Sample numbers indicate durometer (Shore A hardness).

VII Conclusion

In summary these following points are emphasized.

1 The method of error analysis has been so developed to give the upper limit of error; signs were adjusted to produce a totally additive quantity. Statistically, of course, a lesser value exists in actual operation.

In all cases reported on, repeated pressurized runs were made, carefully observing pressure and mercury levels as well as possible hysteresis effects. As an example of one of the precautions taken, at each imposed pressure step the final value was arrived at from a slightly greater one so as to repeatedly guarantee the same meniscus shape (otherwise a meniscus with a distortion in curvature can result). With these procedures no discernible difference in the data could be found and repeatability was excellent.

For the particular case of the nylon sample tested it is believed that the final error $\delta \sigma$ was significantly less than the maximum value established in the foregoing for these reasons: (a) the repetitive nature of executing all aspects of the procedure; (b) the care that was attempted; (c) further numerical substitutions; and (d) bolstered by the overall repeatability of the data, it is believed that the actual value ($0.417$) was determined to within $\pm 0.001$. Similarly, in the case of the elastomers tested it is felt that the inaccuracies experienced were comfortably within the reported limits.

2 The device as described is most appropriately used with low modulus, relatively incompressible materials such as members of the elastomer family for which it was intended. Its use can be extended to the plastics in which case a modification of gauge glass dimensions and read-out instrumentation may be implemented to improve its characteristic accuracy. Samples to be examined should have bulk moduli sufficiently low such that their volume reduction be greater than the system dilatation. They also should be free of internal voids, or fictitious lower values of $\nu$ will result.

3 The device measures bulk compressibility directly and is best suited for measuring properties of materials to be used in applications where elastic compression is dominant.

APPENDIX A

The method used in determining the modulus of elasticity in shear of elastomer samples is shown in Figs. 7 and 8. The following remarks are added for possible clarification.

1 Usually, 50 lb, in 2 lb increments was applied in each test; the mass of each weight was individually determined and accounted for.

2 The maximum displacement, $\delta$, encountered was approximately 0.050 in., resulting in a shear strain of 0.030 radian.

3 An ohmmeter was used, as shown in Fig. 7, to more accurately indicate the actual position of contact.

4 A straight line was fitted through the plot of shear stress ($S$) versus shear strain ($\theta$) and its slope determined to give $G$.

APPENDIX B

The dilatation of the gauge glass section derives from a particular solution of the plane strain problem. As required by it there is no axial strain (the section in the device is fixed at both ends) and the stresses imposed (here the uniform internal pressure) are independent of the axial coordinate. The increase in the bore area of the gauge glass is

$$\delta A_g = \pi [(a + u_r)^2 - a^2]$$

where

$$u_{r|ab} = \frac{ap}{E_g} \left[ \frac{b^2 + a^2}{b^2 - a^2 + \nu} \right]^2$$

where

$u_r$ = radial displacement

$a$ = inner radius

$b$ = outer radius

$p$ = pressure

$E_g$ = Young's modulus of the gauge glass material

$\nu$ = Poisson's ratio of the gauge glass material.

$p = 100$ psi

$a = 0.0625$ in.

$b = 1.0$ in.

$E_g = 450,000$ psi

$\nu = 0.35$

Substitution gives $\delta A_g = 0.7407 \times 10^{-4}$ in.$^2$.

Acknowledgment

Grateful acknowledgment is given to Professors Dudley D. Fuller and Vittorio Castelli of the Mechanical Engineering Department, Columbia University. Without their principal suggestions in the conceptual and following stages, and their special help and interest, the fulfillment of this task would have been particularly difficult.

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DISCUSSION

D. Dowson* and C. M. Taylor*

Mr. Rightmire is to be congratulated on significantly extending the available knowledge of the physical properties of elastomeric materials. Examination of previously published literature to obtain a value of Poisson's ratio for "rubber" results...
in predictions ranging from 0.4 to 0.5. Whilst it could not be claimed that such predictions were accurate it is perhaps surprising to find that for the range of elastomers considered in Mr. Rightmire’s paper the maximum deviation of Poisson’s ratio from 0.5 is only 0.00119 or about 0.24 percent.

Theoretically the value of Poisson’s ratio (ν) for an isotropic elastic material can be determined with a knowledge of any two of the following three properties of the material: Young’s modulus (E), the shear modulus (G), the bulk compressibility modulus (K). For the specific case of a material with a value of Poisson’s ratio very close to 0.5 the value could be determined with good accuracy from any of the following three equations:

\[ \nu = \frac{E}{2G} - 1 \]

\[ \nu = \frac{1}{2} - \frac{E}{6K} \]

\[ \nu = \frac{1}{2} - \frac{G}{2K} \]

The latter expression is effectively the first order solution (equation (6)) used by Mr. Rightmire with some further second order terms neglected. For an elastomer it can be shown that it is impractical to attempt to measure the Poisson’s ratio by determination of the Young’s modulus and the shear modulus, and of the remaining alternatives Mr. Rightmire has combined bulk compressibility measurements with shear modulus determination. Presumably an experimental technique involving measurements of bulk modulus and Young’s modulus would prove equally satisfactory if Young’s modulus were determined with the same precision as the shear modulus.

The experimental work detailed by Mr. Rightmire is admirable in its precision. It has been shown by means of an error analysis that with reasonable experimental techniques and careful data acquisition an accurate value of Poisson’s ratio for an elastomer can be determined.

In a discussion of a paper presented by the present contributors Professors Castelli and Mr. Rightmire stated that most elastomers of engineering interest had Poisson’s ratios in the range 0.47 to 0.50 and to support this quoted evidence from one of their previous publications. Mr. Rightmire’s present work has considered a wide range of elastomers with differing hardness, and only small deviations of Poisson’s ratio from 0.5 have been found. With the advent of this new work it is going to be possible to delineate an even narrower range of Poisson’s ratio for elastomeric materials? In previous experimental work Castelli, Rightmire, and Fuller have tested two neoprene rubber samples with hardness values of durometer 68.5 and durometer 31.0. Since at the time of these experiments a precise determination of the value of Poisson’s ratio for the test elastomers was not available an inverse procedure was used to predict the Poisson’s ratio by comparison of the experimental and theoretical results. The predictions were 0.497 and 0.48 for the respective durometer values 68.5 and 31.0. Has it been possible to correlate this work? It is noted that the softer neoprene sample was predicted to have the lower value of Poisson’s ratio and yet Mr. Rightmire’s present work indicates, in general, that harder elastomers will have a lower value of Poisson’s ratio.

A consequence of the work of Mr. Rightmire is that the simplified approach used by the contributors in the analysis of a “soft liner” circular plate thrust bearing is not applicable for elastomeric materials used as the liner. However for “soft” materials with a Poisson’s ratio of 0.45 or less, the simplified approach can be used and will result in a considerable saving of design time compared with a fuller analysis considering shear effects. This is confirmed by examining the results of the report by Castelli et al., and comparing the exact linear theory with the simplified approach (restrained column model).

The method of manufacture of rubber-like materials would seem to suggest that the materials can often be anisotropic. As such, the values of such quantities as Young’s modulus, the shear modulus and Poisson’s ratio would be expected to be directionally oriented. For example, values of Poisson’s ratio of 0.6 and 0.4 might be expected in orthogonal directions resulting in a mean value of about 0.5. Would Mr. Rightmire like to comment upon this? Was any anisotropy observed in the determination of the shear modulus and how would such an orientation of properties affect the design techniques for compliant surface bearings?

Authors’ Closure

The author is grateful to Professor Dowson and Mr. Taylor for their thorough and constructive discussion of the paper. The following comments are offered in answer to their questions.

The tester will make available much more accurate determinations of ν for elastomers and low-modulus plastics than have been published heretofore. An investigation of values for a greater number of these materials than was presented is contemplated by the author.

The experimental data reported in the paper by Castelli, et al., was obtained with a relatively crude bearing tester, and in view of that machine’s inherent capabilities, the prediction of Poisson’s ratio of 0.497 was good, and of 0.48 was perhaps fair. Since then a more sophisticated experimental apparatus has been designed and built by Professor Castelli and the author, and results from it are presented in their paper.

An example of the degree of correlation obtained between theory and experiment using that tester is shown in Fig. 9. Values for Poisson’s ratio (ν) and shear modulus (G) were obtained from the reported sample of Paracril 45 in the manner described above. Parenthetically, the discrepancy shown is believed to be due to temperature effects arising out of the large thermal coefficient of expansion of organic rubbers (~1.2 × 10^-4°F) and will be the subject of a forthcoming study.

The author agrees that the condition of anisotropy is probably common in rubber-like materials, the degree being a function of...
any imposed preferential direction of loading during the manufacture of the material. Although the Columbia group has not specifically investigated the presence of this condition, it is believed that the samples tested thus far are virtually isotropic, because of the method of their preparation. All elastomer samples have been compression molded from uncured stock using the laboratory facilities and no preferential loading imposed except for milling the material preparatory to curing it.

The additional conditions that should apply in using the simplified Dowson and Taylor method for materials of lower Poisson's ratio are light loads on a thin elastomer surface where the geometric extent of the bearing is determined by the rigid member.