New Method of High Energy Jet Shower Analysis

―Relative Four-Momentum between Secondary Particles―

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A new method of the analysis of high energy jets is developed. This method is related to the study of the correlation of secondary shower tracks observed in the target diagram which introduces the Lorentz invariant quantity; the spectrum of “relative four-momentum between two secondary particles”. The experimental data on high energy jets in emulsion stacks and emulsion chambers (ECC) are analysed by using this method. Comparing the experimental results with the Monte Carlo calculations on the basis of the fire ball model, one can identify fire balls which could not be observed by the knowledge of log tan θ plot only. It is shown that results of a detailed analysis are quite consistent with the H-quantum hypothesis which insists on the existence of energy (or mass) quantum in the violent collision of high energies.

§ 1. Introduction

It is now quite reliable to say that the multiple meson production occurs through the intermediate product called “fire ball” in the high energy collision exceeding around $10^{11}$ eV. Fire balls were studied through the strong correlation of secondary particles in the angular distribution of secondary shower particles, observed mainly in the log tan θ-plot.

On the basis of detailed analysis of high energy jet showers, Hasegawa proposed the hypothesis that there exists an energy (or mass) quantum called “H-quantum”. The mass of H-quantum was estimated as about twice the nucleon rest mass. In the course of justifying this hypothesis, several kinematical approaches of jet analysis have been developed on the basis of the knowledge of the log tan θ-plot. Using the approximate constancy of transverse momentum, Yokoi and Hasegawa obtained a Lorentz invariant formula about the longitudinal momentum transfer, $J_t$, between two groups of secondary mesons. The results of the analysis by this formula revealed that the magnitude of $J_t$ is of the order of H-quantum rest mass. Yajima and Hasegawa generalized this method and proposed a new kinematical method for obtaining relevant physical quantities of

*) Recent observation of extreme high energy nuclear interactions by means of large emulsion chamber experiment at Mt. Chacaltaya by Japan-Brasil collaboration has shown the strong evidence for the existence of this energy quantum. The mass of about thrice the nucleon rest mass was observed.
multiple meson production.\textsuperscript{5)}

When one sees the target diagram of secondary shower particles of high energy jets, we often find strong correlations of shower particles which could not be observed only in the log tan $\theta$-plot. These correlations observed in the target diagram seems to exceed fairly the simple fluctuation expected from the uniform azimuthal distribution of shower particles around the shower axis. We are able to consider that such a correlation is caused by the original character of high energy collision if one stands on the view of productions of the intermediate products, called fire ball. The order of magnitude of the average value of longitudinal four-momentum transfer between two groups of secondary mesons is about the nucleon rest mass, as was shown in reference 4). It is, therefore, quite natural to expect that the average transverse component of four-momentum transfer between fire balls is also about the same value as the order of magnitude. These knowledge of transverse component should be obtained only through the study of target diagram of shower particles and could not be by the kinematical approaches developed so far which are based only on the knowledge of the log tan $\theta$-plot.

One of the authors (T. Shibata) introduced a new Lorentz invariant quantity in order to study the correlations of shower particles in the target diagram; that is “relative four-momentum between two shower particles, $\delta_{ij}$, for arbitrary $i$-th and $j$-th shower tracks. This quantity is the function of relative emission angle between two shower particles, $\Theta_{ij}$, as described in § 2. Therefore, if one considers the spectrum $F_i(\delta_{ij})$ of all secondary shower particles referring to an $i$-th track, the azimuthal correlations observed in target diagram with respect to the $i$-th track are all included through the relative angle, $\Theta_{ij}$, in a Lorentz invariant way. If one calculates this spectrum referring to every shower track, the detailed features of grouping existing in secondary particles become complete.

The grouping nature above mentioned is discussed on the basis of the fire ball model and compared with the experimental results.

\section{Spectrum of relative four-momentum between two particles}

Let us consider the relative four-momentum, $\delta_{ij}$, of the $j$-th track referring to the $i$-th. (We assume that all the secondary particles are pions.) From the definition of this Lorentz invariant quantity, we get (for $E_i, E_j \gg \mu$ and $\Theta_{ij} \ll 1$)

$$\delta_{ij} = (E_i - E_j)^2 - (p_i - p_j)^2$$

$$- \frac{E_i - E_j}{E_i \cdot E_j} 4E_iE_j \sin^2 \frac{\Theta_{ij}}{2},$$

where $\mu$ is pion mass and $\Theta_{ij}$ is the relative angle between $i$-th and $j$-th pions.

As to the practical analysis of the angular distribution of emulsion stack data, we use the approximate constancy of $p_T$, i.e.
Thus one can obtain

\[ |\vec{\sigma}'|^2 = \mu^2 (\sin \theta_i - \sin \theta_j)^2 + 4 \langle p_r \rangle^2 \sin \Theta_{ij}/2. \]  

(1)

On the other hand, in the case of the ECC data, we know the energy in each produced \( \pi^0 \)-meson, then one obtains

\[ |\vec{\sigma}'|^2 = \mu^2 \left( \frac{E_i - E_j}{E_i E_j} \right)^2 + E_i E_j \Theta_{ij}^2. \]  

(2)

The physical meaning of relative four-momentum between two particles can be understood as a quantity approximately proportional to the relative Lorentz factor of one particle to the rest system of another when \( \gamma \approx 1 \). In the case \( \gamma \approx 1 \), this quantity is approximately proportional to the square of relative velocity.\(^*\)

§ 3. Spectrum expected from fire ball

Now let us proceed to study the spectrum of relative four-momentum on the basis of the fire ball model.

i) If one assumes that a fire ball decays isotropically into pions with nearly equal momentum \( \vec{p}_0 \) in the fire ball rest system, the differential spectrum of the \( |\vec{\sigma}'|^2 \) is given by the following (see Appendix A),

\[ f(|\vec{\sigma}'|^2) d|\vec{\sigma}'|^2 = \frac{1}{4 \vec{p}_0^2} d|\vec{\sigma}'|^2. \]  

(3·a)

or, in the integral form,

\[ F(|\vec{\sigma}'|^2) = \frac{|\vec{\sigma}'|^2}{4 \vec{p}_0^2}. \]  

(3·b)

Therefore, the \( \sqrt{|\vec{\sigma}'|^2} \)-spectrum is linear in the log-log plot and the slope is two.

(We should mention that, if one has one fire ball, the \( |\vec{\sigma}'|^2 \)-spectrum should be the same independently of the choice of reference track.)

ii) Next we shall consider two equal fire balls each of which moves (in op-

\(^*\) Professor K. Yokoi suggested that this quantity could be closely connected with the Lorentz invariant relative velocity \( B \) introduced by C. Möller.\(^*\) \( B \) is written in the rest system of one of the particles as follows,

\[ \hat{B} = \mu_1 \mu_2 \frac{v}{\sqrt{1 - v^2}}, \]

where \( \mu_1 \) and \( \mu_2 \) are the rest masses of these particles. The authors are much indebted to Prof. K. Yokoi and Prof. N. Yajima for valuable discussions.
posite direction) with the Lorentz factor $\gamma^*$ in the C.M.S. with arbitrary angles to the incident directions. When we choose, as a reference track, the track with the smallest angle with respect to the moving direction of either the forward or the backward fire ball, we get the following $\sqrt{|\delta^2|}$-spectrum (see Appendix B),

$$f(|\delta^2|) = \begin{cases} \frac{1}{8\bar{p}_0^2} d|\delta^2| & 0 \leq \sqrt{|\delta^2|} \leq 2\bar{p}_0 \\ 0 & 2\bar{p}_0 < \sqrt{|\delta^2|} < 2\mu r^* \\ \frac{d|\delta^2|}{16\gamma^*2\bar{p}_0(\bar{p}_0 + \bar{p}_0)} & 2\mu r^* \leq \sqrt{|\delta^2|} \leq 2(\bar{p}_0 + \bar{p}_0)\gamma^* \end{cases} \quad (4.1)$$

and the integral form

$$F(|\delta^2|) = \begin{cases} \frac{|\delta^2|}{8\bar{p}_0^2} & 0 \leq \sqrt{|\delta^2|} \leq 2\bar{p}_0 \\ \frac{1}{2} + \frac{|\delta^2| - 4\mu^2\gamma^*2}{16\gamma^*2\bar{p}_0(\bar{p}_0 + \bar{p}_0)} & 2\bar{p}_0 \leq \sqrt{|\delta^2|} \leq 2\mu r^* \end{cases} \quad (4.2)$$

When $\bar{p}_0 > \mu r^*$, $F(|\delta^2|)$ is given by

$$F(|\delta^2|) = \begin{cases} \frac{|\delta^2|}{8\bar{p}_0^2} + \frac{1}{16\gamma^*2\bar{p}_0(\bar{p}_0 + \bar{p}_0)}|\delta^2| - \frac{1}{4}(\bar{p}_0)^2 - 1 & 0 \leq \sqrt{|\delta^2|} \leq 2\mu r^* \\ \frac{1}{16\gamma^*2\bar{p}_0(\bar{p}_0 + \bar{p}_0)} - \frac{1}{4}(\bar{p}_0)^2 - 3 & 2\bar{p}_0 \leq \sqrt{|\delta^2|} \leq 2(\bar{p}_0 + \bar{p}_0)\gamma^* \end{cases}$$

In the above formula, $F$ is normalized so as to give $F(4(\bar{p}_0 + \bar{p}_0)\gamma^*) = 1$. We show in Fig. 1 the above integral $|\delta^2|$-spectrum for the case $\bar{p}_0 = 400\text{MeV}/c$. A change in the numerical value of $\bar{p}_0$ causes approximately only a parallel shift of the curves along the abscissa. In Fig. 1, we present also the effect of asymmetry parameter $Z$, defined by

$$Z = (n_f - n_b)/n_s.$$
§ 4. Analysis of the experimental data

In this paper, we analysed the data of ICEF,7) Krakow data ($N_e \leq R$)9) and ECC.9) In the ICEF data, we selected only the events in which the log tan $\theta$-plots re-calculated from the target diagrams were consistent with the original one. 172 emulsion stack data were analysed.

Let us present here some examples to show the characteristic results of this analysis described in § 2. In Figs. 2 a)~e), we present the events which show one-fire-ball type whichever track is chosen as the reference track. On the other hand, in Figs. 3 a)~e) we show the events in which group of mesons can further be sub-divided into two or more fire balls in a systematic manner when one chooses suitable reference tracks, although it would seem to represent an almost isotropic decay of single fire ball if one looked at the original log tan $\theta$-plot only. This could be understood as the fact that such sub-groups,
Figs. 3 a)~e). Examples of differential $\sqrt{|\beta^*|}$-spectra, which are attributed to two fire-ball-events. Under-lined tracks form a strong grouping character.
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\( \gamma_s = 8 \)

\( \gamma_s = 17 \) [1]

\( \gamma_s = 23 \) [1]

\( \gamma_s = 23 \) [2]

Figs. 4 a) ~ c). Normalized integral spectra of \( \Delta \nu_0 = \log (\sqrt{\nu_0} / \sqrt{\nu_{00}}) \) by Monte Carlo calculation of 400 single-fire-ball events.
which we consider as fire balls, have received fairly large recoil momentum in
the direction of the reference track, approximately, where the sub-grouping
is the clearest. In other words, one can find clear-cut sub-groupings in the kicked
direction of fire ball.

In order to study whether or not such sub-grouping as observed in Figs. 3 a)~e)
is expected by only the fluctuation from the decay of single fire ball, the
probability is calculated using the Monte Carlo method as follows.\(^*\)

First, a number of fire balls are made according to the following assumptions:

1) The fire ball moves with the Lorentz factor \( \gamma = 10 \) and decays in isotropic
way to secondary mesons with the definite multiplicity. (The choice of the
numerical value of \( \gamma \) does not affect the analysis described below.)

2) The momentum distribution of secondary mesons in the rest frame of fire
ball, \( f(p^*)dp^* \), is assumed as the distribution,
\[
\frac{dp^*}{dp^*} \propto \exp(-p^*/p_0)
\]
with \( p_0 = 250 \text{ MeV}/c \).

3) Multiplicity of secondary mesons, \( n \), is taken for every case of \( 5 \leq n \leq 25 \).

4) 400 fire balls are made in each case of the multiplicities mentioned above.

After making 400 fire balls in each case, the spectrum \( F(\sqrt{\xi_{ij}}) \) is calcu-
lated using the \( i \)-th track as reference. In this calculation, we use the constant
transverse momentum \( p_T = 400 \text{ MeV}/c \) so as to make it similar to the analysis
of emulsion stack data.

Next, let us consider the frequency spectrum of the ratio of relative four-
momentum, \( A(1/\xi_{ij}) = \log \sqrt{\xi_{ij}} \) between two tracks, \((j+1)\)-th and \( j \)-th,
which give the nearest value of \( \sqrt{\xi_{ij}} \); this means we take the frequency spectrum
of relative distance of neighbouring two tracks in the log-\( \sqrt{\xi_{ij}} \) spectrum shown in
Figs. 2 and 3, where the absolute value \( \sqrt{\xi_{ij}} \) is plotted on the log-scale. As
an example, the frequency spectrum of \( A(1/\xi_{ij}) \) calculated from 400 fire balls is
presented in Figs. 4 a), b), c), for the case of multiplicities of 8, 17, 23, respectively.
The total frequency is normalized to 1. The number put beside each curve, \( j \),
stands for the integral spectrum of \( A(1/\xi_{ij}) \) between the \((j-1)\)-th and \( j \)-th tracks
counted from the reference track. Thus, the fluctuation of \( A(1/\xi_{ij}) \) which should
be expected in the case of the decay of single fire ball is obtained by Monte
Carlo calculations.

When one looks at the sub-grouping as shown in Figs. 3 a)~e), as an exam-
ple, one can estimate the probability that such \( A(1/\xi_{ij}) \) is caused from the
fluctuation of the decay of single fire ball. As the criterion for sub-groupings
of shower tracks, we consider that they belong to different fire balls when one
finds the "Schnitt" with the probability smaller than 1/100 between neighbouring
tracks in the log-\( \sqrt{\xi_{ij}} \) plot. Furthermore, one should find such "Schnitt" con-
sistently even when one changes reference tracks.

\(^*\) Calculation was carried out by the computer OKITAC 5090H of the Institute of Physical
and Chemical Research.
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Fig. 5. Spectrum of the number of fire balls separated by our method.

Fig. 6. Spectrum of the number of pions decayed from a fire ball.

Fig. 7. Spectrum of the number of pions decayed from a fire ball, which is classified for $N_f=2$ and $N_f \geq 3$.

Fig. 8. Frequency of $r_e$ determined by the Castagnoli method, which is parameterized by $N_f=2$ and $N_f \geq 3$. 
After identifying the fire balls by the criterion described above, the distribution of the number of fire balls, \( N_f \), is presented in Fig. 5. In Fig. 6, we present the multiplicity distribution of secondary pions from a fire ball identified as above. The distribution is quite sharp and the mean value is \( 5.10 \pm 0.26 \). In Fig. 7, we show the multiplicity distribution for \( N_f = 2 \) and \( N_f \geq 3 \) separately. One sees that the distributions are quite similar to the two cases. The frequency of \( \gamma_e \) of the events we analysed is presented in Fig. 8 parametrized by \( N_f \). One can observe that the two-fire-ball emission and the emission of more than three fire balls start around \( \gamma_e \approx 5 \) and \( \gamma_e \approx 10 \sim 20 \), respectively, though the inaccuracy of determination of Lorentz factor of C.M.S., \( \gamma_e \), is not negligible. These results are quite consistent with the \( H \)-quantum hypothesis proposed by the one of the authors (S.H.).

Next, let us pick out one-fire-ball events. The average momentum of the emitted pions, \( \bar{p}_0 \), in the rest system of a fire ball is given by (using Eq. (3-a))
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\[ \langle \sqrt{\delta^2} \rangle = \frac{2\alpha}{\beta} \int \sqrt{\delta^2} \cdot f(\sqrt{\delta^2}) d\sqrt{\delta^2} = \frac{4}{3} \bar{p}_0. \] (5)

In Figs. 9-1) and 2), we present the \( \bar{p}_0 \)-distribution when we adopt the top and the second as a reference track, respectively. The difference of the mean value \( \langle \bar{p}_0 \rangle \), in these two cases might be attributed to the effect of residual nucleon. In Figs. 10-1) and 2), the corresponding superposed \( \sqrt{|\delta^2|} \)-spectrum of all one-fire-ball events are shown.

Next let us consider two-fire-ball events. After defining the two fire balls, \( A \) and \( B \), we have the following relations from Eq. (4·a),

\[ \langle \sqrt{\delta^2} \rangle_A = \frac{4}{3} \bar{p}_0, \] (6·a)

\[ \langle \sqrt{\delta^2} \rangle_B = \frac{2}{3} \bar{p}_0 \gamma^*. \] (6·b)

Thus, \( \gamma^* \) is obtained as \( 2\langle \sqrt{\delta^2} \rangle_B / \langle \sqrt{\delta^2} \rangle_A \) (In the case of asymmetric events, \( \gamma^* \) is to be replaced by \( \sqrt{\gamma_A^* \cdot \gamma_B^*} \)). In Fig. 11, we present the relation between \( \gamma^* \) and \( \gamma_e \) determined from the Castagnoli method. One finds that \( \gamma^* \) is almost constant, independently of \( \gamma_e \). Figure 12 gives \( \gamma^* \) distribution and it shows a sharp peak at \( \gamma^* \sim 1.5 \). The mean value is \( 2.02 \pm 0.20 \). Figure 13 shows the superposed \( \sqrt{|\delta^2|} / \langle \bar{p}_0 \rangle \)-spectrum.

At last we treat the emulsion chamber data. In this paper, we select those events in which the number of observed \( \pi^0 \)-mesons are more than four, so that the interpretation is not so ambiguous. The total events satisfying these criteria are 8. We have superposed these \( \sqrt{|\delta^2|} \)-spectra and show this distribution in Fig. 14. This distribution fits remarkably well the \( \sqrt{|\delta^2|} \)-spectrum of single-fire-ball events (Figs. 10-1) and 2)) in the emulsion stack data, and we can say that for these cases the emulsion chamber detect almost all the neutral pions produced by the decay of the fastest fire ball.
§ 5. Conclusion

We investigated the correlation of secondary particles by introducing the relative four-momentum $\beta J$ and got several important physical quantities. These results revealed strongly the existence of H-quantum, and agreement between the $\sqrt{\beta J}$-spectrum from the emulsion stack data and that from the emulsion chamber data (Figs. 10–1) and 2) and Fig. 14) showed that H-quantum plays an important role in multiple meson production in a wide energy region, $10^{13} \sim 10^{14}$eV. As for the energy region of accelerator, $\sim 10^{10}$eV, we are going to make a similar analysis. As extremely high energy events in the emulsion chamber data are now increasing, we shall also analyse these events in the succeeding paper.

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Appendix A

\[ \sqrt{|\delta^2|} \text{-spectrum in the case of one fire ball} \]

Assuming the fire ball decays isotropically into pions with equal momentum \( p_0 \), the distribution function, \( f(\vec{p}, \theta) \), in the fire ball system is given by the following form,

\[
f(\vec{p}, \theta) d\vec{p} d(\cos \theta) = \frac{1}{2} \delta(\vec{p} - \vec{p}_0) d\vec{p} d(\cos \theta). \tag{A.1}
\]

Next, we consider the relative four-momentum, \( \delta^2(\vec{p}, \vec{p}_0) \), of the pion \((\vec{p}_1, \vec{p})\), referring to one, \((\vec{p}_0, \vec{p}_0)(\text{clearly, } \vec{p} = \vec{p}_0, |\vec{p}| = |\vec{p}_0|)\).

\[
\delta^2(\vec{p}, \vec{p}_0) = (\vec{e} - \vec{e}_0)^2 - (\vec{p} - \vec{p}_0)^2 = -2 \vec{p}_0^2 (1 - \cos \theta). \tag{A.2}
\]

From Eqs. (A.1) and (A.2), the distribution function \( f(|\delta^2|) \) is given as

\[
f(|\delta^2|) d|\delta^2| = \frac{1}{4 \vec{p}_0^2} d|\delta^2|. \]

Appendix B

\[ \sqrt{|\delta^2|} \text{-spectrum in the case of two fire balls} \]

For simplicity, we assume two fire balls are emitted in opposite direction with equal Lorentz factor \( \gamma^* \) in the C.M.S. We refer to the pion emitted in the most forward angle referred to the direction of the forward fire ball.

First, we consider \( \delta^2 \) of the pions belonging to the forward fire ball (referring to the above top pion). Similarly to the case of Eq. (A.2), we get the following relation,

\[
\delta^2_f(\vec{p}, \vec{p}_0) = -2 \vec{p}_0^2 (1 - \cos \theta). \tag{B.1}
\]

(the suffix \( f \) denotes the forward fire ball).

Next we calculate \( \delta^2 \) of the pions belonging to the backward fire ball, referring to the above top pion. Now, we look phenomena in the backward fire ball system, then \( \delta^2_b(\vec{p}, \vec{p}_0) \) is given as

\[
\delta^2_b(\vec{p}, \vec{p}_0) = (\vec{e} - \vec{e}_0')^2 - (\vec{p} - \vec{p}_0')^2
\]

(where the suffix \( b \) denotes the backward fire ball, and \( \vec{e}_0', \vec{p}_0' \) are the energy and momentum of the top pion, respectively, from a viewpoint of a backward fire ball system)

\[
\sim 2 \vec{p}_0^2 - 2 \vec{e}_0' (\vec{e}_0 - \vec{p}_0 \cos \theta).
\]

Here, the following relations hold, (the asterisk * denotes the centre-of-mass system)

\[
\vec{e}_0' = \gamma^* (\vec{e}_0 + \beta^* p_0^*) \sim 2 \vec{e}_0^* \gamma^*,
\]

\[
\vec{e}^* = \gamma^* (\vec{e}_0 + \beta^* p_0) \sim \gamma^* (\vec{e}_0 + \vec{p}_0),
\]

then, \( \delta^2_b(\vec{p}, \vec{p}_0) \) is given as
\[ \frac{d\hat{\sigma}^2}{8 \bar{p}_0^2} = \begin{cases} 1 & 0 \leq \sqrt{\hat{\sigma}^2} \leq 2 \bar{p}_0 \\ 0 & 2 \bar{p}_0 < \sqrt{\hat{\sigma}^2} < 2 \mu r^* \\ \frac{d\hat{\sigma}^2}{16 \gamma^{**} \bar{p}_0 (\bar{x}_0 + \bar{p}_0)} & 2 \mu r^* \leq \sqrt{\hat{\sigma}^2} \leq 2 r^* (\bar{x}_0 + \bar{p}_0) \end{cases} \]

Therefore the following equation holds,

\[ |\hat{\sigma}^2|_{\text{min}} = 4 \gamma^{**} (\bar{x}_0 - \bar{p}_0) = 4 \mu r^{**}, \]
\[ |\hat{\sigma}^2|_{\text{max}} = 4 \gamma^{**} (\bar{x}_0 + \bar{p}_0). \]

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