On the Non-Leptonic Decay Process

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A reasoning for the pole approximation of the non-leptonic decay is given on the basis of the viewpoint of the one-hadron-exchange model, and revised numerical results are also given. A discussion of the $\Delta I=1/2$ rule is made.

§ 1. Introduction

One of the main problems in weak interactions is the non-leptonic decay process, particularly its $\Delta I=1/2$ rule. This process has long been a stumbling block for the unified understanding of weak interactions based on the current-current scheme. Some attempts have been proposed for the dynamical approach to the non-leptonic decay, among which the pole approximation model has brought a considerable success by its clear and simple description.\textsuperscript{1,2)

In the following we aim at giving some reasoning on this kind of attempt and a certain numeration of the non-leptonic decay, although our practical calculation contains few things essentially new.

The non-leptonic decay is inevitably subjected to the influence of strong interactions, though it is originally due to weak interactions. Therefore we first point our attention to the characteristics of strong interactions.

Since the proposal of the unitary symmetry based on the Sakata model, a novel feature of strongly interacting system is disclosed, namely the observed hadrons may be composite system consisting of more fundamental entities (Sakatons) and therefore the strong interaction in the hadron level should be traced back to the interaction of the fundamental entity. The dynamical law and the nature of interactions in the level of Sakaton are not yet clear, but a certain indication seems to have been obtained from the study of reactions among hadron systems. It is a considerable success of the one-hadron (which may or may not be Regge particle)-exchange model (abbreviated as the OHE model).\textsuperscript{3}

It means that the Yukawa interaction of hadron system has a certain validity in its lowest order approximation. We may understand this as follows: the creation (and the annihilation) of pairs of Sakatons will be realized, and in any case, being subjected to the “composite mechanism” intrinsic to the Sakaton
system, these will be materialized as hadron in the observed level.\textsuperscript{a)}

Here one point which we should like to emphasize is that the composite mechanism will dominate over the higher order effect of the conventional (Yukawa) interaction of hadron. Though the composite mechanism might be reduced to the primary interaction of Sakaton, the specific character of the primary interaction will be masked by the new dynamical law and only results in the real reaction.

When the weak interaction acts in the hadron system, what role does it undertake? The function of weak interaction is originally the change of particle sorts and the creation (or annihilation) of particles and anti-particles. However, we can expect that the weak interaction of non-leptonic decay could not be free from the overwhelming control of the strong interaction. That is, whatever the primary weak interaction be in the Sakaton level, the realized interaction in the hadron level is the resultant one intervened by the composite mechanism. Further, if the function of creation (or annihilation) in the hadron system is ascribed to the strong interaction, the role of weak interaction is only to change the sort of the particle, that is, to convert one hadron into another one.

To visualize our point it might be appropriate to consider the case of delayed neutron emission in the fission fragment of nucleus as an analogy. After the $\beta$-decay process which converts a neutron into a proton keeping mass number of nucleus unchanged, neutron emission follows as a result of the evaporation process. The former process (conversion) is due to the weak interaction and the latter (decomposition) due to the strong interaction. The difference lies only in that in our case both weak and strong interacting processes are not always real ones and the weak process proceeds just inside the composite system. This is our reasoning of pole approximation in the non-leptonic decay.

We must also refer to the serious criticism which has been cast on the pole approximation in the non-leptonic decay.\textsuperscript{5,6)} We discuss this problem in the next section. In the third section, we shall present the concrete analysis and the final section will be devoted to a further discussion about our standpoint and to the $\Delta I=1/2$ rule problem.

\S 2. \textbf{On the criticism of the pole approximation}

As we understand it, the arguments especially by Iizuka and Miyamoto\textsuperscript{3)} are as follows:

(i) The axis of $p$, $n$ and $\lambda$ in the framework of unitary symmetry is not

\textsuperscript{a)} In order to make the formal feature of "mechanism" more concrete we may recall that the composition and decomposition rule in the framework of unitary symmetry may be regarded as the annihilation and creation rule of Sakaton, respectively.\textsuperscript{4)}
always well settled. When the pole term exists, we should diagonalize the Hamiltonian including it. According to the prescription of the quantum theory that what is observed is the eigenstate of the Hamiltonian, there must remain no pole term in the Hamiltonian of the observed system.

(ii) If the pole term remains, still the other difficulty arises that the usual calculation based on the perturbation method leads to no meaningful result on the non-leptonic decay in the symmetric limit of Sakanon.

Our opinion on these problems is the following. There exists the axis of $\rho$, $n$ and $\lambda$ by which the primary strong interaction is described. In the conventional procedure of field theory the Hamiltonian must be given first to specify the system according to the "correspondence" to the classical theory. That is, what is created or annihilated, or what is represented by the field operator, is the free particle. The interaction is also given in these particle bases, which are the axis of $\rho$, $n$ and $\lambda$ in our case.

When the weak interaction is switched on, the pole term effect will arise just in these bases. What we insist is the realization of such a pole term described in these prefixed particle bases.

Now if there exist pole interactions, the observed particle bases will deviate from the prefixed ones so as to assure a pure exponential time development of the observed particle amplitude. Whether or not the time variation of the decaying particle can be treated by the perturbation method is another problem. Properly we should calculate the effective one-point interactions as a result of elimination of the pole interactions. As to the validity of perturbation treatment, Kaneko, Ohnuki and Watanabe have shown that when the symmetry breaking interaction satisfies the appropriate condition which is really the case in the present problem, it gives a good approximation. The practical calculation in the next paragraph will be performed in the conventional way.

§ 3. Numeration of the non-leptonic decay

In this section, we present a numeration of the non-leptonic decay, taking the quartet model for baryons. We start with the following $\Delta I=1/2$ two-body interactions of mesons.

$$H_{\rho}^{(1)} = G_{\rho}^{(1)} \left( -\pi K + \frac{1}{\sqrt{2}} \pi^0 K^0 \right) + \text{h.c.},$$

$$H_{\rho}^{(2)} = G_{\rho}^{(2)} \left( -K^*_{\alpha, \bar{\alpha}} \bar{\gamma}_{\alpha} \pi + \frac{1}{\sqrt{2}} K_{\alpha}^* \bar{\gamma}_{\alpha} \pi \right) + \text{h.c.}$$

The two-body interaction of baryons becomes necessarily of the $D$-type in this model:

$$H_{\rho}^{(3)} = G_{\rho}^{(3)} \left( -\bar{p} \Sigma^0 + \frac{1}{\sqrt{2}} \bar{p} \Sigma^0 + \frac{1}{\sqrt{6}} \bar{p} A - \bar{\Sigma}^+ \Sigma^- - \frac{1}{2} \bar{\Sigma}^0 \Sigma^0 - \frac{1}{\sqrt{6}} \bar{A} \Sigma^0 + \text{h.c.} \right)$$
In the above expression, $\overline{\psi} \Sigma^+ = \overline{\psi}_f \psi_{\Sigma^+}$, i.e. it contains no $\overline{\psi}_f \gamma_5 \psi_{\Sigma^+}$ term, since our prescription is to choose the weak matrix element which does not change the hadron configuration, so that

$$<\text{hadron pair}|H_{\omega}|\text{vac.}> = 0,$$

which leads to the neglect of the effect of $\overline{\psi}_f \gamma_5 \psi_{\Sigma^+}$ weak vertex. For strong interactions we assume $SU(3)$ symmetry:

$$H_{\omega}^{(a)} = i\sqrt{2}G_{\omega} \text{Tr}(\overline{B}_{\gamma_5} BM + \overline{B}_{\gamma_5} MB) + i\sqrt{2}G_{\omega} \text{Tr}(\overline{B}_{\gamma_5} BM - \overline{B}_{\gamma_5} MB), \quad (3a)$$

$$H_{\omega}^{(b)} = i\sqrt{2}G_{\omega} \text{Tr}(\overline{B}_{\gamma_5} V_{\gamma_5} B - \overline{B}_{\gamma_5} \gamma_5 V_{\gamma_5} + \mathfrak{M}_{\omega} MV_{\gamma_5} - MV_{\gamma_5} \mathfrak{M}_{\omega}), \quad (3b)$$

where by the C.V.C. hypothesis $\Gamma$ couples only to the $F$-type current. The effective interaction then becomes

$$H_{\text{int}} = H_{\omega}^{(a)} + H_{\omega}^{(b)} + H_{\omega}^{(1)} + H_{\omega}^{(2)} + H_{\omega}^{(3)}. \quad (4)$$

This Hamiltonian is essentially the same as that of Kawasaki, Imoto and Fujii and Murayama.\(^5\)

We give the result of the reanalysis using new experimental data\(^6\), although the main point of our discussion is the reasoning of our interaction Hamiltonian (4) as is stated in §§ 1 and 4. Diagrams to be considered for the $S$-wave and $P$-wave decays of hyperons are Figs. 1 and 2, respectively. So far as we take into consideration only the products of the coupling constants referring to the above diagrams, the $A_I = 1/2$ relations for both the $S$ and $P$ waves and the Sugawara-Lee relation\(^9\) for the $S$ wave are obtained exactly together with the result

$$S(\Sigma^-) = 0. \quad (5)$$

Next we present the numerical results of the pole approximation which is followed from the diagrams in Figs. 1 and 2 by using the observed values of masses. In order to obtain $\alpha(A_{\Sigma^+}) > 0$, $\alpha(\Sigma^0) < 0$ and $\alpha(\Xi^-) < 0$ the ratio $G_{\omega}/G_{\rho}$ has to be in the range

$$-4.8 < G_{\omega}/G_{\rho} < -1.5 \quad (6a)$$

or

$$-4.8 < G_{\omega}/G_{\rho} < -4.8 \quad (6b)$$
0.5 < G_p/G_p < 0.7. \quad (6b)

The calculated values of the S-wave amplitudes (A_s) and the P-wave amplitudes (A_p) are listed in Table I, where A_s and A_p are related with the decay matrix by

\[ \langle N\pi|S|Y\rangle = -i(2\pi)^{-1/2}\langle P_y - P_x \rangle \left( \frac{1}{2\omega_p} \right) \tilde{M}, \]

where

\[ \tilde{M} = \tilde{U}\tilde{\Sigma}(B_s + B_p) \tilde{U} = i\tilde{U}\tilde{\Sigma}(A_s + A_p \sigma \cdot \mathbf{n}) \tilde{U}. \quad (7) \]

Carrying out a least square fit to the experimental values of A_s and A_p, we obtain the following most probable set of values for \( G_p G_w(3)/m_p \), \( G_p G_w(1)/m_p \), and \( G_p G_p \):

\[ |G_p G_w(3)/m_p| = (6.47 \pm 0.73) \times 10^{-6}, \]

\[ |G_p G_w(1)/m_p| = (7.26 \pm 0.18) \times 10^{-5}, \quad (8a) \]

\[ |G_p G_w(3)/m_p| = (4.78 \pm 0.12) \times 10^{-6}, \]

\[ G_p/G_p = 0.625. \quad (8b) \]

The calculated values of the asymmetry parameters and the decay rates are also shown in Table I together with the experimental values. From Table I one sees

| Decay process | \( |A_s|_{\text{th.}} \) | \( |A_s|_{\text{exp.}} \) | \( |A_p|_{\text{th.}} \) | \( |A_p|_{\text{exp.}} \) |
|---------------|----------------|----------------|----------------|----------------|
| \( A_s^0 \)   | 3.44           | 3.360 \pm 0.074| 1.38           | 1.274 \pm 0.074|
| \( A_p^0 \)   | 2.42           | 2.30 \pm 0.26  | 0.95           | 1.00 \pm 0.40  |
| \( \Sigma^+ \) | 0.00           | 0.016 \pm 0.02 | 3.89           | 4.04 \pm 0.10  |
| \( \Sigma^+ \) \((\gamma \geq 0)\) | 2.81           | 3.38 \pm 0.83  | 2.29           | 2.54 \pm 0.86  |
| \( \Sigma^+ \) \((\gamma < 0)\) | 2.29           | 2.54 \pm 0.86  | 2.81           | 3.38 \pm 0.83  |
| \( \Sigma^- \) | 4.09           | 4.05 \pm 0.04  | 0.11           | 0.020 \pm 0.088|
| \( \Xi^- \)   | 3.99           | 4.43 \pm 0.11  | 1.66           | 0.75 \pm 0.25  |
| \( \Xi_0^0 \) | 2.73           | 3.39 \pm 0.31  | 1.19           | 0.69 \pm 0.12  |

<table>
<thead>
<tr>
<th>Decay process</th>
<th>( \alpha_{\text{th.}} )</th>
<th>( \alpha_{\text{exp.}} )</th>
<th>( \alpha_{\text{th.}} )</th>
<th>( \alpha_{\text{exp.}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_s^0 )</td>
<td>0.69</td>
<td>0.663 \pm 0.022</td>
<td>2.81</td>
<td>2.65 \pm 0.09</td>
</tr>
<tr>
<td>( A_p^0 )</td>
<td>0.68</td>
<td>0.73 \pm 0.18</td>
<td>1.43</td>
<td>1.34 \pm 0.07</td>
</tr>
<tr>
<td>( \Sigma^+ )</td>
<td>0.00</td>
<td>0.008 \pm 0.037</td>
<td>5.16</td>
<td>5.83 \pm 0.28</td>
</tr>
<tr>
<td>( \Sigma^+ )</td>
<td>-0.98</td>
<td>-0.96 \pm 0.07</td>
<td>4.80</td>
<td>6.52 \pm 0.29</td>
</tr>
<tr>
<td>( \Sigma^- )</td>
<td>-0.05</td>
<td>-0.01 \pm 0.043</td>
<td>6.18</td>
<td>6.06 \pm 0.11</td>
</tr>
<tr>
<td>( \Xi^- )</td>
<td>-0.71</td>
<td>-0.33 \pm 0.10</td>
<td>5.32</td>
<td>5.75 \pm 0.17</td>
</tr>
<tr>
<td>( \Xi_0^0 )</td>
<td>-0.73</td>
<td>-0.391 \pm 0.032</td>
<td>2.47</td>
<td>3.3 \pm 0.6</td>
</tr>
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</table>
that these theoretical values well agree with the experimental values, although there is some discrepancy as for the asymmetry parameters of $E$. Here we should mention a few about the decay of kaon. The decay $K\rightarrow 2\pi$ is given by the diagrams in Fig. 3(a) and the same interaction as in the hyperon decay is participating. Contribution from $\rho$, $\omega$ and $\varphi$ poles such as shown in Fig. 3(b) is negligible owing to the same argument as that of Kawasaki, Imoto and Furui.

That is, the matrix element of Fig. 3(b) is proportional to $(k_1+k_2)(k_1-k_2)$ owing to derivatives at both weak and strong vertices and vanishes on the mass shell. It is clear that we obtain the $\Delta I=1/2$ relations for $K\rightarrow 2\pi$ matrix elements. If we take $(G_{\rho} + G_{\varphi})^2/4\pi \approx 15$ and $G_{\omega(3)}/4\pi \approx 0.5$, which are determined by the $(N\rightarrow N\pi)$ and $(p\rightarrow 2\pi)$ data respectively, the decay probability for $K_i^0 \rightarrow 2\pi$ is calculated using the values (8a) and (8b) as follows:

$$
\begin{align*}
(K_i^0 \rightarrow \pi^+\pi^-) &= \frac{G_{\omega(3)}^2}{4\pi} (G_{\omega(3)}^2)^{1/2} \frac{A}{m_k^2} \left( m_k^2 - m_\pi^2 \right)^{1/2} \sqrt{1 - \left( \frac{2m_\pi}{m_k} \right)^2} \\
&= 5 \times 10^{-8} \text{ sec}^{-1}, \quad (9)
\end{align*}
$$

this value is somewhat larger than experiment, $0.8 \times 10^{-8} \text{ sec}^{-1}$.

So far we have presented the numeration in the quartet model for Sakaton. The similar calculation may be performed in the triplet model, which also gives satisfactory results with slight modification similar to that of Fujii and Murayama in reference 2).

§ 4. Discussion

In the preceding section, starting with the interaction (1a) and (1b), and assuming the quartet model and $SU(3)$ symmetry, we have shown that the pole approximation offers a satisfactory account for the experimental situation.

Now we shall discuss the reasoning of the pole term (4) which supplements the discussion of § 1. We may take one possibility for the origin of Eq. (1a), which requires the specific nature of weak interactions. The only four-fermion interaction which leads to Eq. (1a) is the linear combination of the following two types of interaction:
If the possibility of the interaction (10) is accepted, one needs to reexamine the appropriateness of the current-current picture\textsuperscript{11,12} of the weak interaction. At present the facts supporting the current picture are: i) the universality of the magnitudes of all weak coupling constants, ii) the universality of the weak interactions along with the universality, where the derivation of Eq. (1a) is as follows:

\[
H_i = (\bar{\psi}_p O_{\mu} (1 + \gamma_5) \psi_n) (\bar{\psi}_n O_{\mu} (1 + \gamma_5) \psi_p) + \text{h.c.,}
\]

and

\[
O_{\mu} = 2S + V + A - 2P, \tag{10}
\]

and the derivation of Eq. (1a) is as follows:

\[
\langle \pi^+ | H_i | K^- \rangle = \langle \bar{p} \bar{n} | (\bar{\lambda} \rho) | \lambda \rho \rangle = a,
\]

\[
\langle \pi^0 | H_i | K^- \rangle = (1 / \sqrt{2}) \langle (\rho \bar{p} - n \bar{n}) | (\bar{\lambda} \rho) | \lambda \rho \rangle | n \lambda \rangle
\]

\[
= - (1 / \sqrt{2}) \langle (\rho \bar{p} | (\bar{\lambda} \rho) | n \lambda \rangle - (1 / \sqrt{2}) a.
\]

If the possibility of the interaction (10) is accepted, one needs to reexamine the appropriateness of the current-current picture\textsuperscript{11,12} of the weak interaction. At present the facts supporting the current picture are: i) the universality of the magnitudes of all weak coupling constants, ii) the universality of the weak ‘boson-fermion’ interactions such as $\pi \rightarrow l^+ l^- K^{-} l^-$ and $Y \rightarrow N \gamma$, and iii) the presence of the parity nonconserving nuclear force.\textsuperscript{13} We must note, however, that these facts should be considered as only necessary conditions but not as sufficient conditions for the current-current picture to hold. As is emphasized in reference 12), the non-leptonic interaction in the current-current interaction is essentially nothing but a merely extrapolated one from the pure- and semi-leptonic interactions. Figure 4 represents schematically all types of weak four-fermion interactions along with the universality, where the $(\bar{p} n) (\bar{\lambda} \rho)$ coupling is responsible for the non-leptonic interaction. We should remark that there is a difference between the $(\bar{p} n) (\bar{\lambda} \rho)$ coupling and others in that the pure- and semi-leptonic couplings consist of the four particles which are all different to one another while the non-leptonic coupling $(\bar{p} n) (\bar{\lambda} \rho)$ contains a pair of the identical particles, in other words, conserves proton. This may be stated in another way as follows: pure- and semi-leptonic couplings are characterized by the “conservation of vertex”; this means that each vertex couples to another one which behaves as external fields, but the “conservation” in this sense is not satisfied in the non-leptonic coupling. Taking this difference seriously, and considering the possibility that the interaction type may have a certain relation to the conservation of the particles concerned in the interaction, we may think that the non-leptonic interaction differs in type essentially from the pure- and semi-leptonic ones, therefore Eq. (10) should be the fundamental non-leptonic weak interaction and Eq. (1a) should be derived from this.

Previously the importance of the configuration, that is, of the minimum
number of configuration to specify the system in $U(3)$ symmetry has been emphasized\(^1\) for the hadron system and also an attempt has been given with the idea that the weak interaction keeps the configuration invariant in the non-leptonic decay.\(^2\) Such invariance of the configuration is expressed by using the $SU(3)$ generators and by considering that the transition from initial state to final one is caused by those generators. If we take the property of Eq. (10), the iso-spinor property of Hamiltonian (i.e. Eq. (1a)) may be given through the generators such as

$$\begin{align*}
H_{(1)} N_{13} + N_{12} N_{23} - N_{23} N_{11} \\
= & \sum \left[ \bar{\psi}_{\alpha \beta} \psi_{\beta} d \sigma_{\alpha} - \bar{\psi}_{\alpha \beta} \psi_{\beta} d \sigma_{\alpha} \right] \sum \left[ \bar{\psi}_{\alpha \beta} \psi_{\beta} d \sigma_{\alpha} - \bar{\psi}_{\alpha \beta} \psi_{\beta} d \sigma_{\alpha} \right] (12)
\end{align*}$$

In this line of thought, we should like to mention a few about quite a different possibility. We assume that the function of generators which causes the weak interaction must always be the first order in generators. Then our assumption for non-leptonic weak interactions is that they may be caused by commutators of generators such as

$$|I, N_{13}| = N_{23}$$

(13a)

and

$$|I, I_{-}| = 2 I_{p}$$

(13b)

Then our Hamiltonian (1a) may be followed from (13a) and the parity non-conserving nuclear force from (13b), and this latter property may be able to test by experiment.

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