evaporation is zero.

The relationship between evaporation and the groundwater level is given by

$$Q_e = \begin{cases} Q_m & \text{if } h \geq Z \\ Q_m \dfrac{h - h_0}{Z - h_0} & \text{if } h_0 < h < Z \\ 0 & \text{if } h \leq h_0 \end{cases} \quad (3)$$

where Q_e is the evaporation of phreatic water (m^3/d), Q_m is the maximum evaporation of phreatic water (m^3/d), h is the groundwater level (m), Z is the surface elevation (m) and h_0 is the limit level of phreatic evaporation (m).

Drain excretion

The relationship between drain excretion and the groundwater level (Anderson & Woessner 1992) is as follows:

$$Q_d = \begin{cases} C_d(h - Z) & \text{if } h > Z \\ 0 & \text{if } h \leq Z \end{cases} \quad (4)$$

where Q_d is the amount of drain excretion (m^3/d), C_d is the hydraulic conductivity in drains and backfill materials (m^2/d), h is the groundwater level (m) and Z is the elevation of the drain bottom (m).

GROUNDWATER SIMULATION MODEL WITH COVARIATES

A heterogeneous, isotropic, two-dimensional unsteady unconfined aquifer groundwater flow system was taken as an example to discuss the establishment of a groundwater simulation model that contains covariates and its solution. Assuming the boundaries of the system were the head-dependent flow boundary and no-flow boundary. Assuming the change in the water level was small or even negligible when compared with the thickness of the aquifer, so the aquifer can be calculated by the method of an artesian water aquifer. Under these conditions, the groundwater simulation model with covariates is

$$\begin{cases} \frac{\partial}{\partial x}\left(T\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(T\frac{\partial h}{\partial y}\right) & \\ \quad +\varepsilon - Q - P = \mu\frac{\partial h}{\partial t} & (x,y)\in D,\ t\geq 0 \\ h(x,y,t)|_{t=0} = h_0(x,y) & (x,y)\in D \\ h(x,y,t)|_{\Gamma_1} = h_1(x,y,t) & (x,y)\in \Gamma_1,\ t\geq 0 \\ \left.\frac{\partial h}{\partial \bar{n}}\right|_{\Gamma_2} = 0 & (x,y)\in \Gamma_2,\ t\geq 0 \end{cases} \tag{5}$$

where D is the simulated area of the groundwater system, (x, y) is the plane coordinate (m), t is time (d), h is the groundwater level (m), T is the transmission coefficient (m^2/d), μ is the specific yield, Γ_1 is the head-dependent flow boundary, Γ_1 is the no-flow boundary, $\bar{n}$ is the outer normal direction of the boundary, $h_0(x, y)$ is the initial groundwater level (m), $h_1(x, y, t)$ is the water level of the head-dependent flow boundary (m), ε is the uncontrollable input variable (m^3/d) (for example, rainfall), P is the extraction amount of groundwater (m^3/d) and Q is the covariate (m^3/d).

Equation (5) is solved by the method of finite difference (Xue & Xie 2007) in this study. The partial differential equation in the prediction model is a linear partial differential equation. The equation deals with the implicit difference method of discretization and provides algebraic equations in the method of finite differences:

$$\left(G+\frac{S}{\Delta t}\right)h_{k+1} = W - P - Q + \frac{S}{\Delta t}h_k \tag{6}$$

where G is the hydraulic conductivity matrix, h is the vector of unknown nodal hydraulic heads, S is the storage matrix, P is the vector of groundwater exploitation, Q is the column vector of the covariate and W is the amount of recharge and discharge, or the boundary column vector.

THE STATE TRANSITION EQUATION OF THE GROUNDWATER SYSTEM

The water level h_{k+1} is regarded as a function of the initial water level h_k and the artificial input variable p_k in the state transition equation of the groundwater system (Lu 1999). Assuming the number of planning periods k is N $(k = 1, 2, \ldots, N)$, to reduce calculation truncation error in any planning period, k is divided into n time periods, assuming that each period has equal size. The $(\tau, \tau + 1)$ period in the planning period k is

$$\left(G+\frac{S}{\Delta t}\right)h_{\tau+1} = \frac{S}{\Delta t}h_\tau + W_\tau - P_\tau - C(h_{\tau+1} - Z) \tag{7}$$

where C is the coefficient related to covariates. To consolidate the coefficient in the equation above:

$$Ah_{\tau+1} = Bh_\tau - P_\tau + E_\tau$$

where

$$A = G + \frac{S}{\Delta t} + C, \quad B = \frac{S}{\Delta t}, \quad E_\tau = W_\tau + CZ$$

Then, both sides of the equation are multiplied by A^{-1} to give

$$h_{\tau+1} = M_\tau h_\tau + N_\tau P_\tau + f_\tau \tag{8}$$

Equation (8) is called the state transition equation in period $(\tau, \tau + 1)$ in n time periods, substituting the water head into the next time water head equation one by one, and we get the state transition equation of the planning period k to $k + 1$:

$$h_{k+1} = F_k h_k + V_k P_k + J_k \tag{9}$$

where

$$F_k = M_n M_{n-1} \cdots M_2 M_1$$

$$V_k = N_n + M_n N_{n-1} + \cdots + M_n M_{n-1} \cdots M_3 M_2 N_1$$

$$J_k = f_n + M_n f_{n-1} + \cdots + M_n M_{n-1} \cdots M_3 M_2 f_1$$

The average size of the covariate in each time period can be obtained by substituting $\bar{h}$ from the following formula into Equations (1)–(4):

$$\bar{h} = \sum_{\eta=1}^{T} [h_\eta(0) + h_\eta(t)]/2 \tag{10}$$

where $h(0)$ is the initial water table of the time unit and $h(t)$ is the water table at the end of the time unit.

GROUNDWATER DYNAMIC OPTIMIZATION MANAGEMENT MODEL WITH COVARIATES

The pumping costs of groundwater not only reflect pumping, but also the pumping head (depending on the water table depth). The groundwater management model is established based on the state transition equation of the groundwater system. In this equation, the groundwater level and water demand are considered to be constraint conditions, which enables the establishment of the groundwater management model as follows:

$$\text{Min } Y = \sum_{k=1}^{N}\sum_{i=1}^{M} C_i P_{ik}(hl_i - h_{k+1}) \tag{11}$$

s.t.:

$$h_{k+1} = F_k h_k + V_k P_k + J_k \tag{12}$$

$$\begin{cases} h(j, k+1) \geq h_{\min} & k = 1, 2, \ldots, N \quad j = 1, 2, \ldots, \phi & (13) \\ \sum\limits_{i=1}^{M} P_{ik} \geq w_k & k = 1, 2, \ldots, N & (14) \end{cases}$$

where Y is the total pumping cost (CNY), N is the number of planning periods, M is the unit with an exploiting well, C_i is the cost per unit volume of water that was promoted by a unit height (CNY/m^4), P_{ik} is the pumping during period k in the ith unit (m^3/a), hl_i is the surface elevation in the ith unit (m), h_{k+1} is the pumping water level in the early period $(k+1)$ (m), h_k is the pumping water level during the early period k(m), $h \leq (j,\ k+1)$ is the water level at the control point for the period $k+1$(m), ϕ is the amount of water level control points and w_k is the total water demand of each unit in the period k in the study area. Equation (11) is the objective function expression, and the objective function is the minimization of the pumping costs; Equation (12) is the state transition equation of the groundwater flow system with covariates; Equation (13) are the constraints of the groundwater level and Equation (14) are the constraints of the groundwater amounts.

Substituting the state transition Equation (12) into Equation (11), we obtain a dynamic optimal model and the management model is given as follows:

$$\text{Min } Y = \sum_{k=1}^{N}\sum_{i=1}^{M} C_i(-V_k P_{ik}^2 + P_{ik}(hl_i - F_k h_k - J_k)) \tag{15}$$

s.t.:

$$\begin{cases} h(j, k+1) \geq h_{\min} & k = 1, 2, \ldots, N, \quad j = 1, 2, \ldots, \phi & (16) \\ \sum\limits_{i=1}^{M} P_{ik} \geq w_k & k = 1, 2, \ldots, N & (17) \end{cases}$$

A differential dynamic programming algorithm is used to solve the linear constrained quadratic optimal control problem mentioned above. The entire calculation procedure is divided into three processes including a reverse recursive process, a sequential decision-making process and an iterative process (Jones *et al.* 1987).

Step 1: Reverse recursive process

Based on the basic idea of dynamic programming, to solve the planning problem from the last period to the initial period, that is from period n to the first period, we calculated the expression of the optimal control P^* and the corresponding objective function expression f for each time. Then, substituting pumping amount values (known as the virtual control variable) into Equation (12) gives a set of solutions for the corresponding water table known as virtual state variables. The optimal control in the nth period is P_N^* and the optimal target expression in the nth period is $f_N(\bar{h}_N)$. Then you do double counting in this order, until you reach the first period.

Step 2: Sequential decision-making process

The sequential decision-making process is exactly the opposite of the reverse recursive process described above, being calculated from the first period to the nth period. Because $\bar{h}_1$ (initial condition) is known, substituting it into the expression P_1^* and the optimal target expression $f_1(\bar{h}_1)$, we obtain the value of the sequence decision-making process for the first period. Then solve the expressions in the period of $k = 2, 3, \ldots, N$ according to the above steps.

Step 3: The iterative process

Because $\bar{h}$ is an approximation of h, P^* only approximates the original problem. Therefore, it is necessary to use the iterative solution to account for errors. The errors will become smaller and smaller during the iteration process, indicating that the objective function will converge to a stable value, which is considered to be the end of the calculation.

HYPOTHETICAL GROUNDWATER SYSTEM WITH COVARIATES

Suppose there is a rectangular unconfined aquifer with an area of $16 \times 10\ \text{km}^2$. The east, south and north side are impermeable, while the west side has a fixed river water level with an average water level equal to $H_0 = 35$ m.

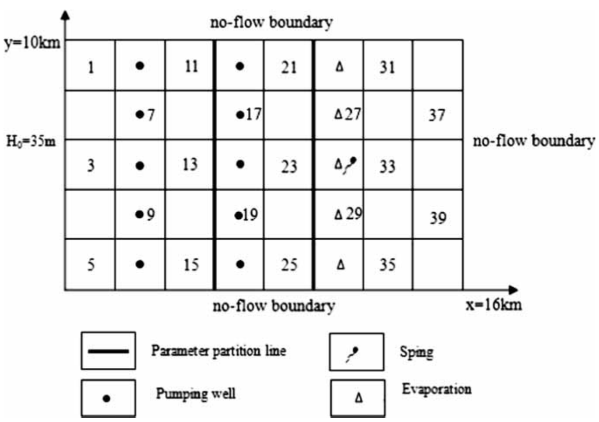

Figure 1 | Finite difference mesh, boundary conditions, locations of pumping wells and covariates in the hypothetical area.

Assuming the aquifer is very thick and the change in the water level is small, the transmissibility for the three zones is $T_1 = 1{,}000\ \text{m}^2/\text{d}$, $T_2 = 900\ \text{m}^2/\text{d}$ and $T_3 = 800\ \text{m}^2/\text{d}$, and the specific yield for all zones is $\mu = 0.3$. There is a uniform distribution of recharge in the aquifer, which is $N = 120$ mm/yr, and it discharges to the river. There are spring groups in the study area and strong evaporation in some areas (Figure 1). The maximum evaporation rate of phreatic water is $Q_m = 3{,}000\ \text{m}^3/\text{d}$ in unit 28 and the limit depth of the phreatic evaporation is $d = 4$ m. According to the planning requirements and taking into account the characteristics of the aquifer shape and the distribution of pumping wells, the aquifer is divided into 45 units ($\Delta x = \Delta y = 2{,}000$ m). Pumping must be conducted in 6–10 or 16–20 units. The surface elevation and the coefficient of pumping costs of the pumping units are shown in Table 1. The planning period is two years, divided into two management periods, which are each divided into two time periods ($\Delta t = 182.5$ d). Under these conditions, the total pumping demand in the first year is $P_1 = 6 \times 10^6\ \text{m}^3/\text{yr}$, while it is $P_2 = 7 \times 10^6\ \text{m}^3/\text{yr}$ in the second year. Assuming the river has been polluted, in order to avoid groundwater contamination, the groundwater level must be higher than 37 m at 1 km, and higher than 38.5 m at 3 km. The objective was to obtain the minimum pumping costs while meeting the required water level and pumping demand in this example.

Construction of the management model

The conceptual model is a heterogeneous, isotropic, two-dimensional, unsteady unconfined aquifer, whose covariates are evapotranspiration and spring discharge. The decision variable is the pumping amount, while the state variable is the groundwater level. The minimization of the objective function gives the lowest pumping cost, given the constraints. Therefore, the groundwater dynamic optimization

Table 1 | Surface elevation and coefficient of pumping costs for the pumping unit

Pumping unit	6	7	8	9	10	16	17	18	19	20
Surface elevation (m)	56	59	56	57	59	71	69	70	69	71
Coefficient of pumping costs (CNY/m^4)	0.020	0.025	0.023	0.026	0.022	0.020	0.022	0.021	0.024	0.023

management model (Model 1) is as follows:

$$\text{Min } Y = \sum_{k=1}^{2}\left[\sum_{i=6}^{10} C_i P_{ik}(hl_i - h_{k+1}) + \sum_{i=16}^{20} C_i P_{ik}(hl_i - h_{k+1})\right] \tag{18}$$

s.t.:

$$h_{k+1} = F_k h_k + V_k P_k + J_k \tag{19}$$

$$\begin{cases} h(j,k+1) \geq 37 \quad k=1,2, \quad j=1,2,3,4,5 & (20)\\ h(j,k+1) \geq 38.5 \quad k=1,2, \quad j=6,7,8,9,10 & (21)\\ \sum_{i=6}^{10} P_{i2} \geq 6\times 10^6 & (22)\\ \sum_{i=16}^{20} P_{i2} \geq 7\times 10^6 & (23)\\ P_{ik} \geq 0 \quad i=6,7,\ldots,10;\ 16,17,\ldots,20, \quad k=1,2 & (24) \end{cases}$$

The symbols are identical to those described for Equations (11)–(14).

The embedding method considers the relationship between the covariate and groundwater level, and the groundwater simulation model that describes the inherent relationship was then added to the optimization model as an equality constraint. Together with the other constraints and the objective functions these models constitute the management model of the groundwater system with covariates. Therefore, this method ensures the correctness of the management model.

Equation (6) is the conversion form of the groundwater simulation model with covariates, so we replace Equation (19) with Equation (6) in Model 1, embed it into the optimization model in the form of a constraint, and together with other constraint Equations (20)–(24) and the objective function Equation (18), this constitutes the groundwater nonlinear programming management model with covariates, which is called Model 2, solved using the Lingo software.

Solution results

The model developed here could efficiently deal with the problem of mutual-feed joint variation in groundwater management models. Substituting the given data into the two management models gives an optimum pumping cost of 1803 597 CNY for optimization management Model 1 and 1803 593 CNY for optimization management Model 2. The distribution of the optimal pumping amounts, the groundwater level at all control points and covariates after optimal exploitation are shown in Table 2.

Table 2 shows that the total pumping amounts meet the planning requirements for all planning times, but that the distribution of the optimum pumping amounts differs according to the coefficients of pumping costs. Additionally, the results produced are similar to those obtained from the

Table 2 | Optimal pumping amounts in pumping unit (units: 10^6 m^3/yr)

Pumping unit	Model 1 Planning time 1	Planning time 2	Model 2 Planning time 1	Planning time 2
6	1.006 603	1.434 528	1.006 598	1.434 524
7	0	0	0	0
8	0.206 862	0.548 754	0.206 863	0.548 752
9	0	0	0	0
10	0	0	0	0
16	0.809 956	1.012 661	0.809 954	1.012 665
17	1.801 382	1.925 394	1.801 390	1.925 389
18	0.850 487	0.850 716	0.850 486	0.850 720
19	1.324 710	1.227 947	1.324 709	1.227 950
20	0	0	0	0
Total pumping amounts	6	7	6	7

Table 3 | Groundwater level at all control points (units: m)

Water level control point	Model 1 Planning time 1	Planning time 2	Model 2 Planning time 1	Planning time 2	Water level control
1	40.208 51	40.143 00	40.208 49	40.142 97	37.0
2	40.254 30	40.061 71	40.254 26	40.061 68	37.0
3	40.259 68	40.162 06	40.259 72	40.162 10	37.0
4	40.270 73	40.150 85	40.270 68	40.150 80	37.0
5	40.274 76	40.182 31	40.274 80	40.182 27	37.0
6	49.037 34	48.805 27	49.037 31	48.805 25	38.5
7	49.411 43	49.016 53	49.411 40	49.016 50	38.5
8	49.399 20	49.387 19	49.399 18	49.387 14	38.5
9	49.481 34	49.453 34	49.481 36	49.453 40	38.5
10	49.503 75	49.483 23	49.503 73	49.483 18	38.5

Table 4 | Covariates after optimal exploitation (units: m^3/d)

Covariates	Covariate unit	Model 1 Planning time 1	Planning time 2	Model 2 Planning time 1	Planning time 2
Evapotranspiration	26	1832.535	1825.401	1832.532	1825.397
	27	2348.606	2119.388	2348.610	2119.392
	28	3000	3000	3000	3000
	29	2296.088	1983.998	2296.086	1983.989
	30	1600.861	1377.833	1600.859	1377.835
Spring discharge	28	1534.856	1040.736	1534.858	1040.741

optimization groundwater management model combined with the parameter of covariates using the embedding method, although there may be some contradictions due to computational errors.

Table 3 shows that all groundwater levels at the control points meet the condition of groundwater level constraints and the groundwater level decreases in Planning time 2 in comparison with Planning time 1.

Table 4 shows the covariate value decrease in Planning time 2 in comparison to Planning time 1. This is mainly due to the groundwater exploitation which leads to a reduction in evapotranspiration and spring discharge.

CONCLUSION

Here, a method of composing and solving the dynamic optimization groundwater management model is provided and applied to a hypothetical groundwater system to obtain an optimal solution. It should be emphasized that the period and scope were very small in this example, therefore the calculation results of the two models are basically identical. However, actual groundwater management systems are often large scale and involve multiple periods; then the advantage of DDP for solving large-scale, multi-period, dynamic, real groundwater systems would be more prominent.

The establishment of the theory and methods dealing with the mutual-feed joint variation in groundwater management models can enhance the degree of approximation of the management model and improve the capacity of the groundwater model for coping with actual problems, as well as provide a foundation for studies in which models of coupling groundwater systems with the surrounding environment are employed. Therefore, this study has important theoretical significance and broad applicability. Due to time constraints, the model developed here was not tested on an actual groundwater system; however, this will be the topic of our next study.

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