Chapter 1

Potential Model Approach

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§1. Introduction

Much progress has been made in the study of nuclear forces following the Taketani theory (1951)\(^1\) which proposes to approach nuclear forces from the outside of a nucleon by verifying meson theoretical predictions in the outer region. In analyzing low energy phenomena, it has achieved remarkable success in establishing the one-pion-exchange potential (OPEP) in region I ($x > 1.5$) quantitatively; it also verified some qualitative pro-
properties of the two-pion-exchange potential (TPEP) in region II ($1.5 < x < 0.7$). ($x$ is the internucleon distance in the unit of the pion Compton wave length.) These investigations were reported in 1956 in the Supplement No. 3 of the Progress of Theoretical Physics. This chapter is devoted to review the analyses carried out in Japan since 1956 using the potential model.

Just about the same time as when the OPEP-tail was established, another powerful approach from a high-energy side was started with the first complete experiment of the $p-p$ scattering at a laboratory energy $E=315$ MeV performed by the Berkeley group (1957). They aimed to determine the nucleon-nucleon scattering matrix by using the data of polarized beam experiments along the line of Wolfenstein’s proposal (1954), which was formulated later by Puzikov, Rindin and Smorodinsky (1957) utilizing the unitarity condition. Some important phenomenological developments stemmed from this experiment, mainly toward lower energy regions below 310 MeV.

A number of works have been performed to obtain a unique set of phase shifts in the nucleon-nucleon elastic scattering region. Recognition of the importance of the OPEP-tail clarified at low energies led the purely phenomenological phase shift analysis to the modified one, by including the effects of the OPEP-tail as the one-pion-exchange pole in scattering amplitudes. Ambiguities involved in the solutions of the phase-shift analysis have been remarkably diminished by replacing phase shifts with large angular momenta with the one-pion-exchange contributions (1959). Also the consideration of energy dependence in the solutions of the modified phase shift analysis served to eliminate spurious solutions. At present, an almost unique solution has been obtained in the two-nucleon isospin $T=1$ state up to about $E=350$ MeV and some reasonable solutions even in the $T=0$ state. Concerning the problems of complete experiments and phase shift analysis, see a review article in Chapter 5.

The phase shift analysis of the $p-p$ scattering data at 310 MeV performed by Stapp, Ypsilantis and Metropolis (SYM) (1957) showed the evidence on the existence of the repulsive core in the singlet $S(^1S_0)$ state and on the spin-orbit effect in the triplet $P(^3P_1)$ states mainly through negative $\delta(^3P_0)$. Among the SYM solutions, No. 1 solution is understandable by a potential picture. Analyzing this solution, Gammel and Thaler (1957) proposed a phenomenological potential (GT potential) with

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1. The existence of the repulsive core in the singlet $S$ state was already pointed out by Iwadare, Otsuki, Tamagaki and Watari (1956) using the low energy data.
2. In this review, nuclear bar phase shifts and mixing parameters are denoted by $\delta(^{2S+1}L_J)$ or $^{2S+1}L_J$ and $\epsilon_J$, where $S$, $L$ and $J$ are the spin, orbital and total angular momentum respectively.
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a strong and short range spin-orbit term. Although this potential has not the correct one-pion-exchange asymptotic form, it has played an important role as an inside phenomenological description. Investigations of searching for semiphenomenological potentials with overall fits in the elastic scattering region have been made by several authors; they combined the phenomenological description (such as the hard core and the strong and short range spin-orbit potential) in the inner region with the OPEP-tail in the outer region: Bryan's potential (1960) with the short tailed spin-orbit potential mainly confined in region III, Hamada and Johnston's potential (HJ potential) (1962) with the quadratic spin-orbit terms and Yale group's potential (Yale potential) (1962) with some angular momentum dependence in addition.

After the establishment of the OPEP in region I in 1956, a program of semiphenomenological approach was started in Japan with investigating the intermediate energy region \(E = 90 \sim 150\) MeV by use of the static potential obtained through the low energy analysis. According to the impact parameter consideration, \(E = 20\) MeV is a critical energy, since at \(E \lesssim 20\) MeV the phase shifts other than that of the S wave are determined mainly by region I (\(x \gtrsim 1.5\)) because their impact parameters are larger than 2 and nonstatic effects are unimportant. The next critical energy was considered to be about \(E = 100\) MeV, because at \(E \gtrsim 100\) MeV the observed quantities are seriously dependent on interactions in region III (\(x \lesssim 0.7\)) mainly through the P-wave phase shifts (their impact parameter \(\lesssim 0.9\)) and besides nonstatic corrections to the outer part of potentials are expected to be appreciable. Otsuki, Tamagaki and Watari propose in 1957 to investigate what features of pion theory play the essential role at intermediate energies around 100 MeV and to what extent the low energy pion theory, hence the static pion theoretical potential, must be modified.

Following this line, Otsuki (1958) analysed the \(p-p\) scattering at 90 MeV and Watari (1958) \(n-p\) scattering at 90 MeV by applying the static pion-theoretical potential to unpolarized cross section \(I_0(\theta)\) and polarization \(P(\theta)\) available at that time. They gave a special notice throughout their works to the fact that one of the characteristics of the pion-theoretical potential is its strong tensor force of the OPEP. The procedures of analysis were as follows: To treat the S-wave phase shifts as free parameters, to use phase shifts determined by the OPEP for higher partial waves than the D wave and to calculate the P-wave phase shifts by using a semiphenomenological potential with the OPEP-tail. They found that the static semiphenomenological potential reproduces the unpolarized differential cross section \(I_0(\theta)\) and the polarization \(P(\theta)\) at 90 MeV without any serious modifications. The analysis along this line was extended up to 150 MeV by Tamagaki (1958) in a somewhat phenomenological way, and it was
found, in accordance with the consequences at 90 MeV, that the experimental data on $I_0(\theta)$ and $P(\theta)$ were well explained by a strong tensor force in the outer part of the interaction.

Concerning the data on $I_0(\theta)$ and $P(\theta)$ up to 150 MeV available at that time, these analyses showed neither serious evidences against the static pion-theoretical potential nor positive evidences for strong nonstatic effects such as the spin-orbit effect. This result was in conflict with the prediction of strong spin-orbit forces made by Gammel and Thaler\(^{29}\) and by Signell and Marshak (1957).\(^{29}\) In the latter case, the spin-orbit term was introduced in Gartenhaus’ potential in order to explain $I_0(\theta)$ and $P(\theta)$ up to 150 MeV. In Signell and Marshak’s potential there was a crucial difficulty, namely that the potential in region III to be phenomenologically treated was represented by unreliable parts of Gartenhaus’ potential, for example, the unreasonably strong attractive core in the odd states, which was removed later (1958) by introducing the hard core.\(^{20}\) The phenomenological GT potential based on the No. 1 solution of the phase shift analysis at 310 MeV was considered as one of the possible ways of explaining the high energy data, since the SYM solutions of the phase shift analysis at 310 MeV were not unique at that time. Both the spin-orbit potentials are as strong in region II as to modify the triplet $P$-wave phase shifts $\delta(3P_1)$ given by the static OPEP greatly at intermediate energies ($E=40\sim100$ MeV) and appreciably even at $E=20$ MeV. Such strong spin-orbit potentials could not be expected as originating from recoil corrections in the two-pion-exchange potential.\(^{21},^{22}\) In these considerations, Hamada, Iwadare, Otsuki, Tamagaki and Watari (HIOTW) (1959)\(^{29}\) analysed $p$-$p$ scatterings at 95 and 150 MeV by using the semiphenomenological potential with the OPEP-tail, allowing some energy dependence for inner parameters, where necessary. They confirmed the previous conclusion that the data on $I_0(\theta)$ and $P(\theta)$ up to 150 MeV can be reproduced by the essentially static potential with the OPEP-tail and with the features characteristic of the TPEP (the triplet odd tensor potential suppressing that of the OPEP and the deep attractive central potential around the pion range). However, this potential predicted a large negative depolarization $D(\theta)$ at a scattering angle $\theta=45^\circ\sim90^\circ$ and an energy $E\approx150$ MeV due to a large positive value of $\delta(3P_0)\sim30^\circ$, while $D(\theta)$ is positive and small for the GT potential due to the small value of $\delta(3P_0)\sim5^\circ$, since $D(\theta)$ is sensitive to the $3P_0$-phase shift $\delta(3P_0)$\(^{23},^{24}\) Thus HIOTW concluded “if it is decisively found that $D(\theta)$ is positive, it may mean something not so simple for the pion theory of nuclear forces, and then various kinds of new factors should be taken into account, for example, a strong spin-orbit coupling in the inner part, large recoil corrections, effects of strong coupling, effect of $K$-mesons, possibility of isoscalar mesons and others”. Although the early experimental data on $D(\theta)$ performed by two
groups (1960) were unfortunately qualitatively different,\textsuperscript{25,30,43} the data revised later (1961)\textsuperscript{25} showed that there exists a strong spin-orbit effect which reduces the large value of $\delta(P_5)$ at $E \approx 150 \text{ MeV}$ due to the OPEP tensor force.

In parallel with the analysis by HIOTW, the following investigations were going on concerning the spin-orbit coupling potential (hereafter denoted by $V_{ls}$). In order to derive the strong $V_{ls}$ many authors reinvestigated the two-pion-exchange recoil corrections. Okubo and Sato (1959)\textsuperscript{26} showed that the $(3/2, 3/2)$-resonance effect in the pion-nucleon interaction cannot be expected to give a large spin-orbit coupling. In the full recoil calculations made by Hoshizaki and Machida (1962)\textsuperscript{27} the $V_{ls}$ due to the two-pion-exchange effects was shown to be weaker by one order of magnitude than phenomenologically needed in the triplet odd state. This conclusion was in agreement with Sugawara and Okubo’s results (1960)\textsuperscript{28}. The possibility of explaining the strong $V_{ls}$ by pion-exchange processes became hopeless. These situations gave rise to a conjecture that the spin-orbit forces are confined to the innermost region III which cannot be understood pion-theoretically, in other words such strong spin-orbit forces may be an indication of dynamics which cannot be explained pion-theoretically, even qualitatively.*

The introduction of the two-nucleon spin-orbit potential $V_{ls}$ is connected with one-particle spin-orbit potential $V_{ai}$ in the nuclear shell model. When the $V_{ls}$ much stronger than that of the TPEP was first introduced, it was investigated to what extent the strong OPEP-tensor potential can explain the $V_{ai}$ in nuclei, and it was shown that about a half of the $V_{ai}$ can be given by three particle correlation effects due to the OPEP-tensor potential through the antisymmetrization (1959)\textsuperscript{29}. Therefore the room in $V_{ai}$ left to $V_{ls}$ is not very large. In fact, several authors found that the strong $V_{ls}$ of the GT potential gives too large shell model spin-orbit effect by 2~3 times larger than the experimental value,\textsuperscript{30} although there is a prediction that the GT-$V_{ls}$ is consistent with $V_{ai}$.\textsuperscript{31} Therefore, the $V_{ai}$ which is related to low energy phenomena may be accounted for by the strong OPEP-tensor force and the weak $V_{ls}$ of the TPEP. This possibility was discussed on $n$-$\alpha$ scattering by Takamura and Tamagaki (1961)\textsuperscript{30} and recently by Kanada, Nagata, Sumi and Otsuki (1963)\textsuperscript{32}.

The recognition of the weakness of the $V_{ls}$ of the TPEP in region II led to a problem how strong $V_{ls}$ is demanded in region II by the

\*\* Furuichi and Watari\textsuperscript{93} investigated the two-pion-exchange contribution in nucleon-nucleon scattering using dispersion relation, and found that the effects of the $(3/2,3/2)$ $\pi N$ resonance and the nucleon-antinucleon pair terms give rather strong $V_{ls}$. For details, see §4 of Chapter 4 in this issue. Very recently, Hiroshige and Semba\textsuperscript{132} confirmed this results by calculating the TPEP assuming $ps$ ($ps$)-coupling for the case where energy conservation holds.
experimental data. To answer this problem, we need to separate the contributions of a potential in region II from those in region III. It became necessary then to perform complete experiments at $E<100$ MeV, e.g. $E\sim 50$ MeV (1960), because at $E=100$ MeV the impact parameter of $P$ wave is about $0.9^{38,40}$. After the examination of the sensitiveness of experimental quantities to a $V_{LS}$ tail, measurement of $C_{pR}(90^\circ)$ at 52 MeV was proposed (see Chapter 5 for details) (1962). The purpose of this measurement (1963) was to examine whether the strong $V_{LS}$ indispensable at high energies remains strong also at low energies in an energy-independent manner, since this strong $V_{LS}$ seemed to give too large shell model spin-orbit effects at least in light nuclei.

Another development was related to the origin of the strong spin-orbit potential. At nearly the same time as when the full recoil calculation was carried out, Fujii (1961) showed the possibility that a strong spin-orbit potential originates from the pion-pion resonance effects in the isospin $I=1$, the total angular momentum $J=1$ state ($\rho$-meson). Owing to the large coupling constant with nucleon to reproduce the strong $V_{LS}$, however, Fujii’s potential accompanied unreasonably strong central and tensor potentials together with the strong $V_{LS}$. Thus a comprehensive study, not restricted only to the problem of $V_{LS}$, has become necessary. In this circumstance, Hoshizaki, Lin and Machida (1961) gave a list of nonstatic one-boson-exchange potentials with full recoil. In 1961 Ogawa, Sawada, Ueda, Watari and Yonezawa proposed that all problems on nuclear forces including that of the spin-orbit forces should be comprehensively considered on the basis of the new situation developed after the full symmetry theory (1959) of the Sakata model (1956), and they presented the basic idea underlying the one-boson-exchange (OBE) model. The first work along this line was performed by Hoshizaki, Otsuki, Watari and Yonezawa (1962), in which the main consequences obtained by the potential model calculations were investigated, and they clarified the existence of the $I=1$ vector meson ($\rho$) and the $I=0$ vector meson ($\omega$), and obtained some indications for the existence of the $I=0$ scalar meson, where $I$ is the isospin of a meson. Later development of the OBE model is underway, as is discussed in Chapter 3.

It has been known for long that spin-orbit potentials can be accounted for by scalar and/or vector meson exchange, as is described in Rosenfeld’s textbook. After the existence of the spin-orbit forces was phenomenologically found, these possibilities were considered again by several authors. Gupta (1961) introduced a heavy neutral scalar meson, Breit (1960) proposed to regard the repulsive core and the spin-orbit potential as originating through a vector field and Sakurai (1960) also proposed that the spin-orbit effects are due to the $I=0$ vector meson (the three pion-resonance
state with \( I=0 \) and \( J=1 \), which was suggested by Nambu (1957) to explain the electromagnetic form factor of nucleon and to be responsible for the hard core. As was discussed by Breit (1960) and by Hoshizaki, Lin and Machida, the other parts of the potential adjusted to the strong \( V_{LS} \) spread its tail around the pion range and nuclear forces cannot be explained by introducing a single heavy meson. Ohnuma (1961) discussed energy dependence of \( C(90^\circ) \), one of Wolfenstein’s parameters, in reference to Sakurai's calculation on this parameter. Also connection of the hard core and the \( V_{LS} \) is not simple (see Chapter 7). It should be emphasized that the one-boson-exchange (OBE) model based on the Sakata model and the attempts by Gupta, Breit and Sakurai were proposed on quite different stand points.

There still remained a problem whether the essential part of the spin-orbit forces is due to the heavy meson exchange (mainly due to \( \omega \)-meson and \( \rho \)-meson exchange). The possibility of making the range of the \( V_{LS} \) smaller than that of the GT potential was investigated. Bryan’s potential is one of such attempts. Also Saylor, Bryan and Marshak (1960) showed, combining a boundary condition description with the outside pion-theoretical potential (they used the TMO potential), that the main features of the experimental data are reproduced by the energy-independent boundary conditions at \( x=0.56 \). The success of this “modified boundary condition model” together with the investigation on the range of \( V_{LS} \) showed the possibility that the spin-orbit effects are completely confined in region III. Although this possibility is interesting, the spin-orbit forces in the intermediate region (\( x=0.5 \sim 1.0 \)) should be considered as determined partly by the one-boson-exchange potential (OBE) since the existence of vector mesons has been verified. The short range property of \( V_{LS} \) seemed to be indicated by the fact that the splitting of triplet F phase shifts \( \delta(F) \) experimentally found is not of the spin-orbit type even at \( E=310 \) MeV. However, the reduction of the range of \( V_{LS} \) does not serve to suppress the steep increase of \( \delta(F) \) above 310 MeV, for example at 660 MeV \( \delta(F) \) is in qualitative disagreement with the experimental phase shift as will be discussed later on. The fact that the very short tailed \( V_{LS} \) such as Bryan’s \( V_{LS} \) gives too small polarization \( P(\theta) \) at \( E\sim 150 \) MeV is an indication that the range of \( V_{LS} \) is not so short. Therefore the problem about the range of \( V_{LS} \) is not considered as related intimately to that of the \( \delta(F) \) splitting. Thus, we can safely say that the outer part of \( V_{LS} \) is mainly described by the one-boson-exchange potential (OBE), while we cannot deny that a part of the inner \( V_{LS} \) is accounted for by some mechanism on the surface of the repulsive core.

When the singlet even phase shifts were determined almost uniquely as the results of the accumulation of solutions of the phase shift analysis,
it was made clear that the angular momentum dependence in the singlet even potential is indispensable, since the static central potentials for reproducing the $^1S_0$ phase shift $\delta(^1S_0)$ give the too large $^1D_2$-phase shift $\delta(^1D_2)$ and $^1G_7$-phase shift $\delta(^1G_7)$. Hamada (1961) introduced the quadratic spin-orbit potential to resolve the angular momentum dependence.\(^{56}\) This introduction of quadratic spin-orbit potential is along the consideration by Puzikov, Rindin and Smorodinsky\(^5\) and by Okubo and Marshak (1958)\(^{60}\) that, generally speaking, a complete potential with all the possible independent terms is necessary in reproducing the phase shifts determined from complete experiments. Taketani and Machida (1960)\(^{60}\) emphasized that theoretical establishment of nonstatic potentials is important, which would play the role of a guide in introducing quadratic spin-orbit potentials. Hoshizaki and Machida\(^{27},^{61}\) calculated the nonstatic OPEP and TPEP with full recoil. The strength of the quadratic spin-orbit potentials needed in the singlet even ($^1E$), triplet even ($^3E$) and triplet odd ($^3O$) states\(^*)\) is of the same order of magnitude with the nonstatic OPEP.\(^{57},^{62}\) As will be discussed in §2, even if quadratic spin-orbit potentials have the asymptotic form of the nonstatic OPEP, it seems reasonable to regard these potentials as phenomenological, because they are parts of nonstatic potentials which can be easily treated in coordinate space. Therefore, the treatments taking the nonstatic effects into account as fully as possible are favourable, such as the momentum space calculations,\(^{63} - {66}\) although the results obtained up to now are not so complete as in coordinate space and one may find some difficulty in dealing with high momentum parts corresponding to region III.

The clearest evidence of the nonstatic effects higher than the spin-orbit effects is present in the $^1E$ state as the indispensable angular momentum dependence of potentials. In the $^3E$ state, too large $^3D_2$-phase shift $\delta(^3D_2)$ given by the strong outside negative tensor potential should be reduced by quadratic or linear spin-orbit potentials. Unless we have no reliable estimate on the quadratic spin-orbit potentials, spin-orbit potentials in the $^3E$ state are largely uncertain.\(^{63}\) In the $^3O$ state, as shown by Tamagaki, Wada and Watari (1965)\(^{97}\) the quadratic spin-orbit potentials modified in regions II and III so as to be effective in the coupled states serve to resolve the difference of splitting between $^3P$-phase shifts $\delta(^3P_I)$ and $^3F$-phase shifts $\delta(^3F_I)$,\(^{97}\) although in the Yale potential this difference is avoided by assuming the spin-orbit potential only for $J \leq 2.11$ $\delta(^3P_I)$ are not strongly affected by these quadratic spin-orbit potentials, while $\delta(^3F_I)$ are remarkably reduced at high energies by those. Thus the problem of the range of spin-orbit forces becomes not serious, and the range should rather be considered as

\(^*)\) We sometimes abbreviate the singlet even, triplet even, singlet odd and triplet odd states to $^1E$, $^3E$, $^1O$ and $^3O$, respectively.
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determined mainly by the meson masses responsible for the spin-orbit effects.

By introducing all the independent potential terms, the semiphenomenological potentials with the OPEP-tail which reproduce the phase shifts below $E \leq 350$ MeV have been found. These potentials well reproduce the recently measured experimental data which are not used in determination of the potential parameters. However, since it is an open question at what extent and at what region these potentials are unique, in the present stage the meaning of the potential model approach is as follows:

1. Each potential term (central, tensor, spin-orbit, quadratic spin-orbit) in region II can be determined more strictly. When different qualitative understandings are possible, limits of such possibilities can be clarified. In analyses, it is important to include nonstatic effects as accurately as possible.

2. The qualitative features in region III are understood on the basis of the potentials well determined in regions I and II, and the characteristics of nuclear forces at short internucleon distances are clarified, which may be expressed in different words in future theory. The utility of a coordinate space potential consists in providing the possible way of representing nuclear forces at and near the nucleon core directly and intuitively. A defect of a coordinate space potential consists in ambiguities in treatment of nonstatic effects and in being restricted in the nonrelativistic energy region. Therefore, the best way of clarifying nuclear forces at short distances seems to be converting the characteristic features found in the coordinate space potential model into those in the models which include nonstatic effects in a more satisfactory way.

3. The repulsive core is probably related to the structure of the nucleon core or the dissociation of meson into fundamental units. If the decisive influence of the structure of the nucleon core on nuclear forces in the core region is real, nuclear forces in this region may be almost independent of the incident energy below about 1 GeV, since the interactions related to the core structure are supposed to be very strong. Then the energy-independent treatment of the core region, such as the introduction of the hard or soft repulsive core and the use of energy-independent boundary conditions, may rather be adequate. It is important to investigate whether nuclear forces in the core region are mostly energy-independent or not. In such investigations into the core region, information from the $S$ wave is of particular importance. At present, the potential model approach in the coordinate space seems most powerful in treating the $S$ wave.

In §2, a general form of the potential in coordinate space is presented and the form of the potential used in this paper is given. In §3, the potential in the singlet even state, especially quantitative features in region II, is discussed. The core region is not discussed in detail since it is dis-
cussed in detail in Chapter 7. The potential in the triplet odd state is discussed in §4. The potentials in the two-nucleon isospin $T=0$ states are briefly discussed in §5.

§2. General form of potential in coordinate space

2.1 Nonlocality and nonstatic terms in potential

The powerful concept "potential" is unambiguously defined in the coordinate space, when it is static and local. Generalization of a nonrelativistic potential can be made in two ways. One is to use a nonlocal potential to be inserted in the Schrödinger equation:

$$[E_{cm} + V^2(r)/M] \psi(r) = \int <r|V|r'>\psi(r') dr',$$

where $r$ is the relative coordinate between two nucleons and $\psi(r) = <r|\psi>$. The potential $<r|V|r'>$ becomes $\delta(r-r')V(r)$ for a local one. The other is to add nonstatic terms. These two ways of generalization are not essentially different, because nonstatic terms are expressed as the nonlocality of potential, as is shown below.

A nonlocal potential $<r|V|r'>$ is written as the Fourier transform

$$<r|V|r'> = (2\pi)^{-6} \int dp' dp'' <p'|V|p'''> exp[i(p' \cdot r - p'' \cdot r')],$$

where $<p'|V|p'''>$ is the potential matrix element for the transition from the two-nucleon state with the momenta $(p'', -p'')$ to that with $(p', -p')$ shown in Fig. 2.1. In constructing the potential in the usual way, $<p'|V|p'''>$ containing also nonstatic terms is given first from field theory and then the transformation Eq. (2.2) is done. By rewriting Eq. (2.2) with the variables $k = p' - p''$ (momentum transfer) and $q = (p' + p'')/2$ (average momentum),

$$<r|V|r'> = (2\pi)^{-3} \int dk dq V(k, q) \exp[iqs + ikR],$$

where $V(k, q) = <p'|V|p'''>$ and $s = r - r'$, $R = (r + r')/2$.

If $V(k, q)$ is independent of $q$, namely a function of the momentum transfer $k$ only ($V(k) = V(k, q)$), then

$$<r|V|r'> = \delta(s) \cdot (2\pi)^{-3} \int dk V(k) \exp[ikR] = \delta(r - r')V(r)$$

with $V(r) = (2\pi)^{-3} \int dk V(k) \exp[ikr]$. 
This means that the $q$-dependence of $V(k, q)$ causes the nonlocality in the coordinate space.

The other description of the $q$ dependence is to exhibit this dependence explicitly in the coordinate-space potential:

$$
\langle r | V | r' \rangle = (2\pi)^{-3} \int dk dq V(k, -i\mathbf{p}) \exp[ik\mathbf{R}] \exp[iqs]
$$

$$
= (2\pi)^{-3} \int dk V(k, -i\mathbf{p}) \exp[ik\mathbf{R}] \delta(s)
$$

$$
= V(r, (-i\mathbf{p} + i\mathbf{p}')/2) \delta(r - r'). \quad (2.3)
$$

Since the momentum operator $\mathbf{p}$ is represented by

$$
\langle r | \mathbf{p} | r' \rangle = \langle r | -i\mathbf{p} \vec{\psi} | r' \rangle = -i\mathbf{p} \psi^*(r),
$$

$$
\langle \psi | \mathbf{p} | r' \rangle = \langle \psi | -i\mathbf{p} \vec{\psi} | r' \rangle = i\mathbf{p} \psi^*(r')
$$
and

$$
\langle r | \mathbf{p} | r' \rangle = -i\mathbf{p} \delta(r - r') = i\mathbf{p} \delta(r - r')
$$

$$
= (1/2) (-i\mathbf{p} + i\mathbf{p}') \delta(r - r'),
$$

the operator $(1/2)(-i\mathbf{p} + i\mathbf{p}')$ in the potential may be considered as the momentum operator $\mathbf{p}$. When we use a nonstatic potential $V(r, \mathbf{p})$, the operator $\mathbf{p}$ appears in place of $\mathbf{q}$ in the potential obtained by the Fourier transform only over $\mathbf{k}$,

$$
V(r, \mathbf{p}) = (2\pi)^{-3} \int dk V(k, \mathbf{p}) \exp[ikr]
$$

$$
= (2\pi)^{-3} \int dk V(k, (-i/2)(\vec{\mathbf{p}} + \vec{\mathbf{p}}')) \exp[ikr]. \quad (2.4)
$$

The last form is convenient in transforming $V(r, \mathbf{p})$ into the original $V(k, \mathbf{q})$, since in $V(r, \mathbf{p})$ the momentum operator $\mathbf{p}$ and a potential function $A(r)$ should be in a definite order. This form means that the operation of $\mathbf{p}$ on the wave function is the average of the operation to the left and that to the right, in other words $\mathbf{p}$ appears in the symmetrized form:

$$
(1/2)(p_j A(r) + A(r)p_j),
$$

$$
(1/4) (p_j p_k A(r) + p_k A(r) p_j + p_j A(r) p_k + A(r) p_j p_k), \quad \text{etc.,}
$$

where $j, k$, etc., specify the components of $\mathbf{p}$.

The above argument shows that the $q$-dependence of potential matrix elements in the momentum space is responsible for the nonstatic (momentum-dependent) terms of potential. Thus the difference in generalizing a
potential from a static and local one comes from the different treatments which express the \( q \)-dependence in the coordinate space. It is usually adopted and is proved useful to generalize a potential by adding nonstatic terms, for example, the spin-orbit coupling term. In order to analyze high-energy nucleon-nucleon scattering, it is inevitable to use nonstatic potentials.

After the proposal of the complete experiments on nucleon-nucleon scattering, the general form of nonrelativistic potentials in the coordinate space was discussed by Puzikov, Rindin and Smorodinsky and by Okubo and Marshak. Later Goto and Machida discussed the general form of the potential in the momentum space (see Chapter 2, §2). Also it has been attempted to replace the hard-core like repulsive effect by the velocity-dependent potential. It is to be noted, however, that in describing the two-nucleon interaction in region III (especially in the core region) we can find no reason for using the nonstatic potential preferentially and rather the nonlocal representation of the interaction may be important in order to describe the effects connected with the structure of the nucleon core (see Chapter 7). At present we can say that the use of nonstatic terms is suitable for describing regions I and II.

2.2 Form of the potential

The general expression for the two-nucleon nonrelativistic potential in the coordinate space is given by Okubo and Marshak in the following form:

\[
V = V_0 + (\mathbf{a}_1 \cdot \mathbf{a}_2) V_\sigma + (\mathbf{L} \cdot \mathbf{S}) V_{LS} + S_{12} V_T
+ \frac{1}{2} \left( (\mathbf{a}_1 \cdot \mathbf{L}) (\mathbf{a}_2 \cdot \mathbf{L}) + (\mathbf{a}_2 \cdot \mathbf{L}) (\mathbf{a}_1 \cdot \mathbf{L}) \right) V_{QLS} + (\mathbf{a}_1 \cdot \mathbf{p})(\mathbf{a}_2 \cdot \mathbf{p}) V_p + \text{h.c.}
\]

with

\[
\mathbf{L} = \mathbf{r} \times \mathbf{p},
\]

\[
\mathbf{S} = (\mathbf{a}_1 + \mathbf{a}_2)/2,
\]

\[
S_{12} = 3(\mathbf{a}_1 \cdot \mathbf{r})(\mathbf{a}_2 \cdot \mathbf{r})/r^2 - \mathbf{a}_1 \cdot \mathbf{a}_2,
\]

where the suffices 1 and 2 specify two nucleons, and h.c. means the Hermitian conjugate. The potential functions \( V_\alpha \) (\( V_0, V_\sigma, V_{LS}, V_T, V_{QLS} \) and \( V_p \)) consist of the isospin-independent part \( V_\alpha^{(i)} \) and the isospin-dependent part \( V_\alpha^{(d)} \):

\[
V_\alpha = V_\alpha^{(i)} + V_\alpha^{(d)}(\mathbf{\tau}_1 \cdot \mathbf{\tau}_2),
\]

where \( V_\alpha^{(i)} \) and \( V_\alpha^{(d)} \) can be regarded as the functions of \( \rho^2 \), \( p^2 \) and \( L^2 \).

The expression for a potential (2.5) is most general, but there are too many degrees of freedom when the \( p^2 \) and \( L^2 \) dependence in \( V_\alpha \) are completely phenomenological. Therefore some restrictions should be imposed on the \( p^2 \) and \( L^2 \) dependence. The six independent components in (2.5) are necessary, since the off-energy shell matrix element is needed to
know the distortion of the wave function. In the scattering matrix related only with the energy shell, five components are independent and the matrix element of the \( V_p \) term is, for example, expressed by those of the other five terms. In order to reproduce the energy dependence of a partial wave phase shift, it is sufficient to assume that \( V_a \) depends on \( r^2 \) and \( L^2 \) and not necessarily on \( \rho^2 \). If inclusion of \( \rho^2 \) dependence or \( V_p \) term is indispensable, it is preferable to make calculation in the momentum space where such explicit momentum-dependent terms are treated in a direct and unambiguous way. Thus, to retain the merit of the potential model in the coordinate space, it is reasonable to adopt the following form of a potential:

\[
V = V_0 + (\sigma_1 \cdot \sigma_2) V_a + (L \cdot S) V_{LS} + S_{LS} V_L + 1/2 \{(\sigma_1 \cdot L)(\sigma_2 \cdot L) + (\sigma_2 \cdot L)(\sigma_1 \cdot L)\} V_{QLS},
\]

where \( V_0 \), etc., are functions of \( r^2 \) and \( L^2 \).

For a sufficiently large \( r \), the potential should reduce to the one-pion-exchange potential (OPEP), because it has been established in our previous review that the outermost part (region I) of the potential is correctly explained by the one-pion-exchange process. The OPEP given by Hoshizaki and Machida is written in the coordinate space including the \( (\mu/M)^2 \) nonstatic correction as follows (see Chapter 3, §8):

\[
V(\text{OPEP}) = \frac{f^2}{4\pi} \mu (r_1 \cdot r_2) \frac{e^{-r}}{x^2} \left[ \frac{1}{3} \left\{ (\sigma_1 \cdot \sigma_2) + S_\mu(1 + \frac{3}{x^2} + \frac{3}{x^4}) \right\} \right.

\left. + \frac{1}{2} \left\{ \frac{\mu}{M} \right\}^2 \frac{1}{x^2} \left( 1 + \frac{3}{x^2} + \frac{3}{x^4} \right) \frac{1}{2} \left\{ (\sigma_1 \cdot L)(\sigma_2 \cdot L) + (\sigma_2 \cdot L)(\sigma_1 \cdot L) \right\} - (\sigma_1 \cdot \sigma_2) L^2 \right],
\]

where \( \mu \) is the pion mass, \( x = \mu r \) and \( M \) is the nucleon mass. The minus sign corresponds to the \( ps(pv) \)-coupling with the Lagrangian density

\[
L' = i \frac{f}{\mu} \bar{\psi} \tau_\alpha \gamma_\mu \gamma_5 \psi \partial_\mu \phi_\alpha,
\]

and the plus sign corresponds to the \( ps(ps) \)-coupling with the Lagrangian density \( (f = g\mu/2M) \)

\[
L' = ig \bar{\psi} \tau_\alpha \gamma_5 \gamma_\mu \phi_\alpha.
\]

The first term is the well-known static OPEP. The \( (\mu/M)^2 \) term, the so-called quadratic spin-orbit term, represents the part of nonstatic effects which corresponds to the form (2.6). There are other nonstatic one-pion-exchange effects which cannot be represented by a potential of the form (2.6). As is shown in Chapter 2, the nonstatic one-pion-exchange effects
vanish altogether in the Born approximation (on the energy shell). These effects appear only when the distortion of wave function due to the total potential takes place. Also the nonstatic term of the OPEP is not uniquely defined, although the quadratic spin-orbit potential in Eq. (2.7) is derived in the most straightforward manner. Therefore the quadratic spin-orbit term in Eq. (2.7) should not be considered as definite, but may be considered as giving some guess about the amount of nonstatic effects. Therefore, we do not use it in its original form, but multiply it by a phenomenological reduction factor $\lambda$. In this sense, although the nonstatic (quadratic spin-orbit) term of the OPEP in Eq. (2.7) provides the theoretical standard for the asymptotic form and the limit of the order of magnitude, its sign and strength should be considered phenomenologically.

The sign of the quadratic spin-orbit potential in Eq. (2.7) differs in the $ps$- and in the $pv$-coupling. Using this fact, Taketani and Machida have proposed the possibility of discriminating between the $ps$- and the $pv$-coupling. As will be discussed in §3, however, we have at present no definite conclusion concerning this point.

In the following of this chapter, we use the following shape of potential. Concerning $V_o$, $V_o$, $V_r$ and $V_{ols}$, the asymptotic forms for a large $x$ are taken so that they become the OPEP (Eq. (2.7)) and modifications in the inner regions (II and III) are represented by phenomenological parameters $a$ and $b$, so that the potentials have the form

$$V(\text{OPEP}) \times (1 + ae^{-x}/x + be^{-2x}/x^2).$$

(2.8)

Usefulness of this parameterization, first proposed by Hamada, has been confirmed by Signell and Yoder. We call these the Hamada-type potentials.

For lack of the spin-orbit term in the OPEP, the outer part of the spin-orbit potential $V_{ls}$ should be given by the two-pion-exchange and the one-boson-exchange processes. The two-pion-exchange process cannot give a strong $V_{ls}$, while a one-scalar- or a one-vector-exchange process gives an appreciable $V_{ls}$ of the form as (see Chapter 3, §3.2):

$$V = -m \frac{g_s^2}{4\pi} \frac{e^{-y}}{y} - m \frac{g_s^2}{4\pi} \left( \frac{m}{M} \right)^2 \frac{1}{2} \frac{1}{y} \frac{e^{-y}}{y^3} (L \cdot S) + \ldots$$

(2.9)

for a scalar meson with mass $m$,

$$V = m \left( \frac{g_s^2}{4\pi} \right)_{\text{eff}} \frac{e^{-y}}{y} - m \left( \frac{f_s^2}{4\pi} \right) \frac{e^{-y}}{y} \left( \frac{1}{y^3} \right) (L \cdot S) + \ldots$$

(2.10)

with

$$-m \frac{m}{M} \frac{g_s^2}{4\pi} (L \cdot S)_{\text{eff}} \left( \frac{1 + 1}{y^3} \right) (L \cdot S) + \ldots$$
Potential Model Approach

\[
\left( g_\pi^2 \right)^{(c)} = g_\pi^2 + \frac{2}{3} \left( \frac{f_\pi^2}{4\pi} \right) (\sigma_1 \cdot \sigma_2)
\]

and

\[
\left( g_\rho^2 \right)^{(LS)} = g_\rho^2 + \frac{8}{3} M \frac{g_\rho f_\pi}{4\pi} + \frac{f_\rho^2}{4\pi}
\]

for a vector meson with mass \( m \). Here \( y = mr = (m/\mu)x \). Coupling constants are defined in the Lagrangian density as

\[
L' = g_\rho \bar{\psi} \rho \phi 
\]
for scalar meson,

and

\[
L' = g_\rho \bar{\psi} \gamma_\mu \gamma_5 \rho \phi \mu + \frac{f_\rho}{2m} \bar{\psi} \gamma_\mu \gamma_5 \gamma_\nu - \frac{f_\rho}{2i} \bar{\psi} (\partial_\mu \phi_\nu - \partial_\nu \phi_\mu)
\]

for vector meson.

In order to include these shapes of \( V_{LS} \), we take the shape of \( V_{LS} \) as

\[
V_{LS} = \mu \sum_i G_i \exp[-\mu, x|x^i].
\]

The operator \( 1/2 \{ (\sigma_1 \cdot L) (\sigma_2 \cdot L) + (\sigma_3 \cdot L) (\sigma_1 \cdot L) \} \) does not vanish in the singlet state. It is convenient to replace this operator with the noncentral operator \( W_{12} \):

\[
W_{12} = 1/2 \{ (\sigma_1 \cdot L) (\sigma_2 \cdot L) + (\sigma_3 \cdot L) (\sigma_1 \cdot L) \} - 1/3 (\sigma_1 \cdot \sigma_2) L^2
\]

\[
= (L \cdot S)^2 - (\partial L) + 1/3 (\sigma_1 \cdot \sigma_2) L(L+1).
\]

The operator \( W_{12} \) is also written as the scalar product of two irreducible tensors, the spin tensor \( S^{(2)}_\mu \) and the space tensor \( L^{(2)}_\mu \) with the rank 2:

\[
W_{12} = \sum_{\mu=2}^2 (-1)^\mu S^{(2)}_\mu L^{(2)}_{-\mu}
\]

with

\[
S^{(2)}_\mu = (\sigma_1 \times \sigma_2)_\mu = \sum_{\alpha+\beta+\mu} (11\alpha\beta | 2\mu) \delta_{\alpha\beta}\delta_{\mu}
\]

and

\[
L^{(2)}_\mu = (L \times L)_\mu = \sum_{\alpha+\beta+\mu} (11\alpha\beta | 2\mu) (L_\alpha L_\beta + L_\beta L_\alpha)/2,
\]

where \((11\alpha\beta | 2\mu)\) is the Clebsh-Gordan coefficient and \(A_{\pm1} = \mp(A_+ \pm iA_-)/\sqrt{2},\) \(A_0 = A_+ + A_- \) for \( A = L, \sigma_1 \) and \( \sigma_2 \). The noncentral operator \( W_{12} \) is zero in the singlet state as well as \( S_{12} \) and \( L \cdot S \). The weighted mean of \( W_{12} \) vanishes as well as \( L \cdot S \) and the diagonal elements of \( S_{12} \) in the triplet state:

\[
\sum_{J=J,L \pm l} <LJ | W_{12} | LJ> = 0. \tag{2.14} \]

* In this chapter, the state is specified by the spin \( S \), the isospin \( T \), the relative orbital angular momentum \( L \) and the total angular momentum \( J \).
Thus the potential form (2·6) is rewritten as

\[ V = V_c + S_{12} V_T + (\mathbf{L} \cdot \mathbf{S}) V_{LS} + W_{12} V_w, \quad (2·6') \]

where \( V_c, V_T, V_{LS} \) and \( V_w \) are functions of \( r^2 \) and \( L^2 \) in each state with the spin \( S \) and the isospin \( T \). \( V_c \) means the central potential involving \( V_0, (\sigma_1 \cdot \sigma_2) V_o \) and the \( L^2 \) term, \( 1/3 \cdot (\sigma_1 \cdot \sigma_2) \not{\mathbf{L}} \cdot \not{\mathbf{L}} V_{QS} \), added owing to the use of \( W_{12} \).

Although the potential (2·6') is not quite general, there are still too many degrees of freedom to determine them effectively by comparison with the experimental data. So, if possible, we prefer to exclude the \( L^2 \) dependence of \( V' \)s except when it is indispensable. The \( L^2 \) dependence in \( V_c \) should be taken into account on equal footing with \( 1/2 \{ (\sigma_1 \cdot \not{\mathbf{L}})(\sigma_2 \cdot \not{\mathbf{L}}) + (\sigma_2 \cdot \not{\mathbf{L}})(\sigma_1 \cdot \not{\mathbf{L}}) \} \). These quadratic spin-orbit terms are small \((\sim (\mu/M)^2)\) of the main parts of potential) and effective only for high partial waves affected mainly by the outer part of potential. It seems reasonable therefore to adopt only the \( L^2 \) term with the asymptotic form (2·7) of the OPEP in \( V_c \), and no \( L \) dependence in other radial functions of potential. It should be noted that, since the \( L^2 \) term comes from the expansion of potential in \( L^2 \), the application of this term and the \( W_{12} \) term is to be limited to not too high \( L \) states (for example, \( L \lesssim 6 \) at \( E \lesssim 300 \) MeV), and that for the higher angular momentum states the phase shifts are to be replaced by the one-pion-exchange contributions (the same as the Born approximation values).

Taking the above considerations into account, we use the following form of the potential in regions I and II for each state specified by the spin \( S \) and the isospin \( T \) (or the spin \( S \) and the parity \( II \)):

\[
\begin{align*}
V &= V_c(x) + S_{12} V_T(x) + (\mathbf{L} \cdot \mathbf{S}) V_{LS}(x) + W_{12} V_w(x) + L^2 V_{LL}(x), \quad (2·15) \\
V_c(x) &= \left( \frac{f^2}{4\pi} \right) \mu \frac{(\tau_1 \cdot \tau_2)}{3} (\sigma_1 \cdot \sigma_2) Y(1 + a_1 Y + b_1 Y^2), \\
V_T(x) &= \left( \frac{f^2}{4\pi} \right) \mu \left( \frac{\not{\mathbf{r}}_1 \cdot \not{\mathbf{r}}_2}{3} \right) X(1 + a_T Y + b_T Y^2), \\
V_{LS}(x) &= \mu \left( G_1 \frac{e^{-i \alpha x}}{x^{s_1}} + G_2 \frac{e^{-i \beta x}}{x^{s_2}} \right), \\
V_w(x) &= \left( \frac{f^2}{4\pi} \right) \mu \left( \frac{\not{\mathbf{r}}_1 \cdot \not{\mathbf{r}}_2}{2M^2} \right) (\sigma_1 \cdot \sigma_2) \frac{X}{x^2} (\lambda + a_w Y + b_w Y^2), \\
V_{LL}(x) &= - \left( \frac{f^2}{4\pi} \right) \mu \left( \frac{\not{\mathbf{r}}_1 \cdot \not{\mathbf{r}}_2}{3M^2} \right) (\sigma_1 \cdot \sigma_2) \frac{X}{x^2} (\lambda + a_{LL} Y + b_{LL} Y^2),
\end{align*}
\]

where \( Y = e^{-\gamma/x} \) and \( X = e^{-\gamma/x} \cdot (1 + 3/x + 3/x^2) \).

The effective potential in each state is given by Eq. (A·5) and summarized in Table A·1 in the Appendix. It should be noted that the
use of $W_{22}V_{w}$ and $L^3V_{LL}$ terms is a phenomenological substitute for the net nonstatic effects other than the $L \cdot S$ term. As shown in Eq. (A·5), $L^3V_{LL}$ is included in a $L$ dependence of $V_c$ and $W_{12}V_{w}$ in that of the diagonal elements of $S_{12}V_{T}$. Excepting the cases where the tensor coupling term plays an important role, the use of such nonstatic terms is equivalent to introducing an angular momentum dependence with a reasonable order of magnitude into $V_c$ and $V_T$.\(^*\)

Potentials in region III are treated phenomenologically. There are a variety of treatments: To extend the shape of Eq. (2·15) inward with introducing a hard or soft repulsive core, to adopt the square well shape with or without a repulsive core, to impose a suitable boundary condition, to assume some velocity dependence, and so forth. In any case, the phenomenological description in region III should be chosen to clarify the basic problems in this region such as: (1) Whether the repulsive core is present in other states than the singlet even state, (2) where spin-orbit potentials are dominant, (3) whether velocity dependence of potential is necessary or not, and whether a nonlocal interaction is necessary or not, and so on. In solving these problems, a certain clue to the future theory may be found. The validity of phenomenology on the interaction in region III depends on the reliability of our knowledge of the potential in region II, because we can hardly take definite information on region III in a way without depending upon the ambiguities involved in region II. In the following of this chapter, we shall show that the quantitative nature of the potential in region II has been established fairly well.

§3. Singlet even state

3·1 Experimental data on the effective range parameters and singlet S phase shifts near 50 MeV

The low energy scattering parameters determined by available data are summarized in Table 3·1. The definitions of the parameters are:

\begin{equation}
np \text{ and } nn: \ n p \cot \delta_0 = - \frac{1}{i a} + \frac{1}{2} r_c k^2 - \frac{i}{2} P \cdot (r_c)^3 k^4 + \frac{i}{2} Q \cdot (r_c)^3 k^6 + \cdots, \tag{3·1}
\end{equation}

\begin{equation}
pp: \ K/R \equiv C k \cot \delta_0^p + h(\eta)/R = - \frac{1}{i a_c} + \frac{1}{2} r_c k^2 - \frac{i}{2} P \cdot (r_c)^3 k^4 + \frac{i}{2} Q \cdot (r_c)^3 k^6 + \cdots, \tag{3·2a}
\end{equation}

where the laboratory energy $E = 2h^2 k^2 / M$, \( \eta = e^2 / \hbar \cdot c \cdot v \cdot \text{rel} \), the Coulomb screening

\(^*\) $V_{w}$ and $V_{LL}$ used here correspond to the quadratic spin-orbit potentials used by several authors as follows: For HJ-potential, $V_{w} = - V_{LL}(HJ)$ and $V_{LL} = (2/3)(\sigma_1 \cdot \sigma_2) V_{LL}(HJ)$. For Yale potential, $V_{w} = 0$ and $V_{LL} = - \delta_{LL} V_{Q}(Yale)$. For the potentials used by Tamagaki, Wada and Wataru (TWW), $V_{w} = - V_{Q}(TWW)$ with $a_{w} = a_{Q}(TWW)$ and $b_{w} = b_{Q}(TWW)$ and $V_{LL} = - (\sigma_1 \cdot \sigma_3 / 3) V_{Q}(TWW) + V_{LL}(TWW)$ with $a_{LL} = 1/2 \cdot (3a_{LL}(TWW) - a_{Q}(TWW))$ and $b_{LL} = \frac{1}{3} \cdot (3b_{LL}(TWW) - b_{Q}(TWW))$.\(^*\)
factor \( C^2 = 2\pi \eta / (\exp 2\pi \eta - 1) \), the proton Bohr radius \( R = \hbar^2 / M e^2 = 28.815 \times 10^{-18} \) cm and \( h(\eta) = \text{Re} \left( \Gamma'(-i\eta)/\Gamma(-i\eta) \right) - \ln \eta \). The nuclear phase shift \( \delta^*_l \) of the \( L \)-th partial wave is defined as

\[
W_L(r) \rightarrow \cos \delta^*_l F_L(r) + \sin \delta^*_l G_L(r) \rightarrow \sin (kr - \eta \ln 2kr + \sigma_L + \delta^*_l),
\]

where \( W_L(r) \) is the radial wave function determined by the pure Coulomb plus nuclear potential, \( F_L(r) \) and \( G_L(r) \) are the regular and the irregular Coulomb wave functions respectively.

For \( p-p \) scattering the vacuum polarization effect should be taken into account. Foldy and Eriksen\(^7\) first pointed out the importance of including this effect in low energy \( p-p \) scattering. The vacuum polarization effect is caused by the vacuum polarization potential: *)

\[
V_{vp}(r) = \frac{2\alpha e^2}{3\pi r} \int_0^\infty d\xi e^{-2\xi r} \left( \frac{1}{\xi^2 + \frac{1}{2\xi^4}} \right) (\xi^2 - 1)^{1/2},
\]

which is very weak but long-ranged with \( \alpha = e^2/hc = 1/137.04 \) (the fine structure constant) and \( \kappa^{-1} = 386.2 \times 10^{-13} \) cm (the electron Compton wavelength). In the presence of \( V_{vp} \), \( \delta^*_l \) is replaced by \( \delta^*_l \), nuclear phase shift in the "electric potential" (Coulomb plus \( V_{vp} \)). If the regular and the irregular solutions of the radial wave function in the electric potential are \( S_L(r) \) and \( T_L(r) \) respectively, then with the nuclear potential on, the radial wave function \( R_L(r) \) becomes

\[
R_L(r) \rightarrow \cos \delta^*_l S_L(r) + \sin \delta^*_l T_L(r).
\]

This in turn becomes

\[
R_L(r) \rightarrow \cos K_L F_L(r) + \sin K_L G_L(r),
\]

at distance large compared with the range of \( V_{vp} \). The phase shift \( K_L \) may be the sum of the vacuum polarization phase shift \( \tau_L \) and \( \delta^*_l \):

\[
K_L = \delta^*_l + \tau_L,
\]

where in the sufficient accuracy the Born approximation gives

\[
\tau_L = - \frac{M}{\hbar^2 k} \int_0^\infty V_{vp}(r) F_L^* (r) dr,
\]

which is estimated as \( 0 > \tau_L > -0.1^\circ \).\(^7\) The effective range expansion for \( \delta^*_l \) similar to Eq. (3.2a) has been given by Heller\(^6\) (treating the vacuum polarization to the first order) as follows:

\[
H(k) = C^2 k \left[ (1 + 2\zeta_0) \cot i\delta^*_0 - \tau_0 \right] + 2\eta k \left[ h(\eta) + l_0(\eta) \right]
= - \frac{1}{\delta ac} + \frac{1}{2} \gamma e k^2 - \frac{1}{2} P \cdot (r_x) e k^4 + \frac{1}{2} Q \cdot (r_x) e k^6 + \cdots,
\]

where \( * \) See reference 74) for a bibliography on the history and derivation of the vacuum polarization potential.
where

\[ z_0 = -\frac{M^2}{\hbar^2 k} \int_0^\infty V_{ss}(r) F_0(r) G_0(r) \, dr, \]

\[ l_0(\eta) = -e^2 \int_0^\infty V_{ss}(r) \left[ (CG_0)^2 - (CG_0)^2_{\eta=0} \right] \, dr. \]

Strictly speaking, the expansion parameters defined in Eq. (3.2a) are not equal to those defined in Eq. (3.2b). However, the vacuum polarization potential is very weak compared with the pure Coulomb plus nuclear potentials, and there is no essential difference between the results obtained from \( \eta \) with Eq. (3.2b) and those from \( \delta \) with Eq. (3.2a) in which

\[ K_L = \delta_l + \Delta_l \]

with

\[ \Delta_l = -\left( \frac{M}{\hbar^2 k} \right) \int_0^\infty V_{ss}(r) W_2(r) \, dr \]

should be used.\(^{33,74}\) According to Heller's estimate,\(^{74}\) only the difference in \( \delta_c \) (\( \delta_c \) from Eq. (2.3b) \(-\delta_c \) from Eq. (2.3a) \(\equiv 0.02 \times 10^{-13} \text{ cm} \)) is to be taken into account. Differences in the other parameters are small in view of the present uncertainties of these parameters shown in Table 3.1. In analyzing the experimental data, the potential model independent treatment in Eq. (3.2b) is preferable, while in the analysis by a potential model the treatment in Eq. (3.2a) is convenient. Therefore, we adopt Eq. (3.2a) in our calculations and take account of the above small difference as uncertainties in \( \delta_c \).

The most accurate values of the effective range parameters for \( p-p \) scattering are obtained by Gursky and Heller.\(^{73}\) Among them the values of the cubic fit including the \( k^6 \) term are probably far from reality, since the ambiguities in \( ^1Q \) strongly affect the other parameters and \( ^1Q = 0.17 \) is too large to be consistent with the values \( (^1Q \sim 0.01) \) calculated from reasonable potentials (see Table 3.2). We adopt the following \( p-p \) values:

\begin{align*}
^1a_c &= (-7.82 \pm 0.02) \times 10^{-18} \text{ cm}, \\
^1r_s(pp) &= (2.80 \pm 0.03) \times 10^{-12} \text{ cm}, \quad (3.3) \\
^1P(pp) &= 0.01 \sim 0.04.
\end{align*}

At present \( ^1P(pp) \) is yet uncertain, since very accurate \( ^1S_0 \) phase shifts with errors less than 0.1% are necessary to determine \( ^1P(pp) \). So we are obliged mainly to use the data on \( ^1a_c \) and \( ^1r_s(pp) \). The data on \( ^1P \) cannot be a condition to restrict the singlet even potential, since the potentials with the OPEP tail consistent with \( ^1a_c \) and \( ^1r_s(pp) \) give \( ^1P(pp) \) in the range of Eq. (3.3), as is shown in §3.2.
Table 3-1. Effective range expansion parameters in the singlet even state. In the first three rows, $a$ means $a_c$, the scattering length with the Coulomb potential. The lowest row shows the parameters calculated from the nuclear potentials shown in Table 3-2 in the absence of the Coulomb potential, which are consistent with the $\delta(S_0)$ data of p-p scattering below 310 MeV.

<table>
<thead>
<tr>
<th></th>
<th>$a(10^{-13} \text{cm})$</th>
<th>$r_c(10^{-13} \text{cm})$</th>
<th>$P$</th>
<th>$Q$</th>
<th>comments and references</th>
</tr>
</thead>
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<td>$p\bar{p}$</td>
<td>$-7.815 \pm 0.008$</td>
<td>$2.795 \pm 0.025$</td>
<td>$0.029 \pm 0.014$</td>
<td>$0$</td>
<td>quadratic fit $75$</td>
</tr>
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<td>$p\bar{p}$</td>
<td>$-7.823 \pm 0.012$</td>
<td>$2.866 \pm 0.078$</td>
<td>$0.114 \pm 0.093$</td>
<td>$0.17 \pm 0.18$</td>
<td>cubic fit $75$</td>
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<td>$p\bar{p}$</td>
<td>$-7.824 \pm 0.0080$</td>
<td>$2.794 \pm 0.026$</td>
<td>$0.026 \pm 0.014$</td>
<td>$0$</td>
<td>quadratic fit $76$</td>
</tr>
<tr>
<td>$nn$</td>
<td>$-20 \sim -26$</td>
<td>$-18 \pm 3$</td>
<td>$-16.1 \pm 1.0$</td>
<td>$-16.4 \pm 1.9$</td>
<td>$n+d \rightarrow n+n+p$ $77a$</td>
</tr>
<tr>
<td>$nn$</td>
<td>$-20 \sim -26$</td>
<td>$-18 \pm 3$</td>
<td>$-16.1 \pm 1.0$</td>
<td>$-16.4 \pm 1.9$</td>
<td>$t+d \rightarrow n+n+He^3$ $78$</td>
</tr>
<tr>
<td>$np$</td>
<td>$-23.678 \pm 0.028$</td>
<td>$2.51 \pm 0.15$</td>
<td>$0.08 \pm 0.08$</td>
<td>$0$</td>
<td>including uncertainty $79$</td>
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<tr>
<td>$np$</td>
<td>$-23.675 \pm 0.095$</td>
<td>$2.69 \pm 0.18$</td>
<td>$0.08 \pm 0.08$</td>
<td>$0$</td>
<td>quadratic search for $E&lt;10$ MeV $80$</td>
</tr>
<tr>
<td>$pp$ (nuclear)</td>
<td>$-16.5 \sim -17.5$</td>
<td>$2.86 \sim 2.94$</td>
<td>$-$</td>
<td>$-$</td>
<td>calculated from potentials in Table 3-2</td>
</tr>
</tbody>
</table>

The nuclear $p-p$ scattering length $a(pp)_N$ in the absence of the Coulomb potential, which is the counterpart of $a(np)$ and $a(nn)$, is calculated from $a_c$. The calculated $a(pp)_N$ depends more or less on the assumed shape of potential and lies in the region $a(pp)_N=(-15 \sim -20) \times 10^{-13}$ cm.$^{13}$

For the square well potential, the relation

$$1/a(pp)_N = 1/a_c - 1/R \cdot \ln(r_c/R) + 0.330$$

leads to $a(pp)_N \sim -17 \times 10^{-13}$ cm.$^{30}$ Recent potential model calculations have given the values very close to this.$^{83}$ In the last row of Table 3-1, we show the nuclear $p-p$ effective range parameters $(a(pp)_N, r_c(pp)_N, etc.)$ calculated from the potentials consistent with the $p-p$ data given in 3-2. The reasonable potentials with the OPEP-tail gives $a(pp)_N = (-16.5 \sim -17.5) \times 10^{-13}$ cm.

The experimental value of the $n-n$ scattering length $a(nn)$ was estimated as $a(nn) = (-20 \sim -26) \times 10^{-13}$ cm by analysing the final state interaction in the deuteron breakup reaction $n+d \rightarrow n+n+p$. This should be compared with the nuclear $p-p$ scattering length $a(pp)_N = (-16.5 \sim -17.5) \times 10^{-13}$ cm and it seemed to imply slight violation of the change symmetry of nuclear forces. However, the recent experiments on $t+d \rightarrow n+n+He^3$ and $\pi^-+d \rightarrow n+n+\gamma$ has shown $a(nn) = (-15 \sim -18) \times 10^{-13}$ cm, which covers the range of $a(pp)_N$. Thus, at present, as far as it is concerned with the two-nucleon problems, the charge symmetry of nuclear forces...
seems to hold fairly well. In order to obtain more detailed information on the charge symmetry of nuclear forces, further improvement on experimental accuracy and the treatment of three-body final state interactions is necessary. Some evidence for the violation of the charge symmetry may be obtained through investigations of the binding energy and the energy level of very light nuclei.

The apparent discrepancy between \( a(pp)_N \) and \( a(np)_N \) indicates that in the singlet even state the \( n-p \) central potential \( V^*_c(np) \) is slightly more attractive than the \( p-p \) central potential \( V^*_c(pp) \). As was already discussed in the previous review,\(^{19}\) this slight but definite charge-dependent effect in nuclear forces is partially explained by the mass difference between the neutral and the charged pions, since for the smaller pion mass the range of the OPEP becomes longer and its strength weaker, and the net effect makes \( V^*_c(pp) \) weaker than \( V^*_c(np) \).\(^8\) The discrepancy between \( r_s(pp)_N \) and \( r_s(np)_N \) may imply a slight charge dependence of nuclear forces. However, the situation is not yet clear, since the possibility \( r_s(np)_N > 2.70 \times 10^{-13} \) cannot be excluded, as recently discussed by Breit, Friedman and Seamon.\(^8\)

As to the data above 10 MeV the results of the modified phase shift analysis are shown in figures in Appendix of Chapter 5. Since the potentials other than the singlet even central potential \( V^*_c \) vanish in the \( 1S_0 \) state and effects of a static potential are dominant below 100 MeV, the \( 1S_0 \)-phase shift \( \delta(1S_0) \) below 100 MeV is important to determine the static central potential \( V^*_c \). In this energy region, experimental data are now being accumulated about the parameters of spin correlations between the scattered and the recoil protons (\( C_{KF} \) and \( C_{KN} \)) and about the triple scattering parameters (\( D, A, R \)). In this chapter we use the solutions of the phase shift analysis near 50 MeV: The experimental value of the \( 1S_0 \)-phase shift is

\[
\delta(1S_0) = -36.3^\circ \sim 38.5^\circ \text{ at } E = 50 \text{ MeV}.\quad (88)-(89)
\]

In the rest of this section, it will be shown to what extent the singlet even central potential \( V^*_c \) in region II \((0.7 \leq x \leq 1.5)\) is determined by the \( pp \) low energy parameters and \( \delta(1S_0) \) at 50 MeV. The nonstatic potential in the singlet even state, \( V^*_{L} \), will be discussed in connection with \( \delta(1D_2) \) and \( \delta(2G_4) \) with use of the \( V^*_c \) in region II thus determined.

3.2*) Quantitative properties of the singlet even central potential in region II

Since the range of the outer part of the potential is fixed by the pion mass, we can obtain, phenomenologically, useful information on the potential shape even at the low energies where the shape-independent approximation

*) Hereafter the natural unit \( \hbar = c = 1 \) is adopted.
is valid. The analysis of the effective range parameters has shown that, owing to the weakness of the OPEP-tail in this state, the singlet even central potential \( V_0 \) is inevitably very strong in region II.\(^{23} \) It is possible to determine the strength of this attraction by the use of the recent experimental values of \( a_c \) and \( r_s(\pi p) \) and the data on \( \delta(\pi S_0) \) below \( E=50 \) MeV, as is shown in the following.

In analyzing the data we use the Hamada-type potential given in Eq. (2.15) outside the hard core radius \( x_c \),

\[
V_0^\ast = \begin{cases} 
-(f^2/4\pi)\mu e^{-x}/x(1+a_0 e^{-x}/x+b_0 e^{-2x}/x^2) & \text{for } x \geq x_c, \\
+\infty & \text{for } x < x_c, \quad (3.4)
\end{cases}
\]

with \( \mu = \mu_0 = 135.1 \) MeV (\( \pi^0 \) mass) and \( x = \mu r \). If we assume some suitable values for \( f^2/4\pi \) and \( x_c \), then the condition that the value of the scattering length calculated by this potential is equal to the experimental value, \( a_c = (-7.82 \pm 0.02) \times 10^{-13} \) cm, imposes a relation between \( a_c \) and \( b_c \). This relation is approximately expressed by a straight line. For example, for \( f^2/4\pi = 0.08 \) and \( x_c = 0.34646 \)

\[
a_c/16.0 + b_c/23.0 = 1 \quad (3.5)
\]

(see Fig. 3.1). This relation means that an increase of attraction in region II should be compensated by a decrease in region III. The effective range \( r_s(\pi p) \) and the phase shift \( \delta(\pi S_0) \) at \( E=50 \) and \( 310 \) MeV calculated from the potentials satisfying Eq. (3.5) are shown in Fig. 3.2. Comparing this result with the experimental values \( r_s(\pi p) = (2.80 \pm 0.03) \times 10^{-13} \) cm and \( \delta(\pi S_0) = 36.3^\circ \sim 38.5^\circ \) at 50 MeV, we obtain the potential parameter \( a_c \) compatible with the experimental data below 50 MeV as

\[
a_c = 6.7 \rightarrow 8.0
\]

and consequently

\[
b_c = 13.1 \rightarrow 11.2
\]

for \( f^2/4\pi = 0.08 \) and \( x_c = 0.34646 \). The results for various values of \( f^2/4\pi \) and \( x_c \) are summarized in Table 3.2. For convenience of explanation, we choose the following potential as the standard hard core potential:

\[
\text{Fig. 3.1. Correlation between the parameters (}a_c \text{ and } b_c) \text{ in the Hamada-type singlet even central potentials } V_0^\ast = -\mu(f^2/4\pi)e^{-x}/x(1+ae^{-x}/x + be^{-2x}/x^2) \text{ of Eq. (3.4)}, \text{ which are restricted to reproduce the experimental value of the } p-p \text{ scattering length } a_0 = (-7.82 \pm 0.02) \times 10^{-13} \text{ cm. The curve is for } f^2/4\pi = 0.08 \text{ and } x_c = 0.34646.}
\]
\[ ^1V_\tau^c = \begin{cases} -0.08 \mu e^{-x}/x(1+7.6e^{-x}/x+11.85e^{-2x}/x^2), & \text{for } x \geq x_c = 0.34646, \\ +\infty, & \text{for } x \leq x_c, \end{cases} \]

which gives \( ^1a_c = -7.858 \times 10^{-19} \text{ cm}, \)
\( ^1a(pp)_c = -17.067 \times 10^{-19} \text{ cm}, \)
\( ^1r_c(pp) = 2.823 \times 10^{-18} \text{ cm}, \)
\( ^1P(pp) = 0.028 \)
\( ^1Q(pp) = 0.010. \)

\( \delta(\Sigma_0) \) at 50 MeV, calculated from the potentials which reproduce the experimental values of \( ^1a_c \) and \( ^1r_c(pp) \), becomes larger as the coupling constant \( f^2/4\pi \) gets larger and/or the hard core radius smaller.

In general, the potentials consistent with the experimental values of \( ^1a_c \) and \( ^1r_c(pp) \) give rather large \( \delta(\Sigma_0) \) at 50 MeV. At present, uncertainties in \( \delta(\Sigma_0) \) are large. However, if \( \delta(\Sigma_0) < 37^\circ \) is definitely confirmed, it indicate that the value of \( f^2/4\pi \) is small and/or the hard core radius is large: (e.g. \( f^2/4\pi \equiv 0.07 \) and \( x_c \equiv 0.35 \)).

The singlet even central potential \( ^1V_\tau^c \) thus determined from the data below 50 MeV has the strength in region II which is more attractive than that of the TMO-potential calculated by Taketani, Machida and Ohnuma up to the fourth order in the perturbation expansion and less attractive than that of the KMO potential derived by Konuma, Miyazawa and Otsuki by taking into account the \( (3/2, 3/2) \)-resonance effects in pion-nucleon system, Fig. 3.4 illustrates the situation:

\[ 0 > ^1V_\tau^c (\text{TMO}) > ^1V_\tau^c > ^1V_\tau^c (\text{KMO}). \]  

(3.6)

This feature of \( ^1V_\tau^c \) indicates the existence of some additional effects on the two-pion-exchange process which make \( ^1V_\tau^c \) in region II more attractive than the ordinary two-pion-exchange potential (TPEP). These effects may be the enhancement of the attractive TPEP due to the \( (3/2, 3/2) \)-resonance in pion-nucleon system or the exchange of a scalar meson (Eq. (2.9)) or a vector meson with tensor coupling to nucleon (Eq. (2.10)) between two nucleons, since the other effects modifying \( ^1V_\tau^c (\text{TMO}) \) are repulsive in
Table 3-2:<sup>a</sup> Values of $a_e$ and $b_e$ of the Hamada-type potentials in Eq. (3·4), $V_z = -\mu_e(f^2/4\pi)(1 + a_e^{e^{-x}}/x + b_e^{e^{-x}}/x^2)e^{-x}/x$ outside the hard core radius $x_a$, which are adjusted to $\lambda_{ac}$ and determined from $r_c(pp)$ and $\delta^{(1S_o)}$ at 50 MeV, for various $x_a$ and $f^2/4\pi$. $^1P$, $^1Q$ and $\delta^{(1S_o)}$ at 310 MeV are also shown. The arrows mean that the left (right) value on an arrow corresponds to each other.

<table>
<thead>
<tr>
<th>$f^2/4\pi$</th>
<th>$x_a$</th>
<th>from $r_c(pp)$</th>
<th>from $\delta^{(50\text{MeV})}$&lt;sup&gt;a&lt;/sup&gt;</th>
<th>from both</th>
<th>calculated from the left two columns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$a_e$</td>
<td>$b_e$</td>
<td>$a_e$</td>
<td>$b_e$</td>
</tr>
<tr>
<td>0.09</td>
<td>0.24660</td>
<td>6.9→8.3</td>
<td>1.1→-0.4</td>
<td>10.1→12.5</td>
<td>-2.4→-5.0</td>
</tr>
<tr>
<td>0.08</td>
<td>0.3985</td>
<td>1.9→4.1</td>
<td>31.0→27.4</td>
<td>1.0→5.5</td>
<td>32.5→25.2</td>
</tr>
<tr>
<td>0.07</td>
<td>0.34646</td>
<td>5.7→8.0</td>
<td>14.5→11.2</td>
<td>6.7→10.9</td>
<td>13.1→7.1</td>
</tr>
<tr>
<td>0.06</td>
<td>0.2953</td>
<td>8.1→10.0</td>
<td>5.0→2.5</td>
<td>10.0→13.5</td>
<td>2.5→-2.1</td>
</tr>
<tr>
<td></td>
<td>0.24660</td>
<td>10.3→11.8</td>
<td>-1.2→-2.9</td>
<td>12.1→15.0</td>
<td>-3.3→-6.7</td>
</tr>
<tr>
<td>0.05</td>
<td>0.34646</td>
<td>10.2→12.7</td>
<td>11.0→8.3</td>
<td>9.5→14.0</td>
<td>13.0→6.4</td>
</tr>
<tr>
<td>0.04</td>
<td>0.2953</td>
<td>12.8→15.1</td>
<td>1.6→1.4</td>
<td>13.8→17.5</td>
<td>0.3→-4.6</td>
</tr>
<tr>
<td>0.03</td>
<td>0.34646</td>
<td>14.4→16.5</td>
<td>-3.9→-6.4</td>
<td>15.4→18.8</td>
<td>-5.1→-9.2</td>
</tr>
<tr>
<td>0.02</td>
<td>0.19790</td>
<td>15.5→17.3</td>
<td>-7.0→-8.8</td>
<td>16.5→19.9</td>
<td>-8.0→-11.7</td>
</tr>
<tr>
<td>0.01</td>
<td>0.34646</td>
<td>15.5→18.5</td>
<td>9.3→4.9</td>
<td>12.9→19.5</td>
<td>13.0→3.4</td>
</tr>
<tr>
<td>0.00</td>
<td>0.2953</td>
<td>18.5→21.1</td>
<td>-2.2→-5.7</td>
<td>17.2→21.7</td>
<td>-0.6→-6.5</td>
</tr>
<tr>
<td></td>
<td>0.24660</td>
<td>20.1→22.6</td>
<td>-8.0→-11.0</td>
<td>19.2→23.9</td>
<td>-7.0→-12.6</td>
</tr>
<tr>
<td>0.001</td>
<td>0.19790</td>
<td>20.9→23.2</td>
<td>-10.7→-13.2</td>
<td>20.5→24.5</td>
<td>-10.3→-14.7</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.14957</td>
<td>21.2→23.4</td>
<td>-11.5→-13.9</td>
<td>20.9→24.7</td>
<td>-11.3→-15.3</td>
</tr>
</tbody>
</table>

<sup>a</sup> Experimental values for $\lambda_{ac}$, $r_c(pp)$ and $^1P(pp)$ are given in Eq. (3·3). The $^1S_o$ solutions of the phase shift analysis are $\delta^{(1S_o)} = 36.3° \sim 38.5°$ at 50 MeV and $-5.7° \sim -11.4°$ at 310 MeV.
Table 3.2a. The results corresponding to those in Table 3.2, for the superposition of one-boson-exchange potential (OBEP):

\[ V'_1 = \mu_1 \left( \frac{f^1}{4\pi} e^{-x} / x + A e^{-x} / x + B e^{-x} / x \right) \]
outside the hard core radius \( x_c \) (Eq. (3.4a)).

<table>
<thead>
<tr>
<th>( f^1 / 4\pi )</th>
<th>( x_c )</th>
<th>( r_c (pp) ) from</th>
<th>( \delta_8 (50 \text{ MeV}) ) from both</th>
<th>( \delta_8 (310 \text{ MeV}) ) calculated from the left two columns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( A )</td>
<td>( B )</td>
<td>( A )</td>
</tr>
<tr>
<td>0.08</td>
<td>0.34646</td>
<td>-13.3→ -9.5</td>
<td>14.9→ 3.8</td>
<td>-13.1→ -8.0</td>
</tr>
<tr>
<td>0.29523</td>
<td>-15.0→ -12.1</td>
<td>24.4→16.2</td>
<td>-15.5→ -11.5</td>
<td>25.9→14.7</td>
</tr>
<tr>
<td>0.24660</td>
<td>-16.0→ -13.6</td>
<td>29.4→23.0</td>
<td>-17.0→ -13.3</td>
<td>32.2→22.2</td>
</tr>
<tr>
<td>0.19790</td>
<td>-16.4→ -14.2</td>
<td>31.8→26.1</td>
<td>-17.8→ -14.2</td>
<td>35.5→26.1</td>
</tr>
<tr>
<td>0.14957</td>
<td>-16.6→ -14.4</td>
<td>32.8→27.3</td>
<td>-18.1→ -14.8</td>
<td>36.7→28.2</td>
</tr>
<tr>
<td>0</td>
<td>-16.7→ -14.5</td>
<td>33.3→27.8</td>
<td>-18.2→ -15.0</td>
<td>36.8→28.9</td>
</tr>
<tr>
<td>0.07</td>
<td>0.29523</td>
<td>-18.4→ -15.6</td>
<td>33.8→25.7</td>
<td>-17.2→ -12.9</td>
</tr>
<tr>
<td>0.24660</td>
<td>-19.7→ -16.9</td>
<td>39.4→31.6</td>
<td>-18.9→ -14.8</td>
<td>37.2→25.8</td>
</tr>
<tr>
<td>0.19790</td>
<td>-20.3→ -17.7</td>
<td>41.9→34.9</td>
<td>-19.6→ -15.8</td>
<td>40.0→29.8</td>
</tr>
<tr>
<td>0.14957</td>
<td>-20.4→ -17.9</td>
<td>42.3→35.9</td>
<td>-19.9→ -16.2</td>
<td>41.2→31.4</td>
</tr>
<tr>
<td>0</td>
<td>-20.4→ -18.0</td>
<td>42.5→36.2</td>
<td>-20.0→ -18.0</td>
<td>41.4→31.9</td>
</tr>
<tr>
<td>0.06</td>
<td>0.34646</td>
<td>-20.1→ -17.7</td>
<td>34.4→27.0</td>
<td>-17.0→ -13.5</td>
</tr>
<tr>
<td>0.14957</td>
<td>-24.5→ -21.7</td>
<td>53.2→45.5</td>
<td>-21.1→ -17.3</td>
<td>43.9→40.9</td>
</tr>
</tbody>
</table>
region II. (1) The potential derived by Inoue, Machida, Taketani and Toyoda\textsuperscript{91} using the normalized Tamm-Dancoff method (FST-method\textsuperscript{92}) is weaker than $V_t^r$ (TMO). (2) The recoil corrections independent of nucleon momentum weaken $V_t^r$ as was confirmed by full recoil calculations by Hoshizaki and Machida.\textsuperscript{27} (3) The exchange of a vector meson with vector coupling to nucleon gives a repulsive potential (Eq. (2·10)). It has been shown by Furuichi and Watari\textsuperscript{93} that the exchange of a neutral scalar meson gives effects almost equivalent to those of the $(3/2, 3/2)$-resonance which produce a nearly state-independent attraction in a central potential.

The solutions of phase shift analysis above 50 MeV restrict the properties of the potential in region III, for example, the data on the singlet $S$-phase shift at 310 MeV, $\delta(^1S_0) = -5.7^\circ \sim -11.4^\circ$, determine the lower and upper limits of the hard core radius. Some of the potentials in Table 3-2 consistent with the data below 50 MeV are excluded by $\delta(^1S_0)$ at 310 MeV. For the Hamada-type potential given in Eq. (3·4), We obtain the allowable region of $x_c$ as (see Fig. 3-3):

\begin{equation}
\begin{aligned}
0.35 \leq x_c \leq 0.30 & \quad \text{for } f^2/4\pi = 0.08, \\
0.35 \leq x_c \leq 0.27 & \quad \text{for } f^2/4\pi = 0.07, \\
0.33 \leq x_c \leq 0.20 & \quad \text{for } f^2/4\pi = 0.06.
\end{aligned}
\end{equation}

The typical $^1S_0$ potentials, namely $V^r_t$ in Eq. (3·4), consistent with all the data below 310 MeV, are shown in Fig. 3-4.

Similar calculations are performed by the use of the superposition of one-boson-exchange potentials (OBEP) with a suitable hard core radius $x_c$.
Fig. 3-4. Singlet even central potential $^1V_0$ (in the unit of pion mass $\mu_0$) of the
Hamada-type: Eq. (3·4). Allowable $^1V_0$ are between the curve for ($f^2/4\pi = 0.08,$
$x_r = 0.35)$ and that for ($f^2/4\pi = 0.06,$ $x_r = 0.2$).

The parameters for (a), (b), (c) and (d) are:

(a): $a_0 = 7.6, b_0 = 11.85,
(b): a_0 = 10.9, b_0 = 1.31,
(c): a_0 = 18.5, b_0 = 2.2,
(d): a_0 = 22.6, b_0 = 11.0.

HM$^{(3)}$: (OPEP+TPEP) with full recoil corrections, TMO$^{(4)}$: static (OPEP+
TPEP) in the standard perturbation calculation, and KMO$^{(5)}$: static (OPEP+
TPEP) involving the pion-nucleon (3/2,
3/2)-resonance effects.

HM$^{(6)}$: (OPEP+TPEP) with full recoil corrections, TMO$^{(4)}$: static (OPEP+
TPEP) in the standard perturbation calculation, and KMO$^{(5)}$: static (OPEP+
TPEP) involving the pion-nucleon (3/2,
3/2)-resonance effects.

The results are shown in Table 3·2a, Figs. 3·3a and 3·4a in parallel with
the Hamada-type potential. The parameters $A$ and $B$ correspond to the
coupling constants of heavy mesons with nucleon: $A = -(g_s^2/4\pi)$ for the scalar
meson with mass $m_s = 4\mu_0$ and $B = (g_v^2/4\pi)\mu_0^2$ for the vector meson with
the mass $m_\pi=5.5\mu_\pi$, by using Eqs. (2.9) and (2.10). The large values of $A$ and $B$ for $x_c=0$ in Table 3.2a mean extraordinary large coupling constants $(g_4^8/4\pi=15\sim 20$, $(g_4^8/4\pi)^{\text{off}}=30\sim 40)$ which are 3$\sim$5 times larger than those obtained in the one-boson-exchange model. Because $m_\pi$ and $m_\pi$ are not very different, the net reasonable potential is constructed as a result of large cancellation between $A$ and $B$ terms. For this OBEP-type potential, a fit to the data below 310 MeV can be obtained only if $f^2/4\pi>0.06$.

The hard core used in this analysis can be replaced by the soft core with a height over 2 GeV. In this case the maximum depth of attraction just outside the core becomes about $-90\sim -150$ MeV. The hard core radius adjusted to the $^1S_0$ scattering is very sensitive to the attraction just outside the hard core, especially when this attraction is singular. If this singular attraction in the narrow region is cut off, the hard core radius is reduced appreciably. Further discussions on repulsive core are not made in this section, since the core region of nuclear forces is discussed in detail in Chapter 7.

3.3 On the pion-nucleon coupling constant

The analysis of the singlet $S$-phase shift in §3.2 shows that the upper and the lower limits of the pion-nucleon coupling constant determined from the singlet $S$ data depend on the assumed potential shape. The upper limit of the pion-nucleon coupling constant $f^2/4\pi$ is determined as $f^2/4\pi<0.08$ for the Hamada-type potential. Otherwise, as is shown in Table 3.2, we can find no allowable region of the potential parameters $a_*$ and $b_*$ in Eq. (3.4). Owing to the rather long-tailed term $\exp(-2x)/x^2$ of this potential which modifies the OPEP for $x=1\sim 1.5$, the OPEP-tail with $f^2/4\pi>0.08$ gives too large singlet effective range $\beta_r(\langle pp\rangle)$. Conversely, this property of the potential makes it difficult to determine the lower limit of $f^2/4\pi$. It is to be noted that this upper limit of $f^2/4\pi$ should become large, when a larger experimental upper limit of $\beta_r(\langle pp\rangle)$ is adopted that than in Eq. (3.3), e.g. when we take the value of the cubic fit in Table 3.1: $\beta_r(\langle pp\rangle)=(2.866 \pm 0.078) \times 10^{-13}$ cm.

In the case of the OBEP-type potential, in contrast to the case of the Hamada-type potential, the lower limit of $f^2/4\pi$ is determined as

$$f^2/4\pi \geq 0.07$$ for the OBEP case,

whereas the upper limit is undetermined. This is due to the rather weak attraction for $x=1\sim 1.5$, where the damping of the term $\exp(-4x)/x$ is fast. Therefore, in this case, the OPEP-tail with $f^2/4\pi<0.07$ gives a too small value of $\beta_r(\langle pp\rangle)$.

Determination of the pion-nucleon coupling constant in the problems of nuclear forces means to determine the strength of the OPEP-tail in a
manner independent of particular choice of inner interactions. From this point of view, in the singlet even state the coupling constant \( f^2/4\pi \) cannot be rigorously determined even with the aid of the recent accurate data, since the upper or lower limit of \( f^2/4\pi \) depends on the potential shape in region II. Thus the situation is not essentially altered since the previous review. What we can say definitely on the pion-nucleon coupling constant is that, on the assumption of the charge independence,

\[
f^2/4\pi < 0.090
\]

from the deuteron data. In this case, too, the lower limit of \( f^2/4\pi \) is determined in a way more or less correlated with the properties of inner parts of interaction, e.g. the hard core radius.

As for the lower limit of the value of \( f^2/4\pi \) we stated previously that in order to reproduce the experimental value of the singlet effective range parameters the possibility of \( f^2/4\pi < 0.070 \) was excluded. This conclusion was derived under the condition \( R < 5 \), where \( R = (\text{the value of the pion theoretical potential (TMO potential) at } x = 1) / (\text{the inner square well depth assumed for } x < 1 \text{ to reproduce the effective range parameters}) \). This criterion depends on the strength of potential near the pion-range, and the lower limit of \( f^2/4\pi \) becomes smaller when the attraction works stronger. This is the reason why \( f^2/4\pi < 0.070 \) is not excluded in this analysis, since the potential determined in §3.2 is stronger than the TMO potential in region II, especially for \( x = 1 \sim 1.5 \). Thus the value 0.070 for the lower limit of \( f^2/4\pi \) stated in the previous report must be revised. In the analysis of the deuteron data, the lower limit of \( f^2/4\pi \) has been obtained as 0.065 under the reasonable condition that the hard core radius \( x_c < 0.40 \). The recent accurate data on \( ^1\alpha_c \) and \( ^1\gamma_c(\overline{p}p) \) cannot improve this lower limit of \( f^2/4\pi \).

Possibilities of determining the pion-nucleon coupling constant from the phase shifts of high partial waves have recently been discussed. The thought in these attempts follows the line of extracting the effect of the OPEP-tail, which was proposed in 1956 by Iwadare, Otsuki, Tamagaki and Watari. In the phase shift analysis the contributions of high partial waves intimately correlate to the detailed angle-dependence of the observed quantities, and the determined phase shifts of high partial waves are apt to be affected by meaningless small fluctuations in data due to experimental errors. Actually at present, the values of the coupling constant determined in the energy-independent analysis are rather scattered between 0.05~0.1 with large standard deviations. In the energy-dependent analysis, however, \( f^2/4\pi = 0.077 \pm 0.011 \) has recently been obtained as the most reliable value, which is consistent with the value mentioned here throughout low energy analysis.

3.4 Nonstatic effects in the singlet even state

The improvement on the accuracy in the phase shift analysis has
clarified the fact that the $^1D_2$-phase shift $\delta(^1D_2)$ calculated from the static singlet even central potential $^1V_0^c$, which reproduces the experimental values of the $^1S_0$-phase shift $\delta(^1S_0)$ below 310 MeV, inevitably becomes larger than the upper limit of the solutions of the phase shift analysis above 200 MeV. The large $\delta(^1D_2)$ breaks down the isotropic angular distribution of $p$-$p$ differential cross section. Bryan$^{10}$ used the potential $^1V_0^c$ whose strength in region II is so weak as to make $\delta(^1D_2)$ as small as possible and avoid the inconsistency between $\delta(^1D_0)$ at high energies and the $^1S_0$ potential, without use of nonstatic effects. This problem has been discussed in detail by Otsuki, Tamagaki and Watari,$^{36}$ using the Hamada-type potentials. The revised results are shown in Fig. 3·5, from which we can confirm the above argument with every reasonable choice of parameters in region III: At 310 MeV, we obtain $\delta(^1D_2) = 14.3^\circ \sim 15.4^\circ$, which are too large by about $3^\circ$.

The value of the impact parameter $b$, defined as

$$b = \sqrt{L(L+1)/k}, \quad (3.8)$$

is $0.9/\mu$ for the $D$ wave and $1.6/\mu$ for the $G$ wave at 310 MeV. To overcome the above-mentioned difficulty in the $^1D_2$-phase shifts, we must mainly improve the potentials in regions I and II. Hamada$^{36}$ introduced the quadratic spin-orbit potentials ($V_{LL}$ in the HJ-potential) in order to suppress $\delta(^1D_2)$ and $\delta(^2G_4)$ at high energies. Only

![Fig. 3·5. Correlation among the singlet even phase shifts $\delta(^1S_0)$, $\delta(^1D_2)$ and $\delta(^2G_4)$ calculated from the $^1V_0^c$ without $^1V_{LL}$. This figure is a revised one of Fig. 1 in reference 36). The boundary line means that the correlation found in the phase shift analysis cannot be obtained by $^1V_0^c$ only.](https://academic.oup.com/ptps/article-abstract/doi/10.1143/PTPS.39.23/1864050/1864050)
Potential Model Approach

Potential $V_{\text{TL}}$ contributes in the singlet state because of $W_{\text{TL}} = 0$. For the Hamada-type potential, the singlet even potential becomes

$$V^e = V^e_T + L(L+1) V_{\text{TL}}$$

$$= -\mu \left( \frac{f^2}{4\pi} \right) e^{-x}/x \left( 1 + a e^{-x}/x + b e^{-2x}/x^2 \right)$$

$$+ L(L+1) \left( \frac{\mu}{M} \right)^2 \frac{e^{-x}}{x^2} \left( 1 + \frac{3}{x} + \frac{3}{x^2} \right) \left( \lambda + a_{LL} \frac{e^{-x}}{x} + b_{LL} \frac{e^{-2x}}{x^2} \right),$$

outside the hard core radius $x_c$.

Due to the factor $L(L+1)$, the effects of a rather weak $V_{\text{TL}}$, which is a correction to $V^e_T$ with $(\mu/M)^2$-order, are appreciable. To make $V_{\text{TL}}$ repulsive, it is necessary that

$$\left( \lambda + a_{LL} \frac{e^{-x}}{x} + b_{LL} \frac{e^{-2x}}{x^2} \right) > 0$$

Table 3.3. The singlet even phase shifts (in degrees) calculated from the nuclear potentials with different quadratic spin-orbit terms:

(I) $\lambda = 0.25$, $a_{LL} = 1.75$, $b_{LL} = 0$ and

(II) $\lambda = 1$, $a_{LL} = 0.2$, $b_{LL} = -0.2$.

The other parameters are: $\hbar c = 135.1$ MeV, $f^2/4\pi = 0.08$, $a_e = 7.6$, $b_e = 11.85$ and $x_e = 0.34646$ for both cases. The case (I) potential is referred as the standard potential in the singlet even state and the phase shifts are shown in figures in Appendix of Chapter 5.

<table>
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The $^1S_0$-low energy parameters are: $a_0 = -7.858 \times 10^{-13} \text{cm}$, $r_e(pp) = 2.823 \times 10^{-3} \text{cm}$, $P(pp) = 0.028$, $Q(pp) = 0.011$ and $a_0(p p) = a_0(n n) = -17.067 \times 10^{-13} \text{cm}$, $r_e(nn) = 2.933 \times 10^{-3} \text{cm}$, $P(nn) = 0.020$, $Q(nn) = 0.009$. 

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in regions I and II. If we assume only the OPEP term \((\lambda = \pm 1, a_{LL} = b_{LL} = 0)\), the \(\rho\nu\)-coupling case \((\lambda = +1)\) is favorable. However, due to the singular nature of \(1^1V_{LL}^\ast\), the contribution of \(1^1V_{LL}^\ast\) in region II is important. In fact, as is shown in Table 3-3, the \(1^1V_{LL}^\ast\) with
\[
\lambda = 0.25, a_{LL} = 1.75, b_{LL} = 0
\]
gives similar results for \(\delta(1^1D_2)\) and \(\delta(1^3G^\ast)\) to that of the HJ potential with
\[
\lambda = 1, a_{LL} = 0.2, b_{LL} = -0.2.
\]

Thus the effects from region I can be complemented by an increase of \(1^1V_{LL}^\ast\) in region II. The \(1^1V_{LL}^\ast\) to reproduce the singlet even scattering is shown in Fig. 3.6.

There are also nonstatic effects other than \(V_w\) and \(V_{LL}\). The analysis by Otsuki, Tamagaki and Watari\(^{36}\) using the OPEP and the TPEP with full recoil corrections derived by Hoshizaki and Machida has shown that the main cause to reduce \(\delta(1^1D_2)\) and \(\delta(1^3G^\ast)\) is \(1^1V_{LL}^\ast\) in the OPEP for the \(\rho\nu\)-coupling and the velocity-dependent term in the TPEP for the \(ps\)-coupling. If the \(\rho\nu\)-coupling is assumed, the whole nonstatic effects of the OPEP plus TPEP are sufficient to reduce \(\delta(1^1D_2)\) and \(\delta(1^3G^\ast)\). But if with the \(ps\)-coupling alone, the nonstatic effects are not sufficient to reduce them to the experimental values. Consequently, we could probably say at present that the pion-nucleon interaction may not be only of the \(ps\)-type. The most crucial point on the discrimination of the coupling type along this line is to extract the OPEP-nonstatic effects in region I definitely. As to the singlet even potential, this discrimination is difficult, since the fit can be obtained also by a small \(\lambda\).
The momentum space calculation with the OPEP\(^{64,65}\) (see Chapter 2) showed that the net nonstatic effect reduces \(\delta^{(1D_2)}\) by 10~15\% at 310 MeV but its difference between two coupling types is very small. Thus it seems adequate to say that the positive \(L^2\) potential presented here is a reasonable phenomenological substitute to the net nonstatic effect in the singlet even state.

§4. Triplet odd state

4.1 Main features of phase shifts*)

At low and intermediate energies, the triplet \(P\)-wave phase shifts \(\delta^{(3P_J)}\) \((J=0,1,2)\) show the tensor-type splitting owing to the properties of the static OPEP-tail. The OPEP is

\[
^3V^-(\text{OPEP}) = (f^2/4\pi)µ/3\cdot e^{-x}/x\{1 + S_{11}(1 + 3/x + 3/x^2)\}
\]

in the triplet odd state, and consists of the very weak repulsive central part and the moderately strong positive tensor part. The effective potentials in the \(^3P_J\) states (Table A·1) are

\[
\begin{align*}
V^{(3P_0)} &= (^3V_c + 2\cdot ^3V_{LL}) - 4\left(^3V_T - \frac{5}{6}\cdot ^3V_{w}\right) - 2\cdot ^3V_{LS}, \\
V^{(3P_1)} &= (^3V_c + 2\cdot ^3V_{LL}) + 2\left(^3V_T - \frac{5}{6}\cdot ^3V_{w}\right) - ^3V_{LS}, \\
V^{(3P_2)} &= (^3V_c + 2\cdot ^3V_{LL}) - (2/5)\left(^3V_T - \frac{5}{6}\cdot ^3V_{w}\right) + ^3V_{LS} + (6\sqrt{6}/5)\cdot V_T^*,
\end{align*}
\]

where \((6\sqrt{6}/5)\cdot V_T^*\) means the coupling term with the \(^3F_2\) state. The typical OPEP-tensor type splitting of \(\delta^{(3P_J)}\) appears at low energies (below 20 MeV) as

\[
\delta^{(3P_0)} : \delta^{(3P_1)} : \delta^{(3P_2)} = 4 : -2 : 0.4 + r_3,
\]

since the centrifugal potential screens the inside strong potentials and each phase shift is well estimated by the Born approximation:

\[
\delta^{(3P_J)} = -\left(Mk/\mu\right)\int_0^{r_3} \tilde{h}(kx/\mu) \cdot V^{(3P_J)} \cdot x^2 \, dx.
\]

\(r_3 (1 > r_3 > 0)\) stands for the effect of the coupling with the \(^3F_2\) state through the tensor potential. Direct experimental verifications of the feature (i) have not yet been performed. Measurements of the scattering parameters other than the unpolarized cross section \(L_O(\theta)\) are needed at such low energies in order to obtain the evidence about the OPEP in the triplet odd state.

*) The results of the modified phase shift analysis are shown in Appendix of Chapter 5.
In the energy region $E = 20 \sim 100$ MeV, the feature (i) holds no longer and turns to a less pronounced tensor-splitting:

$$(4.2b) \quad \delta(^3P_0) > |\delta(^3P_1)| > \delta(^3P_2) > 0,$$

because the effects of the two-pion-exchange potential and others in region II and nonstatic effects begin to reveal, although the contributions of the static OPEP are large. Therefore, in this energy region, it is possible to investigate the properties of the potential in region II through modifications to be added to the static OPEP contributions.

In the energy region $E = 100 \sim 200$ MeV, the feature of the tensor-type splitting remains only in the signs of $\delta(^3P_1)$:

$$(4.2c) \quad \delta(^3P_1) > \delta(^3P_0) > 0 > \delta(^3P_2).$$

Nuclear forces in region III contribute considerably and the effects from regions II and III compete with that of the OPEP-tail. Further above $E \geq 200$ MeV, effects from region III and nonstatic effects are dominant and the splitting of $\delta(^3P_1)$ becomes of the following type:

$$(4.2b) \quad \delta(^3P_2) > 0 > \delta(^3P_0) > \delta(^3P_3).$$

This means that a negative spin-orbit potential $^3V_{\tilde{s}}$ is predominant inside and a positive tensor potential $^3V_{\tilde{t}}$ outside. On the other hand, even at high energies above 300 MeV, the triplet $F$-wave phase shifts $\delta(^3F_{1-3,4})$ show the splitting of tensor type:

$$\delta(^3F_1) > \delta(^3F_2) > 0 > \delta(^3F_3).$$

This fact suggests that the triplet odd nonstatic effects cannot be ascribed to the spin-orbit potentials with the strength independent of angular momentum $L$.

At the present stage where such main features of nuclear forces in the triplet odd state are clarified, the problems to be investigated hereafter are: (1) How to establish region II quantitatively, (2) whether the hard-core-like repulsion exists in the triplet odd state and (3) whether nonstatic effects other than the spin-orbit force exist. The average $P$-wave phase shift at very low energies will be discussed in §4·2. Nonstatic effects will be discussed in §4·3 in connection with the $\delta(^3F_1)$ splitting. Results of phase shift analysis at 50 MeV will be discussed in §4·4 and the interaction at the core region in §4·5.

The parameters of potentials used for explanations in §§4·2-4·5 are given in Table 4·2. The phase shifts calculated from the standard hard core potential denoted by HC-81, which is often referred in this section, are given in Table 4·3.
4.2 Triplet P wave at very low energies

From the analysis of the nuclear-Coulomb interference in p-p scattering cross section at low energies, one estimates the value of the average $^3P$-phase shift $\langle \delta_1^T \rangle$, defined as

$$\sin \langle \delta_1^T \rangle = (1/9) \{ \sin \delta(^3P_0) + 3 \sin \delta(^3P_1) + 5 \sin \delta(^3P_2) \}. \quad (4.3)$$

As reported in the previous review, Otsuki and Tamagaki were the first to explain the small and negative value of the experimental $^3P$-phase shift $\langle K_i \rangle$ at about 2 MeV as the consequence of the weak repulsive central potential of the OPEP at the outermost region ($x \gtrsim 2$). The establishment of the OPEP in region I was motivated by this success. Ohmura suggested later to reinvestigate this problem, because the vacuum polarization correction in the Coulomb potential should be subtracted from $\langle K_i \rangle$, following Foldy and Eriksen's argument, as mentioned in §3.1. Since the distortion of the P wave due to nuclear forces is small at very low energies ($E \lesssim 10$ MeV), the vacuum polarization phase shift $\tau_i$ and $\Delta_i$ as defined in §3.1 are essentially the same. Thus the nuclear phase shift can be written as

$$\langle K_i \rangle - \tau_i = \langle \delta_i^T \rangle = \langle \delta_i^C \rangle.$$ 

Hereafter we omit the affix E or C. At energies of 1 to 4 MeV, $\tau_i$ varies $-0.05^\circ$ to $-0.04^\circ$. The nuclear phase shift $\langle \delta_i \rangle$ is reduced from $\langle K_i \rangle$ by one order of magnitude, that is, $\langle \delta_i \rangle \sim -0.01^\circ$ at $E=1\sim 4$ MeV. Otsuki, Taketani, Tamagaki and Watari reinvestigated the problem and showed that even such small $\langle \delta_i \rangle$ is reasonably reproduced in the presence of the weak repulsive tail of the OPEP-central potential. The inside potentials necessary to explain high energy p-p scattering are very strong and become comparable with the P-wave centrifugal potential at $x \sim 0.7$. Effects of the inside potentials to the average value $\langle \delta_i \rangle$ of the $^3P_j$-phase shifts are appreciable, although effects to each $^3P_j$-phase shift are small at these low energies. The main reason for such situation is the weakness of the central potential $^3V_c$ of the OPEP in the triplet odd state. Since noncentral contributions to the $\langle \delta_i \rangle$ due to the $^3V_{ls}$, $^3V_T$ and $^3V_W$ are cancelled (completely in the Born approximation), contributions of the $^3V_c$ are of primary importance for the $\langle \delta_i \rangle$ at very low energies. Now after the establishment of the OPEP, it is adequate to use the average P-wave phase shift $\langle \delta_i \rangle$ as an information source to clarify the $^3V_c$ (strictly speaking, the $^3P$-effective central potential $^3V_c + 2^3V_{ls}$) in region II.

We discuss $\langle \delta_i \rangle$ using the following two forms of $^3V_c$, which are the same as those used for the triplet odd analysis by Tamagaki, Wada and Watari.
\[ V_c^3 = \begin{cases} \mu \left( \frac{f^2}{4\pi} \right) \frac{1}{3} \frac{e^{-x}}{x} \left( 1 + a_c e^{-x} + b_c e^{-2x} \right) & \text{for } x \geq x_c, \\ +\infty & \text{for } x < x_c, \end{cases} \] (4.4)

which is denoted by the hard core (HC) case, and

\[ V_c^3 = \begin{cases} \mu \left( \frac{f^2}{4\pi} \right) \frac{1}{3} \frac{e^{-x}}{x} \left( 1 + a_c e^{-x} + b_c e^{-2x} \right) & \text{for } x \geq x_3, \\ V_c^0 & \text{for } x < x_3, \end{cases} \] (4.5)

which is denoted by the square well (SQ) case. The results are shown in Fig. 4.1. The values of \( a_c \) are restricted as follows: In order to reproduce the experimental \( \langle \theta_i \rangle \) at \( E \leq 3 \) MeV given by Noyes,\(^{16}\)

\[ a_c = -10 \sim -16. \] (4.6)

The triplet odd central potential \( V_c^3 \) in region II indicated by these \( a_c \), is attractive and moderately strong, as shown in Fig. 4.2, which confirms the previous results.\(^{10}\) Also \( V_T^3 \) and \( V_L^3 \) have a small contributions to \( \langle \theta_i \rangle \). As the positive \( V_T^3 \) and/or negative \( V_L^3 \) becomes stronger, the \( \langle \theta_i \rangle \) becomes larger. The values in Eq. (4.6) are determined with these ambiguities taken into account. The \( \langle \theta_i \rangle \) at \( E = 10 \) MeV is positive for \( a_c \geq -12 \) and negative for \( a_c \leq -8 \). More precise values of the \( \langle \theta_i \rangle \) at energies about 10 MeV are desirable for determining the properties of the \( V_c^3 \) in region II.

![Fig. 4.1. Triplet average P-wave phase shifts \( \langle \theta_i \rangle \) at low energies calculated from the Hamada-type hard core potential (HC) and the potential with the inside square well (SQ) in the absence of Coulomb field. For the cases denoted by HC, the entries in parentheses mean \( (a_0, b_0, x_c) \) in Eq. (4.4) and for the cases denoted by SQ, \( (a_c, b_c, V_c^0) \) in Eq. (4.5). The other parameters are given in Table 4.2. The points \( \bullet \) are the solutions obtained by Noyes.\(^{76}\) OPEP-curve is calculated from the OPEP with the hard core \( x_c = 0.32628, \mu c^2 = 139.4 \) MeV and \( f^2/4\pi = 0.08. \)

\(^{40}\) The main parts of §§4 and 5 are based on the results of references 57) and 62), where the value \( \mu c^2 = 139.4 \) MeV is used.
Below 20 MeV, information on nuclear forces can be obtained only through data on the angular distribution, $L_0(\theta)$. Only the $\langle \delta \theta \rangle$ is determined from such data, and since it is a small and delicate quantity, it may be unreasonable to demand a further improvement on its accuracy. To determine each phase shift $\delta(\ell P_I)$, even at very low energies, complete experiments are necessary, then the splitting of $\delta(\ell P_I)$ will give information on noncentral potentials $\langle \ell V_\ell \rangle$ and $\langle \ell V_{ls} \rangle$ in region II.

4.3 Nonstatic potentials in the triplet odd state

At the energy $E=200$ MeV the impact parameter of the $P$ wave is about 0.7, the boundary between regions II and III. Therefore, in the scattering above 200 MeV, the $\ell P_I$ waves are affected by the spin-orbit potential $\langle \ell V_{ls} \rangle$ as a whole in inner regions II and III. The spin-orbit potentials necessary to explain the behaviour have the order of magnitude

$$\langle \ell V_{ls} (x=0.7) \rangle \sim -50 \text{ MeV.} \quad (4\cdot7)$$

The spin-orbit potentials so far proposed have this feature, as shown in Fig. 4.3, although their shape is at variance. The difference in these potentials are related to the relative proportion of these shape in regions II and III. In order to determine the shape of $\langle \ell V_{ls} \rangle$, it is necessary to determine that in region II.

As stated in §4.1, it is found from the splitting of $\ell P_I$-phase shifts that in the triplet odd state the negative $\langle \ell V_{ls} \rangle$ is predominant inside and the positive tensor potential $\langle \ell V_{\ell} \rangle$ outside. While the splitting of $\ell P_I$-phase shifts show no evidence for spin-orbit potential, in particular the $\ell P_{1/2}$-phase shift $\delta(\ell P_{1/2})$ remains rather small even above 200 MeV. In the light of this fact,
it was emphasized that the spin-orbit potential should be of very short range. Bryan’s work\(^{10}\) was an attempt to shrink the tail of \(^3V_{LS}\). However, as was discussed by Tamagaki, Wada and Watari,\(^57\) such a potential predicts \(\delta(3F_i) \approx 4^\circ\) at 310 MeV and shows a rapid increase of \(\delta(3F_i)\) above 300 MeV even though the \(^3V_{LS}\) almost confined in region III gives a small \(\delta(3F_i)\) at \(E<200\) MeV. At 660 MeV this potential predicts \(\delta(3F_i)\) larger than about 15\(^\circ\), and this value is qualitatively in disagreement with the small experimental values of \((3^\circ \sim 4^\circ)\).\(^{100-102}\)

Probably this discrepancy cannot be removed by relativistic effects and those of inelastic channels. Therefore, the splitting between \(^3F_i\)-phase shifts should not be considered as closely connected with the range of \(^3V_{LS}\).

The range of the spin-orbit potential generated by a meson exchange is related with the exchanged meson mass and is expected to be larger than those proposed in phenomenological analysis. If we assume that the \(^3V_{LS}\) is generated only through the vector meson exchange with mass \(m_V = 5.5 \mu\) (this value is nearly equal to the masses of \(\omega\)-meson and \(\rho\)-meson), the effective coupling constant \((g_{\pi}^2/4\pi)^{(L8)}\) responsible for the spin-orbit potential in the triplet odd state is estimated by comparing Eq. (2.10a) with Eq. (4.7):

\[
(g_{\pi}^2/4\pi)^{(L8)}_{\text{eff}} = (g_{\pi\alpha}^2/4\pi)^{(L8)}_{\text{eff}} + (g_{\pi\beta}^2/4\pi)^{(L8)}_{\text{eff}} = (g_{\pi\alpha}^2/4\pi + f_{\pi\alpha}/4\pi + (8M/3m_{\pi})(g_{\alpha\alpha}f_{\alpha\alpha}/4\pi)) + (g_{\pi\beta}^2/4\pi + f_{\pi\beta}/4\pi + (8M/3m_{\pi})(g_{\beta\beta}f_{\beta\beta}/4\pi)),
\]

where the suffix \(i = 0\) means the isoscalar meson, \(i = 1\) the isovector meson,
and $g_{vi}$ and $f_{vi}$ are the coupling constants of vector interaction and tensor interaction, respectively. The comparison leads to $(g^2_v/4\pi)_{el}^{(LS)}=35$ for $m_v=5.5\mu$. For the Hamada-type potential, $G_1=-6$ is a reasonable choice for $\mu_v=m_v/\mu=5.5$. Substituting $y=(m_v\mu/\mu)$ and $G_1=-(3/2)(m_v\mu/M^2)\times(g^2_v/4\pi)_{el}^{(LS)}$ into Eq. (2·10), we obtain $(g^2_v/4\pi)_{el}^{(LS)}=32$. Sawada, Ueda, Watari and Yonezawa\textsuperscript{105} have recently analyzed the nucleon-nucleon scattering by the OBEC model and found that the nucleon-nucleon scattering can be explained by the superposed contribution of one-$\pi$, one-$\rho$, one-$\omega$ and one-scalar meson exchange. (For detailed discussion, see Chapter 3 of this report.) From their result, the effective coupling constant as defined above is

\begin{align}
(g^2_v/4\pi)_{el}^{(LS)}&=12\sim15, \\
\frac{g^2_v}{4\pi} &= -3 \sim -4, \text{ with } m_v=5.5\mu,
\end{align}

(4·9a)

and the coupling constant of isoscalar scalar meson is

\begin{equation}
\frac{g^2_{5\sigma}}{4\pi} = 12\sim14 \quad \text{with } m_s=4.4\mu, \quad (4·9b)
\end{equation}

which gives the additional contribution $-(m_s\mu/2M^2)(g^2_{5\sigma}/4\pi)$ to $G_1$. If $m_s=5.5\mu$, the corresponding value of $g^2_{5\sigma}$ is estimated as $(12\sim14)\times(5.5/4.4)^2=15\sim18$. Then we find $(g^2_v/4\pi)_{el}^{(LS)}+(1/3)g^2_{5\sigma}/4\pi=14\sim17$ which is small as compared with the value $(32\sim35)$ obtained above in the potential analysis. This difference comes mainly from a difference in treating nonstatic effects which play the more important role for the large meson mass.\textsuperscript{105,106} Also the difference comes partly from a difference in treatment of higher order effects. In the OBEC model phase shifts are calculated by the so-called Born approximation, whereas in the potential analysis phase shifts are obtained by solving the Schroedinger equation. The repulsive effects in the $^3P_0$ and $^3P_1$ states are mainly concerned with the determination of these coupling constants, and the Born approximation overestimates effects of repulsion. In any case, the range of the spin-orbit potential should not be shorter than those given by such heavy one-meson-exchange contributions.

There are two attempts to explain the small $\delta(^3F_s)$. One is to introduce angular momentum dependence into the $^3V_{ls}$ so that $^3V_{ls}$ is absent in the states with $J\geq3$, as is assumed in the Yale potential.\textsuperscript{115} Although this angular momentum dependence is allowable in the general form of a potential in Eq. (2·6), there is no reason why only $^3V_{ls}$ should be strongly angular momentum-dependent. The other attempt is to use quadratic spin-orbit potentials as Tamagaki, Wada and Watari have done.\textsuperscript{97} The quadratic spin-orbit potentials have already been necessary in the singlet even state (§3·4) and indicated also in the triplet even state (§5·2). The parameters of these...
Fig. 4.4. Triplet odd quadratic spin-orbit potentials. Their parameters are given in Table 4.2. OPEP (pv) means the potential (2.7) with positive HJ-curve has this asymptotic form and the curves of HC-81 and SQ-1 have the asymptotic form of a quarter of this term.

Fig. 4.5. Effective potentials in the $^3P_J$ and $^3F_J$ states for HC-81 case with $^1V_W$ and $^1V_{LL}$ (solid lines) and without these terms (dashed lines). Their potential parameters are gives in Table 4.2. Introduction of these nonstatic terms changes only the qualitative feature of $V(^3P_J)$, which is strongly attractive due to the $^3V_{LS}$ without these terms.
Fig. 4-6. The triplet odd phase shifts calculated from the potentials with and without the nonstatic terms $^3V_w$ and $^3V_{LL}$ whose parameters are given in Table 4-2.

HC-81: with $^3V_w$ and $^3V_{LL}$ in Fig. 4-4.
HC-84: without $^3V_w$ and $^3V_{LL}$ in HC-81 and with a somewhat larger hard core radius $x_c = 0.33287$ than that of HC-81.
SQ-1: with $^3V_w$ and $^3V_{LL}$ in Fig. 4-4. The similar potential without these terms to HC-84 is SQ-3 in reference 57).

$x$ : Hamma and Hoshizaki, 101)
$+$ : Azhgivey et al., 109)
$-$ : Arndt and MacGregor, 94(d)
1 : Values of the solutions near 50 MeV, 85-89)

quadratic spin-orbit potentials have been chosen so as to be effective in uncoupled states. Contrary to these states, in the triplet odd state the main role of the quadratic spin-orbit potential $^3V_w$ and $^3V_{LL}$ is to reduce the value of the phase shift of the $^5F_i$ state (coupled state) at high energies, therefore its parameters should be chosen so as to be effective in the coupled states. The effective potential in the $^5F_i$ state is

$$V(^5F_i) = V_c - (2/3)^3V_f + 3^3V_{LS} + 5^3V_w + 12^3V_{LL} + (4V/3) V_f^*.$$  

Since the spin-orbit potential $^3V_{LS}$ is negative in explaining the behaviour of $^5P_f$-phase shifts, this potential acts very attractively in the $^5F_i$ state as $3^3V_{LS}$. This is the reason why $\delta(^5F_i)$ become very large at high energy in the previous calculations. To cancel these attractions, the potential $5^3V_w$
+12 $V_{ll}$ must be repulsive and its magnitude must be about $3 \, V_{ls}$. If we assume a suitable quadratic spin-orbit potential, the energy dependence of $\delta^{(F_4)}$ becomes reasonable. The above argument holds good regardless of whether the repulsive core is present or not. These situations are shown in Figs. 4.4, 4.5 and 4.6. Figure 4.4 shows the quadratic spin-orbit potentials introduced to explain $\delta^{(F_4)}$. Figures 4.5 and 4.6 illustrate that the introduction of $V_w$ and $V_{ll}$ does not affect the qualitative behaviour of the $^3P_J$ and $^5F_J$-effective potentials other than the $^3F_4$ state, and consequently that of the $^3P_J$- and $^5F_J$-phase shifts other than $\delta^{(F_4)}$.

The $V_w$ and $V_{ll}$ used to reduce the $\delta^{(F_4)}$ are of the same order of strength with those in the singlet and triplet even states, and the quadratic spin-orbit effects introduced into nuclear forces are consistent as a whole. In the potential model in the coordinate space, quadratic spin-orbit potentials are the phenomenological expression for all the nonstatic effects other than the $L \cdot S$ term, because all the explicit velocity-dependent terms are dropped ($\S$2). As is discussed in Chapter 3, the one-boson-exchange contribution (OBEC) model predicts the weak quadratic spin-orbit effects and the rather strong nonstatic corrections to the central and tensor potentials in the momentum space, which are strongly energy-dependent. Also these recoil corrections and the contact interaction terms corresponding to the delta function and its derivatives give the effects dependent on $L$ (the magnitude of an orbital angular momentum). This OBEC model reproduces $\delta^{(D_2)}$, $\delta^{(G_4)}$ and $\delta^{(F_4)}$ at high energies in a natural way without any modification. Although it is not easy to show in an apparent form the one-to-one correspondence between the nonstatic effects in the coordinate space and the momentum space, we can say that the energy- and angular-momentum-dependent effects in momentum space are described in a phenomenological way as the quadratic spin-orbit potentials $V_w$ and $V_{ll}$ in coordinate space, and these nonstatic effects other than the energy- and $L$-independent $V_{ls}(r)$ become indispensable above 200 MeV.

Summarizing the nonstatic effects in the singlet even and triplet odd states, we can say that the effects of nonstatic potentials appear in the following manner: At $E \leq 20$ MeV the static potential is predominant, at $E \approx 20 \sim 200$ MeV at least the spin-orbit potential becomes necessary, and at $E \geq 200$ MeV the nonstatic potential such as the quadratic spin-orbit ones is to be added.

4.4 Triplet P-wave phase shift at intermediate energies ($E \sim 50$ MeV)

The energy region about 50 MeV is suitable for obtaining information on nuclear forces in region II, since the $\delta^{(P_J)}$ splitting is in a transition from type (i) in Eq. (4.2a) to type (ii) in Eq. (4.2b). The unpolarized
cross section $I_\theta(\theta)$ and the polarization $P(\theta)$ have been explained by the static central plus tensor potentials$^{23}$ and also by the potentials with the strong $L \cdot S$ term.$^{13,12}$ Therefore, measurements of triple scattering parameters and spin correlation ones are indispensable to determine each $\delta(3P_0)$ definitely. In particular $\delta(3P_0)$ at this energy region shows particular sensitivity to the potential in region II because of the competition of the repulsive $^5V_{L\bar{S}}$ effect with the attractive $^5V_T$ effect:

$$V(3P_0) = ^5V_{\bar{T}} - 4^5V_T - 2^5V_{L\bar{S}} + (10/3)^5V_{\bar{W}} + 2^5V_{LL}.$$  

From the above consideration, Nisimura et al.$^{57}$ measured the spin correlation parameter $C_{KP}(90^\circ)$ at 52 MeV, which is sensitive to $\delta(3P_0)$ at this energy, in order to get information on the tail of $^5V_{L\bar{S}}$, and later $C_{NN}(90^\circ)$ in order to reduce ambiguities in the singlet even phase shifts. Griffith et al.$^{105}$ measured $A(70^\circ)$ at 50 MeV. Ashmore et al.$^{108}$ measured $A(\theta)$ at 47.5 MeV and $A(\theta)$ and $R(\theta)$ at 47.8 MeV mainly in order to determine $\delta(3P_0)^{107}$ where $A(\theta)$ is sensitive to $\delta(3P_0)$ as well as $C_{KP}(\theta)$. Discussions of these experiments are given by Nisimura in Chapter 5. The results of the phase shift analysis of these data are

$$\delta(3P_0), \delta(3P_2), \delta(3P_0), \epsilon_2$$  

with $A$-data,  

$$11^\circ \sim 13^\circ, -7.5^\circ \sim -8.5^\circ, 5.6^\circ \sim 6.2^\circ, -3.0^\circ \sim -1.9^\circ$$  

without $A$-data,  

$$14^\circ \sim 17^\circ, -6.0^\circ \sim -8.2^\circ, 4.4^\circ \sim 6.0^\circ, -2.2^\circ \sim -2.7^\circ$$  

provided that all the data used in the analysis are normalized at 50 MeV.

The significance of $\delta(3P_0)$ at 50 MeV was discussed by Hoshizaki, Otsuki, Tamagaki and Watari$^{105}$ and later by Tamagaki, Wada and Watari.$^{57}$ Their conclusions are summarized as follows (Figure 4.7 serves to illustrate the situation):

(a) If the $3P_0$-phase shift $\delta(3P_0)$ is really greater than $15^\circ$, we cannot explain the energy dependence of this phase shift up to 310 MeV by using an energy-independent potential. The polarization data at the energy about 150 MeV demand this phase shift to be small. To explain values of $\delta(3P_0)$ at 50 MeV, 150 MeV and 310 MeV by a potential, we are obliged to introduce some energy dependent terms in the potential.

(b) If $\delta(3P_0)>11^\circ$ is confirmed, the potential in region II cannot have too weak tensor part and too strong $LS$ part. The $\delta(3P_0)$ calculated by the HJ potential (with $a_T=-1.29$) is just on this lower limit. Thus a somewhat stronger $^5V_T$ in region II (with $a_T=-0.7 \sim -1.0$) such as HC-81 and HC-71 is favourable to explain the energy dependence of $\delta(3P_0)$. Too strong $^5V_T$ in region II such as HC-83 is unfavourable, since this tensor potential and a negative $^5V_{L\bar{S}}$ give too small values of $\delta(3P_0)$.
If \( \delta(3P_0) \approx 11^\circ \sim 14^\circ \), the negative tail of the \( ^3V_\pi \) with the range \((1/5\mu \sim 1/5.5\mu)\) determined by the heavy meson mass is consistent with \( \delta(3P_0) \). In the combination of the \( ^3V_\pi \) discussed in (b), too long-ranged \( ^3V_\pi \) is excluded.

Fig. 4.7. Energy dependence of triplet \( P \)-wave phase shifts for various potentials with the OPEP-tail: HC-81, HC-82, HC-83, HC-71 and HJ.\(^{115}\) Their potential parameters are given in Table 4·2.

The solutions of the phase shift analysis at fixed energies are shown:

1. at \( E=50 \) MeV are the allowable values of \( \delta(3P_0) \) and \( \epsilon_2 \) obtained in the phase shift analysis near this energy.\(^{115}\)-\(^{117}\)

at the other energies are the solutions by Arndt and MacGregor.\(^{99}\)

The dashed line at 52 MeV are the solutions obtained without use of \( A(\theta) \)-data.

If the phase shifts at \( E \sim 50 \) MeV are more precisely determined, the consequences concerning the potentials in region II such as those mentioned above can be derived. However, there may still be some ambiguities in the solutions of the phase shift analysis and inconsistencies among the experimental data. For example, \( C_\pi(90^\circ) \) and \( D(70^\circ) \) calculated from these solutions lie outside the upper limits of the data by two and one standard deviations.\(^{89}\) Thus further experiments must be carried out. Similar situations seem to be present with other energies.
4.5 Core region in the triplet odd state

In the potential model representation of nuclear forces it is usual to introduce a hard core in the triplet odd state. But this is only an assumption for simplicity.

In order to obtain information about the core region, we need the quantities not shielded by the surrounding repulsive potentials. The \(^3P_0\) state, where the attraction due to the negative \(^3V_{\text{trp}}\) acts strongly, is of particular importance, since in the \(^3P_0\) and \(^3P_1\) states the effects of the core region are almost shielded by the repulsion due to the negative \(^3V_{\text{trp}}\) just outside the core region. Also, the centrifugal potential prevents the \(^3F_1\) waves from entering into the core region. The situation is illustrated in Fig. 4·8 by the sensitivity of the phase shifts to the hard core radius. In order to conclude the discussion of the existence of the triplet odd repulsive core, therefore, we must show that the repulsive core is indispensable to reproduce the phase shifts of the \(^3P_2 + ^3F_2\) state. Discussion of the repulsive core in the triplet odd state is not so easy as in the singlet even state, because of the effects of the centrifugal potential and the noncentral ones. However, owing to the establishment of the main features of region II as is discussed in §4·2–§4·4, we can say something about the core region in this state.

Sawada conjectured the possibility of the nonexistence of the triplet odd repulsive core, on the basis of the successful explanation of \(p-p\) scattering by the OBEC calculation. Decomposing the OBEC amplitude (the same as that in the Born approximation), Matsuda and Sawada discussed this possibility in detail and found that a hard-core-like repulsive amplitude plays no important role in producing the OBEC fit. This problem was investigated by Tamagaki, Wada and Watari by using the potential model. Taking the form in Eq. (2·15) for \(x > 0.698\) and assuming the square well coreless
potential in region III;
\[ ^3V^- = V_0^e + V_0^n S_{12} + V_0^{2L} \mathbf{L} \cdot \mathbf{S} + V_0^3 W_{12} + V_0^{3L} L^2 \] for \( x \leq 0.698 \),

they showed that the phase shifts can be reproduced provided that

- \( V_0^e = 130 \sim 140 \) MeV,
- \( V_0^2 = -8 \sim -13 \) MeV,
- \( V_0^3 = -165 \sim -205 \) MeV,

or

- \( V_0^e = 0 \),
- \( V_0^2 = 25 \sim 70 \) MeV,
- \( V_0^3 = -170 \sim -180 \) MeV,

In the first case the central potential is repulsive, but in the second case it nearly equals zero. Therefore, in the triplet odd state the hard core-like repulsion is not indispensable in contrast with the singlet even state. The calculation using the OBEP-type potentials with zero cutoff also supports this conclusion.\(^{108,109}\) In the second case the \( V_0^e \) and \( V_0^2 \) act repulsively in the \( ^3P_2 \) state, and also in the \( ^3F_i \) state, where they suppress the steep increase of the \( \delta(^3F_i) \) at high energies. The nonstatic effects represented by such quadratic spin-orbit forces play a role equivalent to that of the hard core in the triplet odd state.

Thus, in the triplet odd state, two distinct interpretations of the core region are possible: One is hard core-like repulsion plus surrounding singular potential, and the other is nonsingular potential plus nonstatic effects. It is interesting to know in what manner and at what energies differences between the two possibilities reveal. Below 310 MeV and even at 660 MeV, no qualitative difference occurs, as is shown in Table 4·1. Figure 4·6 seems to indicate that the large hard core radius gives too low values of \( \delta(^3P_2) \) at 660 MeV (\( x_c > 0.3 \)) and may be unreasonable, unless effects of inelastic channels act attractively in \( \delta(^3P_2) \). Since the phase shifts

<table>
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<th>( ^3P_2 )</th>
<th>( ^3P_1 )</th>
<th>( ^3P_2 )</th>
<th>( \epsilon_2 )</th>
<th>( ^3F_2 )</th>
<th>( ^3F_3 )</th>
<th>( ^3F_4 )</th>
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<td>0.91°</td>
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<tr>
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<td>-28.30°</td>
<td>17.46°</td>
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<td>-45.37°</td>
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Table 4·1. Triplet odd phase shifts at 310 MeV and 660 MeV for the hard core potential and for the coreless square well potential (HC–1 and SQ–1 in reference 57)). Their parameters are given in Table 4·2.
Table 4.2. Parameters of potentials (2.15) used in the triplet odd state. Some of them are fixed as $\mu_1=5.5$, $\mu_2=6.7$, $n_1=2$ and $n_2=3$.

<table>
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<th>$\mu^2$ (MeV)</th>
<th>$f^2/4\pi$</th>
<th>$a_s$</th>
<th>$b_s$</th>
<th>$a_T$</th>
<th>$b_T$</th>
<th>$G_1$</th>
<th>$G_2$</th>
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<td>6.92</td>
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** In SQ-1 potential, $x_c$ is the square well cutoff radius $x_1$ and the depths of wells are $V_1=-75.65$, $V_{1S}=-182.48$, $V_{1S}=6.19$, $V_{1S}=20.0$ and $V_{1S}=33.31$ in MeV.

** In the HJ potential, $\mu_1=2$ and $\mu_2=3$ for $V_{1S}$.
Table 4.3. Triplet odd phase shifts (in degrees) calculated from the HC-81 potential, whose parameters are given in Table 4.2. These values are shown in Appendix of Chapter 5.

<table>
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<th>$E$ (MeV)</th>
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<th>$^{3}P_{1}$</th>
<th>$^{3}P_{2}$</th>
<th>$\epsilon_{2}$</th>
<th>$^{3}F_{3}$</th>
<th>$^{3}F_{2}$</th>
<th>$^{3}F_{s}$</th>
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are well reproduced with their essential features by either a nonsinglar potential or a singular potential with a rather small core radius \( x_c < 0.3 \), discrimination between the two possibilities seems to be difficult even if the experiment becomes complete below 1 GeV. The answer will probably be obtained by solving the basic problem as to whether the mechanism causing the repulsive core in the singlet even state acts similarly in this state or not.

§5. Potential in the \( T=0 \) state

We can obtain information on the interactions in the triplet even state and singlet odd state \((T=0\) states\) only from the \( n-p \) data. The present experimental situation on \( n-p \) scattering is far from completeness. The data on the spin polarization and the spin correlation are scanty and not quite accurate. Accordingly, the solutions of the phase shift analysis involve a large ambiguity at present. Therefore, we can only discuss qualitative properties in region II and the limitation of various possibilities in explaining the data, and we can derive no definite quantitative conclusions for region II.

5.1 Deuteron problem

Investigating the deuteron pion-theoretically, Iwadare, Otsuki, Tamagaki and Watari\(^{15,94}\) verified the OPEP-tail and determined the value of the pion-nucleon coupling constant as \( f^2/4\pi = 0.065 - 0.09 \). They utilized the tensor-dominant feature, characteristic of the OPEP in the outer region \((x > 1.0)\), without any special assumption for the inner region \((x < 1.0)\). The coefficient of the tensor potential in the outer region, the coupling constant, can be determined by the deuteron quadrupole moment \( Q_d \) which is concerned mainly with the outside behaviour of the deuteron wave function, if the inner wave function is taken so as to be consistent with the boundary condition at the origin or at the hard core radius and with the triplet effective range \( 3^r \). Hamada\(^{110}\) confirmed this conclusion, by using the semi-phenomenological potential with the static OPEP-tail and with the hard core.

Since the deuteron is a loosely bound system, we obtained the conclusions above-mentioned for the outer part irrespective of ambiguities in the inner part. This means that in the deuteron problem we can hardly obtain anything definite about the properties of potentials in the inner region. If the nonstatic potentials are introduced, even the conclusion that \( 3^V_{t} \) is strongly negative\(^{13,94}\) should be reinvestigated. Gammel and Thaler\(^{111}\) and Glendenning and Kramer\(^{112}\) showed various potentials having the central part, tensor part, and \( LS \) part, which are compatible with the deuteron data, mainly for the cases with negative \( 3^V_{t} \). However, the deuteron data can be easily reproduced by the potentials with positive \( 3^V_{t} \) and quadratic spin-orbit potentials.\(^{13}\)
The use of nonstatic potential causes the correction to the magnetic moments of nuclei, because nonstatic operators contain the nucleon momentum operator $p_i$ explicitly or implicitly. In the deuteron, the additional Hamiltonian $-\mu \cdot H$ is obtained by the substitution $p \to p - eA(r)/c$, where a constant magnetic field $H$ is related to the vector potential $A(r) = -r \times H/2 = -r \times H/4$. $r_p$ is the coordinate of proton in the center of mass system, and $r$ is the distance between proton and neutron. The deuteron magnetic moment operator $\mu_d$ becomes

$$
\frac{\mu_d}{(\hbar/2Mc)} = (\mu_s + \mu_n) S + L/2 
+ \frac{M}{\hbar^2} [\frac{-3V_{ls}^+}{4}(S \times r) \times r - \frac{3V_{lw}^+}{8}(σ_1 \cdot L)(σ_2 \cdot r) \times r

+ (σ, \times r) \times r(σ_2 \cdot L) + 1 \leftrightarrow 2] + \left(\frac{3V_{ls}^+}{2} - \frac{3V_{lw}^+}{3}\right) r^2 L].
$$

Thus the nonstatic potentials $^3V_{ls}^+$, $^3V_{lw}^+$ and $^3V_{lw}^+$ contribute to the deuteron magnetic moment $\mu_d$ in addition to the proton magnetic moment $\mu_p$, the neutron magnetic moment $\mu_n$ and the term due to the D-state probability $P_D$.

The deuteron wave function is written as

$$\psi_{nl}(x) = \frac{u(x)}{x} Q_{nl}^m \frac{w(x)}{x} Q_{nl}^m,$$

where $x = \mu r$ and the radial wave functions $u(x)$, $w(x)$ are normalized by

$$\int_0^\infty (u^2(x) + w^2(x)) dx = 1.$$

By the use of $u(x)$ and $w(x)$, $\mu_d$ is expressed in the form

$$\frac{\mu_d}{(\hbar/2Mc)} = \mu_s + \mu_n - \frac{3}{2}(\mu_s + \mu_n - \frac{1}{2}) P_D + \Delta \mu + \Delta \mu_{LS} + \Delta \mu_{tw} + \Delta \mu_{tw},$$

with

$$\Delta \mu_{LS} = \frac{M}{6\mu} \int_0^\infty (u^2 - \frac{uw}{\sqrt{2}} - w^2) x^2 (3V_{ls}^+ / \mu) dx,$$

$$\Delta \mu_{tw} = \frac{M}{4\mu} \int_0^\infty (\sqrt{6} \mu w + \frac{13}{5} w^2) x^2 (3V_{lw}^+ / \mu) dx,$$

$$\Delta \mu_{lw} = \frac{3M}{2\mu} \int_0^\infty w^2 x^2 (3V_{lw}^+ / 2\mu - 3V_{lw}^+ / 3\mu) dx,$$

where in the presence of a hard core we take $u(x) = w(x) = 0$ for $x < x_c$ (hard core radius). $\Delta \mu$ are other corrections such as the relativistic and the exchange current corrections, and estimated to be $|\Delta \mu| < 0.02$. $\Delta \mu_{tw}$ and $\Delta \mu_{lw}$ are small because of the absence of the main $^3S_1 - ^3S_1$ term, since the
quadratic spin-orbit operators contain the angular momentum operator twice. \( \Delta_{\mu LS}, \Delta_{\mu W} \) and \( \Delta_{\mu LW} \) have been calculated by using Hamada and Johnston’s wave function.\(^{10}\) For the OPEP with the pseudovector coupling \((\lambda=1, a_W = b_W = a_{LL} = b_{LL} = 0)\), \( \Delta_{\mu W} + \Delta_{\mu LW} = -0.0035 \) and for various \( ^3V_{W}^* \) and \( ^3V_{L}^* \) introduced to adjust high energy phase shifts their contributions to \( \mu_d \) are estimated as \( \Delta_{\mu W} + \Delta_{\mu LW} = 0 \sim -0.007 \). The \( p^2 \) term reduces this contribution.\(^{113}\)

Therefore, \( \Delta_{\mu W} + \Delta_{\mu LW} \) is completely masked by the large ambiguity due to \( \Delta_{\mu} \). Assuming \( |\Delta_{\mu} + \Delta_{\mu W} + \Delta_{\mu LW}| < 0.02^{114} \) and the value of the \( D \)-state probability is \( P_d = 0.06 - 0.08 \) (the large value of \( P_d \) is characteristic of the strong tensor potential of the OPEP), we estimate the \( \Delta_{\mu LS} \), which is compatible with the experimental value of \( \mu_d \), to be

\[-0.01 < \Delta_{\mu LS} < 0.04.\]

If the spin-orbit potential is negative and strong as the triplet odd one \( (^3V_{LS}^* = ^3V_{LS} < 0) \), it gives \( \Delta_{\mu LS} < -0.01 \), and is unfavourable to \( \mu_d \), because it demands an unreasonably small or negative \( D \)-state probability.\(^{115}\) Then a weakly negative or positive \( ^3V_{LS}^* \) is probable. An additional evidence against the strong negative \( ^3V_{LS}^* \) was discussed by Arking\(^{110}\) in connection with the magnetic moment of \( \text{H}^3 \) and \( \text{He}^3 \), and it was confirmed that the negative spin-orbit potentials such as that proposed by Gammel and Thaler\(^{117}\) and by Signell and Marshak\(^{19}\) are too large to be reconciled with the present experimental data of the magnetic moment on a reasonable assumption of the exchange moment effects. The possibility of large exchange moment is not completely excluded.\(^{110}\) and this objection against the negative \( ^3V_{LS}^* \) should not be taken too seriously. Thus there are many possibilities for the potentials in the inner part which are consistent with the deuteron data.

The deuteron wave function calculated by Hamada and Johnston\(^{10}\) is given in Table 5·1. The entries are the radial \( S \) and \( D \) wave functions (multiplied by \( x \)) which are normalized to \( \int_{x_e}^{\infty} (u^2 + w^2) dx = 1 \) with \( x_e = 0.343 \). They predict the following deuteron data:

- **Binding energy**: \( \varepsilon = 2.226 \text{ MeV} \),
- **Electric quadrupole moment**: \( Q_d = 2.85 \times 10^{-27} \text{ cm}^2 \),
- **Effective range**: \( \rho(-\varepsilon, -\varepsilon) = 1.77 \times 10^{-13} \text{ cm} \),
- **\( D \)-state probability**: \( P_d = 6.97 \text{ percent} \).

The values of the wave function for \( x < 1.2 \) may have some ambiguity, but the values for \( x > 1.2 \) are not far removed from reality.

The experimental data on the deuteron are given in Table 5·2.
Table 5.1. The deuteron wave function calculated from Hamada and Johnston's potential \( \int_{0.341}^{1}(u^2(x) + w^2(x))dx = 1 \).

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Table 5.2. Experimental data on the deuteron.*

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<td>Electric quadrupole moment:</td>
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<td>Magnetic moment:</td>
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<td>Scattering length:</td>
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<td>Effective range:</td>
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<td>( p(-\varepsilon, -\varepsilon) = (1.82 \pm 0.05) \times 10^{-13} \text{ cm} )</td>
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** The following value of \( Q_d \) is obtained in another analysis:**
\( Q_d = (2.7965 \pm 0.005) \times 10^{-27} \text{ cm}^2 \).
5.2 Triplet even scattering at high energies

In order to obtain information on inner interactions in the triplet even state, a high energy scattering should be treated to clarify the following problems concerning:

(i) the sign and strength of the $^3V_{ls}^z$,
(ii) quadratic spin-orbit effects,
(iii) qualitative features of the $^3V_{c}^z$ and $^3V_{t}^z$ in regions II and III, and
(iv) the triplet even repulsive core.

The static OPEP with the suitable hard core (e.g. $f^2/4\pi = 0.0755$, $x_c = 0.322$) explains all the low energy data. However, this potential gives a very large value of the $^3D_s$-phase shift $\delta(^3D_s)$ at high energies ($E \gtrsim 100$ MeV) because of its strong negative tensor potential, which is apparently inconsistent with the solutions of the phase shift analysis.\cite{121-124}

To suppress the $\delta(^3D_s)$, Hamada introduced the quadratic spin-orbit potential and obtained the fit of $\delta(^3D_s)$.\cite{117} In Gammel and Thaler's potential for high energy $n-p$ analysis,\cite{117} a negative $^3V_{ls}^z$ serves to reduce the magnitude of $\delta(^3D_s)$. Tamagaki, Wada and Watari\cite{52} extensively investigated the limits of various possibilities for the triplet even potentials by analysing the $n-p$ scattering up to 310 MeV. Their conclusions are summarized as follows (see Fig. 6-3).

(1) The effective potential in the $^3D_s$ state is

$$V(^3D_s) = ^3V_{c}^z + 2^3V_{t}^z - ^3V_{ls}^z - 7^3V_{w}^z + 6^3V_{LL}^z,$$

where $^3V_{c}^z + 2^3V_{t}^z$ gives very strong attraction and causes a large value of $\delta(^3D_s)$. To compensate this effect $- ^3V_{ls}^z - 7^3V_{w}^z + 6^3V_{LL}^z$ must be positive. If $^3V_{c}^z$ is negative and $^3V_{LL}^z$ is positive and they have appreciable strength (that is, somewhat stronger than that of the OPEP for the $pc$-coupling case), $^3V_{ls}^z$ can take a weak or positive value of about $(0.\sim 1.5) \times |^3V_{ls}^z|$. On the other hand, if the quadratic spin-orbit potential is really weak, we need the negative tail of $^3V_{ls}^z$ which is of the same order as the spin-orbit potential in the triplet odd state. If the negative $^3V_{ls}^z$ becomes too strong, $\delta(^3D_s)$ becomes too large, so such a strong value of the negative $^3V_{ls}^z$ is excluded. To get rid of the objection against the negative $^3V_{ls}^z$, the modification in order to make the $^3V_{ls}^z$ positive in region III serves to make the contribution of the $^3V_{ls}^z$ to $\mu_d$ negligibly small. The potentials with such behavior are given as NMLS in reference 62).

(2) Since the effective potential in the $^3D_s$ state is

$$V(^3D_s) = -^3V_{c}^z + 2^3V_{t}^z + 8^3V_{c}^z - 3^3V_{ls}^z + 7^3V_{w}^z + 6^3V_{LL}^z,$$

the strongly negative tensor potential as in the OPEP gives a too low value of $\delta(^3D_s)$ at high energies ($E \gtrsim 150$ MeV). The $^3V_{c}^z$ and $^3V_{LL}^z$ chosen to reproduce the experimental values of $\delta(^3D_s)$ are effectively cancelled in the
The $^3V_{T}^\dagger$ should therefore be weaker than that of the OPEP and a strongly attractive $^3V_{T}^\dagger$ is needed in the inner region ($x<1.0$) in order to complement the decrease of attractive effect of the tensor potential. The positive $^4V_{Ts}$ also gives a similar effect. The potentials with these features are apt to give $\epsilon_1\sim0$ or slightly negative $\epsilon_1$ at high energies, which is not inconsistent with the solutions of the phase shift analysis at present except the 310$-$330 MeV points. Since any reasonable potential model cannot predict such rapid energy dependence that $\epsilon_1=0\sim5^\circ$ at $E=100\sim200$ MeV and $\epsilon_1>20^\circ$ at $E=310\sim330$ MeV, the more complete experimental data are necessary to determine $\epsilon_1$. To determine $\epsilon_1$ more precisely is useful for obtaining the relative weight of the $^3V_{T}^\dagger$ and $^4V_{T}^\dagger$ in the inner region.

(3) No definite evidence has been shown about the existence of the repulsive core in the triplet even state. Probably it exists, because the $^3S_1$-phase shift $\delta(^3S_1)$ changes the sign at $E\sim350$ MeV. However, it is an open problem whether the triplet even repulsive core is as strong as the singlet even one. It may be possible to reproduce $\delta(^3S_1)$ by a much weaker repulsive core than the singlet even one. In a preliminary analysis by one of the authors (R.T.), the Gaussian repulsive core with the height of about 500 MeV is allowable, while the height of about 2 GeV is necessary in the singlet even state.

The allowable region for the triplet even potentials is large; in particular the spin-orbit potential is largely uncertain even in region II. The main reason why there exist such large ambiguities lies in the situation that the physical quantities of the $^3S_1+^3D_1$ state are adjusted to experimental values by a suitable combination of $^3V_{T}^\dagger$ and $^3V_{T}^\dagger$, and the $\delta(^3S_1)$ and $\delta(^3D_1)$ are fitted to experimental values by a suitable choice of $^3V_{Ts}$, $^3V_{T}$, and $^3V_{Ls}$, even if the data from the phase shift analysis become complete. Therefore, in order to reduce the ambiguities in region II, it is necessary, first of all, to obtain theoretically reliable estimate of the quadratic spin-orbit or more generally the nonstatic effects. In the OBEC analysis,$^{103}$ it was shown that the quadratic spin-orbit effect is small compared with the recoil correction to the central and tensor terms and with the spin-orbit effect. The effective strength of the $^3V_{Ls}$ is estimated from Eqs. (2·9) and (2·10) as

\[
\left(\frac{g_7^2}{4\pi}\right)_{\text{eff}} + \frac{1}{3} \left(\frac{g_8^2}{4\pi}\right)_{\text{eff}} = \left(\frac{g_7^2}{4\pi}\right)_{\text{eff}} - 3 \left(\frac{g_8^2}{4\pi}\right)_{\text{eff}} + \frac{1}{3} \left(\frac{g_8^2}{4\pi}\right)_{\text{eff}} \sim 25.
\]

This value means that the OBEC fit in the $n$-$p$ scattering is obtained by the negative spin-orbit potential.

Uncertainty in the triplet even potentials leads to uncertainty about the calculated binding energies in nuclei. Some suitable modification of the triplet even potentials may serve to improve the calculated values of the binding energies of the nuclear matter,$^{120,125}$ and the three-body system,$^{126}$
which are too small. The triplet even potentials, which have the smaller core radius, the stronger central parts and the weaker tensor parts than those of Hamada and Johnston and of the Yale group, may be able to give reasonable nuclear binding energies.\textsuperscript{103, 104} Examples of such potentials are shown in Fig. 6·3; PLS4 (for positive \(^{3}V_{L}^{+}\)), NLS2 (for negative \(^{3}V_{L}^{-}\)) and NMLSl (for outer-negative and inner-positive \(^{3}V_{L}^{-}\)) given in reference 62). The calculated phase shifts for NMLS1 are given in Table 5·3 and in Appendix of Chapter 5.

<table>
<thead>
<tr>
<th>CASE</th>
<th>(\mu c^2) (MeV)</th>
<th>(a_0)</th>
<th>(b_0)</th>
<th>(a_T)</th>
<th>(b_T)</th>
<th>(G_1)</th>
<th>(\mu_1)</th>
<th>(n_1)</th>
<th>(G_2)</th>
<th>(\mu_2)</th>
<th>(n_2)</th>
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</thead>
<tbody>
<tr>
<td>PLS4</td>
<td>139.4</td>
<td>7.0</td>
<td>8.75</td>
<td>-0.6</td>
<td>-0.4</td>
<td>4.5</td>
<td>5.5</td>
<td>2</td>
<td>6.75</td>
<td>6.7</td>
<td>4</td>
</tr>
<tr>
<td>NLS2</td>
<td>139.4</td>
<td>1.6</td>
<td>12.9</td>
<td>-0.9</td>
<td>0.1</td>
<td>-3.0</td>
<td>&quot;</td>
<td>&quot;</td>
<td>-4.5</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>NMLSl (^{b})</td>
<td>138.04 (^{b})</td>
<td>2.0</td>
<td>13.0</td>
<td>-0.26</td>
<td>-0.68</td>
<td>-6.0</td>
<td>&quot;</td>
<td>&quot;</td>
<td>35.0</td>
<td>10.0</td>
<td>&quot;</td>
</tr>
<tr>
<td>HJ (^{1})</td>
<td>139.4</td>
<td>6.0</td>
<td>-1.0</td>
<td>-0.5</td>
<td>0.2</td>
<td>0.0743</td>
<td>2.0</td>
<td>&quot;</td>
<td>-0.0074</td>
<td>3.0</td>
<td>3</td>
</tr>
</tbody>
</table>

\(^{b}\) This potential is the same with NMLS1 in reference 62) except \(\mu c^2\) and \(x_0\). The effective pion mass \(\mu = (2\mu_1 + \mu_0) / 3\) in the \(T=0\) OPEP is used and \(x_0\) is slightly increased to secure the low energy fit.

(b) Triplet even nuclear bar phase shifts (in degrees) calculated from the NMLSl in Table (5·3a). These values are shown in figures in Appendix of Chapter 5. The low energy parameters are \(\delta a = 5.340 \times 10^{-13}\) cm, \(\delta r_{z} = 1.754 \times 10^{-13}\) cm, \(\delta P = -0.009\), \(\delta Q = 0.002\), \(q = 0.31 \times 10^{-24}\) cm\(^2\), where \((\tan \epsilon f/k^2) = q(k \to 0)\) for Blatt-Biedenharn mixing parameter.

<table>
<thead>
<tr>
<th>(E) (MeV)</th>
<th>(^{3}S_1)</th>
<th>(^{3}D_1)</th>
<th>(^{3}D_2)</th>
<th>(^{3}D_3)</th>
<th>(^{3}G_3)</th>
<th>(^{3}G_4)</th>
<th>(^{3}G_5)</th>
<th>(\epsilon_5)</th>
<th>(I_5)</th>
<th>(I_6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.02</td>
<td>1.00</td>
<td>-0.70</td>
<td>0.88</td>
<td>0.00</td>
<td>0.07</td>
<td>-0.00</td>
<td>0.01</td>
<td>-0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>20</td>
<td>86.38</td>
<td>1.27</td>
<td>-2.15</td>
<td>2.87</td>
<td>0.02</td>
<td>0.39</td>
<td>-0.03</td>
<td>0.09</td>
<td>-0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>40</td>
<td>68.69</td>
<td>1.24</td>
<td>-5.29</td>
<td>7.32</td>
<td>0.18</td>
<td>1.29</td>
<td>-0.17</td>
<td>0.51</td>
<td>-0.03</td>
<td>0.13</td>
</tr>
<tr>
<td>60</td>
<td>57.62</td>
<td>1.03</td>
<td>-8.16</td>
<td>11.45</td>
<td>0.52</td>
<td>2.21</td>
<td>-0.41</td>
<td>1.07</td>
<td>-0.08</td>
<td>0.33</td>
</tr>
<tr>
<td>80</td>
<td>49.34</td>
<td>0.82</td>
<td>-10.71</td>
<td>14.96</td>
<td>1.01</td>
<td>3.07</td>
<td>-0.71</td>
<td>1.70</td>
<td>-0.13</td>
<td>0.55</td>
</tr>
<tr>
<td>100</td>
<td>42.61</td>
<td>0.63</td>
<td>-12.97</td>
<td>17.84</td>
<td>1.62</td>
<td>3.81</td>
<td>-1.06</td>
<td>2.34</td>
<td>-0.19</td>
<td>0.80</td>
</tr>
<tr>
<td>120</td>
<td>36.90</td>
<td>0.46</td>
<td>-15.01</td>
<td>20.14</td>
<td>2.31</td>
<td>4.47</td>
<td>-1.43</td>
<td>2.97</td>
<td>-0.24</td>
<td>1.05</td>
</tr>
<tr>
<td>140</td>
<td>31.91</td>
<td>0.33</td>
<td>-16.85</td>
<td>21.95</td>
<td>3.05</td>
<td>5.03</td>
<td>-1.83</td>
<td>3.59</td>
<td>-0.28</td>
<td>1.29</td>
</tr>
<tr>
<td>160</td>
<td>27.44</td>
<td>0.21</td>
<td>-18.53</td>
<td>23.35</td>
<td>3.80</td>
<td>5.51</td>
<td>-2.24</td>
<td>4.18</td>
<td>-0.31</td>
<td>1.52</td>
</tr>
<tr>
<td>180</td>
<td>23.40</td>
<td>0.11</td>
<td>-20.08</td>
<td>24.39</td>
<td>4.55</td>
<td>5.91</td>
<td>-2.65</td>
<td>4.75</td>
<td>-0.34</td>
<td>1.75</td>
</tr>
<tr>
<td>200</td>
<td>19.68</td>
<td>0.03</td>
<td>-21.52</td>
<td>25.15</td>
<td>5.29</td>
<td>6.24</td>
<td>-3.07</td>
<td>5.28</td>
<td>-0.35</td>
<td>1.98</td>
</tr>
</tbody>
</table>
5.3 Singlet odd state

In the singlet odd state, the one-pion-exchange potential gives a strong repulsion owing to the large factor \((\sigma_1 \cdot \sigma_2) (\tau_1 \cdot \tau_2)\) = 9. This repulsion plays an important role in reproducing the backward peak in \(I_6(\theta)\) of \(n\)-\(p\) scattering. It seems necessary to weaken this OPEP repulsion in region II and to strengthen the repulsion in region III from the energy dependence of \(\delta(1P_1)\). Both the Hamada and Johnston and Yale group potentials have this property. The difference between these two potentials is attributed to the quadratic spin-orbit term: In the former a large quadratic spin-orbit term is used and in the latter this term is not introduced. Concerning this point, it is important to reduce ambiguities in the phase shift analysis, and at present we can only say that the modification to the OPEP in region II is attractive and a strong repulsion probably exists in region III. Such an example is illustrated as A in Fig. 6·4 and the phase shifts are given in Table 5·4 and in figures in Appendix of Chapter 5.

### Table 5·4. Singlet odd phase shifts (in degrees) calculated from the potential without \(1V_{fi}\) (A in Fig. 6·4):

\[
1V = 3\mu e^2 (f^1 / 4\pi) \\
\times (1 - 4Y + 12Y^2) Y,
\]

where \(Y = e^2/x\) and \(\mu e^2 = (2\mu_1 + \mu_2)/3 = 138.04\) MeV

and \(x_0 = 0.29523\).

<table>
<thead>
<tr>
<th>(E) (MeV)</th>
<th>(\vec 3S_1)</th>
<th>(\vec 3D_1)</th>
<th>(\vec 3D_2)</th>
<th>(\vec 3D_3)</th>
<th>(\vec 3G_2)</th>
<th>(\vec 3G_3)</th>
<th>(\vec 3G_4)</th>
<th>(\vec 3I_5)</th>
<th>(\vec 3I_6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>220</td>
<td>16.25</td>
<td>-0.03</td>
<td>-22.87</td>
<td>25.66</td>
<td>5.99</td>
<td>6.52</td>
<td>-3.49</td>
<td>5.79</td>
<td>-0.35</td>
</tr>
<tr>
<td>240</td>
<td>13.04</td>
<td>-0.09</td>
<td>-24.13</td>
<td>25.98</td>
<td>6.64</td>
<td>6.74</td>
<td>-3.90</td>
<td>6.26</td>
<td>-0.35</td>
</tr>
<tr>
<td>260</td>
<td>10.04</td>
<td>-0.13</td>
<td>-25.33</td>
<td>26.13</td>
<td>7.25</td>
<td>6.92</td>
<td>-4.30</td>
<td>6.71</td>
<td>-0.33</td>
</tr>
<tr>
<td>280</td>
<td>7.20</td>
<td>-0.17</td>
<td>-26.46</td>
<td>26.15</td>
<td>7.79</td>
<td>7.05</td>
<td>-4.70</td>
<td>7.12</td>
<td>-0.31</td>
</tr>
<tr>
<td>300</td>
<td>4.51</td>
<td>-0.19</td>
<td>-27.54</td>
<td>26.04</td>
<td>8.27</td>
<td>7.15</td>
<td>-5.09</td>
<td>7.51</td>
<td>-0.27</td>
</tr>
<tr>
<td>320</td>
<td>1.96</td>
<td>-0.21</td>
<td>-28.58</td>
<td>25.85</td>
<td>8.70</td>
<td>7.22</td>
<td>-5.47</td>
<td>7.87</td>
<td>-0.23</td>
</tr>
<tr>
<td>360</td>
<td>-29.86</td>
<td>0.06</td>
<td>-42.66</td>
<td>16.40</td>
<td>7.80</td>
<td>6.54</td>
<td>-10.30</td>
<td>10.34</td>
<td>1.65</td>
</tr>
</tbody>
</table>

\[0.11x741]
§6. Concluding remarks

We have so far discussed many evidences and informations which have been obtained by the potential model approach under the condition that the OPEP-tail with the pion-nucleon coupling constant \( f^2/4\pi = 0.065 \sim 0.090 \) exists in region I (\( x > 1.5 \)). We summarize them in Table 6·1.

The first (second) column indicates our knowledge of the potential in region II (III) for each two-nucleon state. In the upper part (above the dotted lines of each column) are shown the properties of the potentials clarified by comparison with the experimental data up to \( 350 \text{ MeV} \). In the lower part (below the dotted lines) are shown the possibilities that have not been well determined because of the correlation with ambiguities in the other parts of potentials. The potentials thus determined are shown in Figs. 6·1~6·4, here the uncertainties are illustrated by the shaded area.

Fig. 6·1. Singlet even potentials restricted by the data below \( E \sim 350 \text{ MeV} \). The shaded area denotes the allowable region of \( V^e_x \) and \( V^e_{LJ} \). Figures 3·4, 3·4a and 3·6 should be referred to for details.
Table 6.1. Summary of the results obtained by the potential model approach, using

The upper part of each column shows the properties of potentials clarified by

The lower part of each column shows the possibilities that have not been unsolved.

<table>
<thead>
<tr>
<th></th>
<th>region II ((x = \mu r = 1.5 \sim 0.7))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Singlet even</strong> ((^1E))</td>
<td>(S = 0) (T = 1)</td>
</tr>
<tr>
<td>(1V_0)</td>
<td>The strongly attractive central potential (1V_0) exists in the domain (1V_0) (TMO) (&gt; 1V_0 &gt; 1V_0) (KMO), and its allowable region is shown in Fig. 6.1 by the shaded area. (§3·2)</td>
</tr>
<tr>
<td>Nonstatic effect to decrease (\delta) ((1D_2)) at (E \geq 150) MeV should be present, which can be represented by the weak repulsive potential (1V_{ll}) with the reasonable strength expected pion-theoretically. (§3·4)</td>
<td></td>
</tr>
<tr>
<td><strong>Triplet odd</strong> ((^3O))</td>
<td>(S = 1) (T = 1)</td>
</tr>
<tr>
<td>(1V_F)</td>
<td>The tail of (1V_{ls}) is negative and the strength is about (1V_{ls}(x = 0.7) \sim -50) MeV. This potential has the range parameter (\sim 1/5.5) determined by the mass of the vector mesons ((\omega, \rho)) and of the strength determined by the effective coupling constant ((\frac{g_\pi^2}{4\pi^2}) \sim 32), if the main part of the (1V_{ls}) is due to the one vector meson exchange. (§4·2)</td>
</tr>
<tr>
<td>(1V_F)</td>
<td>(1V_F) is positive but weaker than (1V_F) (OPEP), and its average strength is of (1V_{ll} \sim 8) MeV. (§4·3)</td>
</tr>
<tr>
<td>(\delta) ((3P_0)) at (E \geq 200) MeV is reasonably reproduced by positive (1V_{0}) and positive (1V_{ll}) modified in region II so as to be effective in the coupled states. (§4·3)</td>
<td></td>
</tr>
<tr>
<td><strong>Triplet even</strong> ((^3E))</td>
<td>(S = 1) (T = 0)</td>
</tr>
<tr>
<td>(1V_0)</td>
<td>Negative (1V_\uparrow) is weak erhan (1V_\uparrow) of the OPEP: (1V_\uparrow) (OPEP) (\geq 1V_\uparrow) (OPEP).</td>
</tr>
<tr>
<td>(1V_0)</td>
<td>(1V_0) is strongly attractive: (1V_0 \leq 3V_\uparrow &lt; 3V_\uparrow) (OPEP). (§5·2)</td>
</tr>
<tr>
<td>(1V_{ll})</td>
<td>(1V_{ll}) is largely uncertain. There are two possibilities. If negative (1V_\uparrow) and positive (1V_{ll}) are of the same order of magnitude as in (^1E) and (^3O), the spin-orbit potential is positive or small: (1V_{ls} = 0 \sim 1.5) (1V_{ls}). If (1V_\uparrow) and (1V_{ll}) are negligibly small, the tail of (1V_{ls}) should be negative: (1V_{ls} \sim -1V_{ls}). (§5·2)</td>
</tr>
<tr>
<td><strong>Singlet odd</strong> ((^1O))</td>
<td>(S = 0) (T = 0)</td>
</tr>
<tr>
<td>(1V_0)</td>
<td>Repulsive (1V_0) is weaker than (1V_0) of the OPEP. (§5·3)</td>
</tr>
</tbody>
</table>
Potential Model Approach

\[ V = V_e + V_T S^1_1 + V_{LS}(LS) + V_{W12} + V_{LL} L^2 \] with the OFEP-tail.

comparision with the experimental data up to 350 MeV.

uniquely determined owing to the correlation with other parts and the problems that remain

between region III \((x = \mu r \leq 0.7)\)

The repulsive core exists in the \(1S_0\) state, and its radius is \(x_0 = 0.35 - 0.20 (f^2/4\pi = 0.08 - 0.06)\) for the hard core potentials. For the Gauss soft core, its height is larger than 2 GeV. If \(f^2/4\pi \geq 0.07\), the Yukawa soft core with the range \(\approx 1/5.5\), is allowed, where large cancellation takes place between the large coupling constants responsible for the repulsive core and the intermediate attraction. (§3.3)

In the soft core representation, the maximum depth of the attraction is about \(-100 - 150\) Mev just outside the core (see Chapter 7).

Negative \(^1V_{LS}\) is abruptly strong inside: \(^3V_{LS}\) \((x = 0.5) < -200\) MeV. (§4.3)

\(^3V_{\delta}\) is not strongly attractive, but positive or weak. (§4.5)

\(^3V_{\bar{\tau}}\) is not strongly positive but negative or weak. (§4.5)

The effective repulsive potential is necessary at short distances so as to cancel the attraction due to \(^3V_{LS}\) in the \(J = L + 1\) states \((^3P_2\) and \(^3F_4\)\), while in the other states the core region is completely masked by the repulsion due to \(^3V_{LS}\). (§4.5)

There are two possibilities: One is the repulsive core with surrounding singular potentials and the other is the nonsingular potential with the nonstatic effects acting repulsively in the \(J = L + 1\) states \((^3P_2\) and \(^3F_4\)). (§4.5)

\(^3V_{\delta}\) is strongly attractive, although \(^3V_{\bar{\tau}}\) is uncertain. (§5.2)

For negative \(^3V_{LS}\) tail, \(^3V_{LS}\) at short distances should be positive, unless the exchange moment correction to the deuteron magnetic moment \(\mu_d\) is positive and large. (§5.1)

The repulsive core probably exists in the \(^4S_1\) state, although not proved.

Repulsion is probably strong. (§5.3)
Fig. 6-2. Triplet odd potentials restricted by the data below $E \sim 350$ MeV. The shaded area denotes the allowable region well confirmed. The dashed lines illustrate some reasonable possibilities. Figures 4-2, 4-3 and 4-4 should be referred to for details of (A) central, (C) spin-orbit and (D) $V_{W}$ and $V_{LL}$, respectively.
Fig. 6·3. Triplet even potentials consistent with the data below $E\sim 350$ MeV. The shaded area is not so definite as in the singlet even and the triplet odd state, but reasonable triplet even potentials will probably exist in this area. The parameters of the potentials (HJ, PLS4, NLS2, NNLS1) are given in Table 5·3(a).
Fig. 6-4. Singlet odd potentials with $L^2$-term (HJ) and without $L^2$-term (A). A drastic modification of the OPEP in HJ-curves is compensated by the use of $V_{LL}$. No positive evidence for $V_{LL}$ can be found. Parameters of two cases are: For HJ, $a_0 = -8$, $b_0 = 12$, $\lambda = 1$, $a_{LL} = 2$, $b_{LL} = 6$ and $x_0 = 0.343$; and for A, $a_0 = -4$, $b_0 = 12$, $\lambda = a_{LL} = b_{LL} = 0$ and $x_0 = 0.29523$.

Acknowledgements

One of the authors (R. T.) would like to express his sincere thanks to Professor H. Tanaka for his kind hospitality. He is also very grateful to Dr. M. Wada for many helpful discussions. Some parts of numerical calculations were carried out by HITAC 5020 at Computer Center, Tokyo University.

Appendix

Here we shall show the matrix elements of potential operators over the spin and angular parts. The matrix elements are diagonal with respect to $S$, because the two-nucleon potential should be symmetric for the interchange $\sigma_1 \leftrightarrow \sigma_2$.

For the tensor force operator $S_{12}$, the matrix elements are

$$
S_{12} = \begin{pmatrix}
q_{J-1,1,I} & q_{J,1,I} & q_{J+1,1,I} & q_{J,0,1} \\
-\frac{2(J-1)}{2J+1} & 0 & \frac{6\sqrt{J(J+1)}}{2J+1} & 0 \\
0 & 2 & 0 & 0 \\
\frac{6\sqrt{J(J+1)}}{2J+1} & 0 & -\frac{2(J+2)}{2J+1} & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
$$

(A.1)

The $4 \times 4$ matrix is $\langle LSJ | S_{12} | L' S' J \rangle$,

where $|LSJ\rangle = q_{LSJ}^\dagger = \sum (LSm_l m_s | Jm_J \rangle Y_l^m(Q) \chi_s^{m_s}$

(A.2)
composed of the angular wave function \( Y_{\ell}^{m} \) and the spin wave function \( \chi_{s} \).

The operator \( \mathbf{L} \cdot \mathbf{S} \) conserves \( L \) and vanishes in the singlet states:

\[
\mathbf{L} \cdot \mathbf{S} = \frac{1}{2} \left[ \mathbf{J}^{2} - \mathbf{L}^{2} - \mathbf{S}^{2} \right] = \frac{1}{2} \left[ J(J+1) - L(L+1) - S(S+1) \right]
\]

\[
\begin{align*}
J-1 &= L & \text{for } S=1, \ L=J-1, \\
-1 &= - (L+1) & \text{for } S=1, \ L=J+1, \\
-(J+2) &= - (L+1) & \text{for } S=0.
\end{align*}
\]

The operator \( W_{12} \) given in Eq. (2·12) conserves \( L \) and vanishes in the singlet states. The matrix elements in the triplet states become

\[
W_{12} = (\mathbf{L} \cdot \mathbf{S})^{2} - (\delta_{LJ} + 1/3) L(L+1)
\]

\[
\frac{(J-1)(2J-3)}{3} = L(2L-1)/3 & \quad \text{for } L=J-1, \\
-4J(J+1)/3 = -(2L-1)(2L+3)/3 & \quad \text{for } L=J, \\
(J+2)(2J+5)/3 = (L+1)(2J+3)/3 & \quad \text{for } L=J+1.
\]

Thus the potential (2·15)

\[
V = V_{c} + S_{12}V_{T} + (\mathbf{L} \cdot \mathbf{S})V_{Ls} + W_{12}V_{w} + L^{2}V_{LL}
\]

gives an effective potential for each partial wave as follows:

\[
\begin{align*}
^{1}V(J) &= V_{c} + J(J+1) \cdot V_{LL}, \\
^{2}V(L=J-1) &= V_{c} - 2(J-1)/(2J+1) \cdot V_{T} + 6\sqrt{J(J+1)}/(2J+1) \cdot V_{w}^{*} \\
&\quad + (J-1) \cdot V_{Ls} + (J-1)(2J-3)/3 \cdot V_{w} + J(J-1) \cdot V_{LL} \\
&= [V_{c} + L(L+1) \cdot V_{LL}] - 2L/(2L+3) \cdot [V_{T} - (2L-1)(2L+3)/6 \cdot V_{w}] - V_{Ls}, \\
^{3}V(L=J) &= V_{c} + 2V_{T} - V_{Ls} + [1 - 4J(J+1)/3] \cdot V_{w} + J(J+1) \cdot V_{LL} \\
&= [V_{c} + L(L+1) \cdot V_{LL}] + 2[V_{T} - (2L-1)(2L+3)/6 \cdot V_{w}] - V_{Ls}, \\
^{4}V(L=J+1) &= V_{c} - 2(J+2)/(2J+1) \cdot V_{T} + 6\sqrt{J(J+1)}/(2J+1) \cdot V_{w}^{*} \\
&\quad - (J+2) \cdot V_{Ls} + (J+2)(2J+5)/3 \cdot V_{w} + (J+1)(J+2) \cdot V_{LL} \\
&= [V_{c} + L(L+1) \cdot V_{LL}] - 2L/(2L-1) \cdot [V_{T} - (2L-1) \cdot V_{Ls} + (2L+3)/6 \cdot V_{w}] - (L+1) \cdot V_{LL} + 6\sqrt{J(J+1)/(2J+1) \cdot V_{w}^{*}},
\end{align*}
\]

where the mark \( V_{w}^{*} \) indicates the coupling term between the \( L=J-1 \) and \( L=J+1 \) states. These effective potentials are listed in Table A·1.
Table A.1. Effective potential in each state with the spin $S$, parity $\Pi$, isospin $T$ and total angular momentum $J$, given by a potential,

$$ V = V_0 + S_{12} V_T + (\mathbf{L} \cdot \mathbf{S}) V_{LS} + W_{12} V_W + \mathbf{E} V_{LE}. $$

The coupling terms due to a tensor potential are denoted by $V_T$. 

$T=1$

<table>
<thead>
<tr>
<th>$J$</th>
<th>$S=0$, $\Pi=+$</th>
<th>$S=1$, $\Pi=+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$^1S_0$</td>
<td>$^1P_0$</td>
</tr>
<tr>
<td></td>
<td>$V_C$</td>
<td>$V_0 + 2V_T$</td>
</tr>
<tr>
<td>1</td>
<td>$^1P_1$</td>
<td>$^3P_1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-V_{LS} - (5/3)V_W + 2V_{LL}$</td>
</tr>
<tr>
<td>2</td>
<td>$^1D_2$</td>
<td>$^3P_2$</td>
</tr>
<tr>
<td></td>
<td>$V_C + 6V_{LL}$</td>
<td>$V_0 - (8/5)V_T + (6\sqrt{6}/5)V_T^# - 4V_{LS} + 12V_W + 12V_{LL}$</td>
</tr>
<tr>
<td>3</td>
<td>$^1F_3$</td>
<td>$^3F_3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-V_{LS} - 15V_W + 12V_{LL}$</td>
</tr>
<tr>
<td>4</td>
<td>$^3G_4$</td>
<td>$^1F_4$</td>
</tr>
<tr>
<td></td>
<td>$V_C + 20V_{LL}$</td>
<td>$V_0 - (4/3)V_T + (4\sqrt{5}/3)V_T^# - 6V_{LS} + 26V_W + 30V_{LL}$</td>
</tr>
<tr>
<td>5</td>
<td>$^3H_5$</td>
<td>$^1H_5$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-V_{LS} - 39V_W + 30V_{LL}$</td>
</tr>
<tr>
<td>6</td>
<td>$^1I_6$</td>
<td>$^3H_6$</td>
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</tbody>
</table>

$T=0$

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<th>$S=1$, $\Pi=+$</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>$^1P_1$</td>
<td>$^3S_1$</td>
</tr>
<tr>
<td></td>
<td>$V_C + 2V_{LL}$</td>
<td>$V_0 - 2V_T + \sqrt{8} V_T^#$</td>
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<tr>
<td>$V_0 + 2V_T$</td>
<td>$-3V_{LS} + 7V_W + 6V_{LL}$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$^3D_4$</td>
<td>$^3D_4$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-V_{LS} - 7V_W + 6V_{LL}$</td>
</tr>
<tr>
<td>3</td>
<td>$^3F_3$</td>
<td>$^3D_3$</td>
</tr>
<tr>
<td></td>
<td>$V_C + 12V_{LL}$</td>
<td>$V_0 - (10/7)V_T + (12\sqrt{3}/7)V_T^# - 5V_{LS} + (55/3)V_W + 20V_{LL}$</td>
</tr>
<tr>
<td>4</td>
<td>$^3G_4$</td>
<td>$^1G_4$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-V_{LS} - (77/3)V_W + 20V_{LL}$</td>
</tr>
<tr>
<td>5</td>
<td>$^1H_5$</td>
<td>$^3G_5$</td>
</tr>
<tr>
<td></td>
<td>$V_C + 30V_{LL}$</td>
<td>$V_0 - (14/11)V_T + (6\sqrt{30}/11)V_T^# - 7V_{LS} + 35V_W + 42V_{LL}$</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>$-V_{LS} - 55V_W + 42V_{LL}$</td>
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</table>

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