Soft Meson Approximation and the Decays of Tensor Mesons

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December 12, 1967

The soft pion approximation has been used to calculate the decay rate of \( \rho \) meson by Kawarabayashi and Suzuki.\(^1\) It is in reasonable agreement with experiment in spite of its large energy release. In this note, we compare the observed partial decay widths of tensor mesons, i.e. \( A_0(1320) \), \( K_\rho(1420) \), \( f(1250) \) and \( f'(1500) \), with theoretical predictions based on the soft meson approximation, the nonet theory and the PCAC hypothesis. By making use of the reduction technique, the PCAC hypothesis and the soft meson approximation, we obtain

\[
\langle k^\alpha k^\beta | T_{\mu \nu} | \rangle = (2\pi)^4 \delta^4 (P_i - P_f) \\
\times \frac{k^\mu k^\nu t^{(\alpha \beta)}}{\sqrt{8\omega_\mu \omega_\nu m_\pi V}} \frac{4}{F_\alpha F_\beta} C^{\alpha \beta} (AA),
\]

for the decays of the \( J^P=2^+ \) mesons into two pseudoscalar mesons,\(^2\)\(^,*\) where the definition of \( C^{\alpha \beta} (AA) \) is the following:

\[
\int \int d^4x d^4y \langle 0 | T[A_\alpha^a(x) A_\beta^a(y)] | T^+ \rangle = (2\pi)^4 \delta^4 (P_i - P_f) \frac{t_{10}^{(\alpha \beta)}}{\sqrt{2m_\pi V}} C^{\alpha \beta} (AA)
\]

and \( t_{10}^{(\alpha \beta)} \) is the polarization tensor of tensor meson. We assume the PCAC hypothesis

\[
\phi_\alpha (x) = \frac{2}{F_\alpha m_\pi} \partial_\mu A_\mu^a (x); \quad \alpha = 1, 2, \ldots, 9. \quad (2)
\]

* The derivation of Eq. (1) is the same as that of reference 1, and the terms except for the right-hand side of Eq. (1) vanish for the decays of tensor mesons.
Table I. Decays of tensor mesons into two pseudoscalar mesons.

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Phase space</th>
<th>Predicted rate (MeV)</th>
<th>Glassow &amp; Socolow (MeV)</th>
<th>Observed rate (MeV)</th>
<th>Observed rate (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td></td>
</tr>
<tr>
<td>$f \to 2\pi$</td>
<td>55.0</td>
<td>110</td>
<td>110</td>
<td>110</td>
<td>110</td>
</tr>
<tr>
<td>$f \to KK$</td>
<td>5.2</td>
<td>4.4</td>
<td>2.5</td>
<td>1.7</td>
<td>4.2</td>
</tr>
<tr>
<td>$f \to \eta\eta$</td>
<td>1.5</td>
<td>0.14</td>
<td>0.07</td>
<td>0.04</td>
<td>0.2</td>
</tr>
<tr>
<td>$f' \to 2\pi$</td>
<td>99</td>
<td>1.7</td>
<td>1.7</td>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>$f' \to KK$</td>
<td>27</td>
<td>31.4</td>
<td>18</td>
<td>11.8</td>
<td>31</td>
</tr>
<tr>
<td>$f' \to \eta\eta$</td>
<td>17</td>
<td>11.3</td>
<td>5.5</td>
<td>3.4</td>
<td>9</td>
</tr>
<tr>
<td>$A_2 \to \pi\pi$</td>
<td>23.5</td>
<td>8.8</td>
<td>6.0</td>
<td>4.7</td>
<td>11</td>
</tr>
<tr>
<td>$A_2 \to \pi\eta'$</td>
<td>0.9</td>
<td>1.5</td>
<td>1.2</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>$A_2 \to KK$</td>
<td>8.1</td>
<td>5.5</td>
<td>3.1</td>
<td>2.0</td>
<td>6</td>
</tr>
<tr>
<td>$K_w \to K\pi$</td>
<td>42.0</td>
<td>42.5</td>
<td>32.0</td>
<td>26.0</td>
<td>42</td>
</tr>
<tr>
<td>$K_w \to K\eta$</td>
<td>12.0</td>
<td>1.8</td>
<td>1.0</td>
<td>0.6</td>
<td>1.4</td>
</tr>
</tbody>
</table>

In the first column all charged states are included and input values are underlined.
Phase space is $k^2/M_2^2$ in units of $10^{-3} \text{BeVs}$.
Misprinted value of $\Gamma(f\to KK)$ in reference 4) is corrected.

has been obtained by Kawarabayashi and Wada.\textsuperscript{a,b} According to their discussion we obtain

$$F_K^2 = K^2 F_\pi, \quad F_w = \frac{4K^2 - 1}{3} F_\pi^2$$

(3)

and

$$F_{w_0} = \frac{2K^2 + 1}{3} F_\pi^2.$$ 

Therefore, if $F_K^2 > F_\pi^2$, $F_w^2 > F_\pi^2$, $F_{w_0}^2 > F_\pi^2$. C\textsuperscript{ao}r(\textit{AA}) in Eq. (1) is nothing but D-coupling parameter, because it conserves C.\textsuperscript{b} We take into consideration the unitary octet-singlet mixing for $f$ and $f'$ (; the mixing angle $\theta_f \approx 30'$) and for $\eta$ and $\eta'$ (; the mixing angle $\theta \approx 10'$)

$$f' = \cos \theta_f f_0 - \sin \theta_f f_9,$$

$$f = \sin \theta_f f_0 + \cos \theta_f f_9,$$

$$\phi_\eta = \cos \theta_{\phi_\eta} - \sin \theta_{\phi_\eta},$$

and

$$\phi_{\eta'} = \sin \theta_{\phi_{\eta'}} + \cos \theta_{\phi_{\eta'}}.$$ 

We discuss the following cases for $K = F_K/F_\pi$:

(a) $K \approx 1.00$ (Kawarabayashi and Wada),
(b) $K \approx 1.15$ (Glassow et al.),\textsuperscript{4}

(c) $K \approx 1.28$ (Cabbibo).\textsuperscript{5}

Thus we obtain the decay widths shown in Table I which are based on the input $\Gamma(f\to 2\pi) = 110 \text{MeV}$. In Table I our results are compared with those by Glassow and Socolow. The agreement between our results and experiments is good except for $\Gamma(f\to KK)$ in each case. The idea may be applied to another decay mode, the decay of tensor meson into a vector meson and a pseudoscalar meson, by making use of the field-current identity hypothesis.\textsuperscript{6} This will be discussed elsewhere.

The author would like to express his thanks to Professor Y. Miyamoto for his encouragement and Professor Y. Hara for his encouragement and reading of this manuscript.

3) S. L. Glashow and R. H. Socolow, Phys. Rev. Letters 15 (1965), 329. The main difference between their result and ours is the inclusion of the ratio $F_K^2/F_w^2$ and an effect.

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of $\nu-\nu'$ mixing. In both formulations the absolute rate cannot be obtained.


7) A. H. Rosenfeld et al., preprint (UCRL 8030 (Rev.) September 1967).